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A

COMPLEAT TREATISE

ON

PERSPECTIVE,

IN

THEORY AND PRACTICE;

ON THE TRUE PRINCIPLES OF

DR. BROOK TAYLOR.

MADE CLEAR, IN THEORY, BY

VARIOUS MOVEABLE SCHEMES, AND DIAGRAMS;

AND

REDUCED TO PRACTICE,

IN THE

MOST FAMILIAR AND INTELLIGENT MANNER;

SHEWING

HOW TO DELINEATE ALL KINDS OF REGULAR OBJECTS, BY RULE.

THE THEORY AND PROJECTION OF SHADOWS, BY SUN-SHINE, AND BY CANDLE-LIGHT.

THE EFFECTS OF REFLECTED LIGHT, ON OBJECTS;

THEIR REFLECTED IMAGES, ON THE SURFACE OF WATER,

AND ON POLISHED, PLANE SURFACES,

IN ALL POSITIONS.

THE WHOLE EXPLICITLY TREATED;

AND ILLUSTRATED, IN A GREAT VARIETY OF FAMILIAR EXAMPLES;

IN FOUR BOOKS.

EMBELLISHED WITH

AN ELEGANT FRONTISPIECE, AND FORTY-EIGHT PLATES.

CONTAINING

DIAGRAMS, VIEWS, AND ORIGINAL DESIGNS, IN ARCHITECTURE, &c. NEATLY ENGRAVED.

ALL ORIGINALS;

INVENTED, DELINEATED, AND, GREAT PART, ENGRAVED BY THE AUTHOR,

THOMAS MALTON.

THE SECOND EDITION, CORRECTED AND IMPROVED; WITH LARGE ADDITIONS.

LONDON:

Printed for the Author; and ſold by Meſſrs. ROBSON, in Bond-ſtreet; BECKET, Adelphi, Strand; TAYLOR, near Great Turn-ſtile, Holborn; DILLY, in the Poultry; and by the AUTHOR, No. 56, Poland-ſtreet, Oxford Road, near the Pantheon.

MDCCLXXVIII.

TO THE

PRESIDENT AND MEMBERS,

OF THE

ROYAL ACADEMY,

For PAINTING, SCULPTURE, and ARCHITECTURE;

INSTITUTED AT LONDON,

By, and under the Aufpices of, his moft gracious MAJESTY

GEORGE THE THIRD.

Gentlemen,

IF the unremited labour and affiduity, with which I have profecuted the ftudy of the Science and Art of Perfpective, have thrown any new Light on that moft neceffary branch of the Polite Arts, fo, as to render its Principles clearer and better underftood, in Theory, more eafily applicable, in Practice, more generally ufeful and fubfervient to the Arts, of which, it muft be allowed to be the foundation, 'tis what I have chiefly aimed at; and prefume, this Work may not be wholly undeferving your Patronage and Encouragement, although it may not merit your entire Approbation. Such as it is, Gentlemen, I fubmit to your Candour, and claim your Protection.

The propriety of dedicating fuch a Work to Gentlemen, who are, undoubtedly, the moft competent Judges of it, will plead an excufe for my prefumption; if, on an accurate examination of its Contents, it be found, that I have rendered an apparently intricate Science more familiar, and better adapted to the capacities of young Students, in fo effential a part of their Studies; the neglect of which, amongft the rifing Artifts, is much to be lamented. Perfpective feems to be looked on as an Appendage, only, which may be difpenfed with, inftead of the firft requifite; in which, the Student, who would be a Candidate for Fame, fhould be well grounded.

It

DEDICATION.

It is with reluctance I add, but it is too obvious, that, in many fine Pieces, in respect of Design, Composition, Drawing, or Colouring, there seems to be a want of a just Idea of Perspective.

Pardon me, Gentlemen, I do not mean to depreciate the Merits of such experienced Artists, in their several Performances; for, in my opinion, there only wants a thorough knowledge of Perspective, to render the present Age as famous as any former Æra; which, now, is established on the most permanent and infallible Principles; whereby, the trouble of projecting Objects, perspectively, is, in the process, greatly abridged and facilitated.

I am,

Gentlemen,

with great respect,

Your most obedient,

and obliged humble Servant,

THOMAS MALTON.

PREFACE.

IN every Age, and in every Climate, where the Polite Arts are cultivated and encouraged, Emulation, and a deſire of Fame, inſpire the Breaſts of the riſing Generations; the ingenious Mechanic catches the ardent and laudable Flame; Commerce is extended, the People are neceſſarily enriched, and the State becomes potent and formidable. Architecture rears her ſtately Domes and lofty Turrets to the Skies; the ſumptuous Edifices, raiſed by the hand of the able Mechanic, and embelliſhed by the ingenious Sculptor, with breathing Statues and almoſt animated Marble, proclaim abroad the Magnificence of the Founder, and juſtly immortalize the ſkilful Architect; whoſe Works may vie with the moſt celebrated relicks of Antiquity, which ſeem to be of more than human Compoſition. Whilſt Painting, their Siſter Art, aſſiſts and unites in decorating the interior Domes, Cielings, &c. or, in well-choſen Pieces and Subjects, and maſterly Performances, adorns the Stair-caſe, the Gallery, the Drawing-room, and the Cabinet.

Perſpective is allowed by all, who are well acquainted with it, to be the Baſis of all the Polite Arts which have their foundation in Drawing; particularly Painting, or Delineating; for, Colouring does not come within its Rules. The Sculptor and Architect may receive great aſſiſtance from Perſpective. I am ſenſible it is not abſolutely neceſſary to the Art of Deſigning, but it is eſſentially ſo to ſee the effect of the Deſign. An Architect may, doubtleſs, be famous in his Art without Perſpective, but more ſo if he had that accompliſhment; as he would be better able to judge of the effect it would produce, and convey a juſt Idea of it to others, before the Deſign is executed. Drawings, in Perſpective, from particular Stations, where the Building will be moſt conſpicuous, would anſwer in that reſpect, the ſame purpoſe as a Model, and at a much leſs expence.

At a Time when the Arts are in ſuch a Degree of perfection, a Treatiſe on Perſpective may be thought an unneceſſary Publication; ſeeing there are, already extant, many valuable Books on the Subject, containing all the Rules, for practice, which are either neceſſary, or uſeful; and conſequently, unleſs ſome new Principles are propoſed, by which, the trouble of projecting Objects, perſpectively, may be leſſened, it is uſeleſs to increaſe the number. For ſome, with truth, alledge, that, unleſs Perſpective be comprized in a ſmall compaſs, it will never be ſtudied by thoſe, for whoſe uſe it is chiefly intended. On Perſpective, as on other Subjects, there are, indeed, a ſufficient number of Authors; and yet, I may venture to affirm, that, no Subject, whatever, has been worſe handled, in general.

I do not pretend to have found out new Principles, nor do I think, there can or need be any other; thoſe given, by Dr. Brook Taylor, being ſufficient for any purpoſe, whatever; and that, the Principles, on which he has founded his Syſtem, are the moſt ſimple and perfect that can poſſibly be conceived. Notwithſtanding what many, who have not a true Idea of Perſpective, imagine, that there is imperfection in it; that the Rules, preſcribed, do not always produce a true or pleaſing Repreſentation of Nature; I maintain, that all, which can be done by Rule, is performed on the moſt perfect and infallible Principles that can be deviſed; that Perſpective is, abſolutely, at its *ne plus ultra*; where then, it may reaſonably be aſked, is the neceſſity for, or uſe of this Treatiſe?

There are, generally, two Motives, which induce every Perſon to publiſh to the World, his Inventions or Improvements, or Studies of any kind. The firſt is Vanity; or, to give it a milder Appellation, a laudable thirſt of Fame; (I believe, I ſhall not err wide if I affirm, that, Vanity is the firſt, and chief ſpring of all that is great and laudable) the other is the lucrative Emolument, if not always the Conſequence, at leaſt, it may reaſonably be expected an Attendant on Fame; it is ſcarce determinable, whether Vanity or Intereſt ſtimulates moſt to action. To pretend that we are not actuated by either of theſe Motives is the height of folly; for though our Station, in Life, may ſet us above the ſordid views of Gain, yet no Station, whatever, is exempt from Vanity; which, moſt wiſely, for the beſt of purpoſes (the good of the whole) is made a neceſſary ingredient in our Compoſition.

Now,

Now, although I have not the leaſt pretence to the invention of new Principles, yet I am firmly perſuaded, that I have made uſe of thoſe we have to the beſt advantage; that, from the irregular and imperfect Order, as they are given by Dr. Brook Taylor, I have digeſted it into an uſeful and practical Syſtem; not involved in a labyrinth of mathematical Demonſtration, of things which are to little purpoſe in the Art of Delineating; as the voluminous Work of Mr. Hamilton, publiſhed in the Year 1738; which, though a moſt elaborate and valuable Production, has not been of the leaſt uſe, to the Arts it was intended to promote. In the mathematical part, I have not entered further than is really neceſſary, to evince and enforce the Principles, on which the Practice is founded; and, in Practice, I have ſhewn its immediate and abſolute dependance on the Theory. I have by means of an Apparatus, contrived for the purpoſe, united the Theory and Practice ſo together, that he muſt have very little penetration, and a ſhallow Capacity, who does not in a ſhort Time, and with little Study, conceive a clear and comprehenſive Idea of the rationale of Perſpective. I have, every where, made ſuch remarks, and thrown ſuch light on the Subject, as, I flatter myſelf, will make it juſtly deſerve the Title I have given it: my Deſign, throughout, having been to make the ſtudy and practice of Perſpective, at the ſame time, eaſy and entertaining. The Subjects I have choſen to diſplay, and embelliſh it with, are ſuch as are common and familiar; for, to what purpoſe is an intricate and puzling delineation of the Dodecahædron, &c. except, to ſhew that the Author underſtood it? I am of opinion, that the Reader will be much better pleaſed, with the deſcription of ſuch Objects as are frequently before his Eyes, and ſuch as are fit Subjects to introduce into a Picture; or to form a Picture on. I have not been ſatisfied with the common Leſſons for plane Figures, and Solids compoſed of or bounded by Planes, and left the Student to decorate his Buildings with the embelliſhments of Columns, Entablatures, &c. of the ſeveral Orders; but I have, minutely, ſhewn, how to delineate them, ſingly, and to compoſe a regular Building of the ſeveral Materials, deſcribed and treated ſeparately of before.

There are, at this time, but few Gentlemen who are not pleaſed with a fine Picture, which truly repreſents Nature, on Canvas; the Deception being ſo well managed, that, we can almoſt imagine we ſee the real Objects, themſelves, through the aperture of the Frame. For, it is poſſible, in perſpective Delineations, in which there is a true gradation of Light and Shade, and judicious Colouring (on Cielings, &c. which we cannot come near) to deceive the Eye and Judgment, ſo, as to imagine, what we ſee are real, and ſolid Exiſtences.

Although Perſpective is not conſidered as a neceſſary part of the Education of a Gentleman, it muſt be looked on as a genteel and polite Accompliſhment, a Qualification for a Prince. For, has not our moſt gracious Monarch cultivated and particularly encouraged it? nay, deſcended to learn its Rules, and to delineate, with his own Hand, pieces of Architecture? Is it poſſible, for a Gentleman to ſee, as he ought, much leſs to judge of a Picture, with true taſte, and with the diſcernment of a Connoiſſeur, without having ſome notion of Perſpective? it expands the Ideas, and makes us ſee Objects as they really appear, to a judicious Obſerver; it makes a Perſon a judge of the diſpoſition, and proportion of the ſeveral parts of a Picture, to each other; the Symetry and Harmony of the whole are perceived; if there be any diſcordance, or unſeemly diſtortion of the Parts, owing to an injudicious choice of the Artiſt, either in the Diſtance or Situation of the Object, or in the Poſition of the Picture, they will be quickly ſeen; eſpecially, when there is introduced regular Architecture, in Buildings of any kind.

The manner, in which Perſpective has hitherto been treated, ſcientifically, is too rigidly and mathematically ſo, to be entertaining, to a Perſon not converſant with the Elements of Euclid; which, though a branch of Univerſity Education, has been treated, in thoſe ſeminaries of Learning, in a manner, which deters numbers from the ſtudy of that moſt uſeful and neceſſary Science, the foundation of all mathematical reſearches. Every Science, and neceſſary Art, ſhould be ſo treated as to allure a Perſon to the purſuit; by making him (in a familiar way)

relifh a ftudy, for which he has not a natural propenfity. Many Gentlemen imagine, if they neither paint nor draw, that, Perfpective is wholly ufelefs, and unneceffary to them; they are greatly miftaken who think fo, if they defire to be accomplifhed, and would beftow a little time and pains to be fo.

Of all the mathematical Sciences, the ftudy of Perfpective is perhaps the moft entertaining; the pleafure and fatisfaction which refult from it, in delineating, can only be felt, not defcribed. Aftronomy (to which Perfpective greatly conduces) is the moft fublime of all, and next to it I fhould rank Perfpective. The entertainment, refulting from the former, is of a higher kind; we are, as it were, lifted from the Earth, in contemplation of the fuperior Works of the great Creator, in fuch wife, as to look on all below not worthy of our notice. The advantages to Society, which are deduced from this moft exalted Science, are many and great; and, being well known, would be impertinent to enumerate. On the other hand, the ftudy of Perfpective, fcientifically, is highly entertaining to a rational Mind; to thofe who would purfue it, I recommend the Works of Mr. Hamilton. I have not chofen to go further into it, than is really ufeful; and yet, to fome Perfons, I fhall be thought prolix enough; who want to know it, without the labour of Study; to which, Artifts, in general, have great antipathy; I mean, to the ftudy of any thing mathematical.

It is ufual, with moft writers on Perfpective, to introduce fo much of Geometry into their Work, as they judge neceffary for the knowledge of Perfpective. There would be, in my opinion, as much propriety in prefacing every Book, on Literature, with Grammar; left the Reader fhould not be acquainted with Syntax, and the Idiom of the Language. I fairly own it above my fkill, to draw a line between the neceffary and unneceffary, for all is more or lefs fo. Some Theorems, in Geometry, are only neceffary for the attaining of others; which, if they cannot be obtained without them, are neceffary. In fhort, if the Student be quite unacquainted with Geometry, he is very unfit to ftudy Perfpective; let him firft become intimate with Euclid, and then he may fafely purfue Perfpective; the practical part of one is fufficient for the other. Wherefore, feeing I have compiled a Syftem of Geometry, to which this Work is fubfervient, I have wholly omited the Definitions of geometrical Terms, here ufed; becaufe, it is reafonable to fuppofe the Student already acquainted with them; otherwife, he muft firft ftudy that Grammar, of the Science of Perfpective.

Perfpective being a branch of the Science of Optics (as it is founded on direct Vifion) it becomes neceffary to confider, in the firft place, the ftructure of the human Eye, and the nature of Vifion; to conceive (as far as our Intellects can trace) an Idea, how that extraordinary and aftonifhing Senfe is performed; which being, in fome degree, underftood, we fhall be well prepared for a clear underftanding of Perfpective, and enabled to diftinguifh between the Reprefentation of an Object on a Plane, and its real Appearance; a circumftance, which is a great ftumbling-block to many, who have not rightly confidered the difference.

I have prefaced this Work with a Section on Light and Colour; and as a more effential requifite, to Perfpective, the fecond contains a brief defcription of the Eye, and of Vifion. In the next is contained the foundation of Perfpective; it treats on direct Vifion, comprifed in two Theorems, of univerfal application; which are the very Effence of Perfpective. The two remaining Sections, of the firft Book, are wholly digreffive; they not being at all neceffary, to the underftanding of what is contained in the following Work; the Reader may therefore pafs them over, if he be fo difpofed, without breaking the thread of the Subject. They contain matter of mere Opinion; refpecting the materiality of Light, Reflection, Refraction, &c.

Book the fecond contains the whole ufeful Theory of Perfpective, rectilinear and curvilinear; which is fomewhat copious, by reafon of the Examples given for illuftration; and Corollaries, deducible from the Theorems.

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The firft Section is a general Introduction, containing many preliminaries, neceffary to be known, previous to what follows.

The fecond is alfo introductory, and contains a full explanation of all the various kinds of Projection, ichnographic, orthographic and ftereographic; with a circumftantial and comprehenfive Definition of Perfpective, and other introductory matters.

The third is more elementary; it contains full, yet brief Definitions of all the Terms made ufe of in the Theory.

The fourth Section contains the whole Theory of right-lined Perfpective, in fourteen Theorems; from which are deduced feveral ufeful and practical Leffons, in Corollaries, Scholia, &c.

In the firft feven Theorems, is contained all the neceffary knowledge of Perfpective, relative to Interfections and Vanifhing Lines, in general; the firft and moft effential requifite in Practice.

The eighth Theorem, though felf-evident, is intended as a refutation of an abfurd opinion which many entertain, that continued Right Lines, feen direct, cannot be reprefented by right Lines; feeing that, parallel right Lines always appear to incline towards each other. The ninth teaches all that relates to Planes and Lines parallel to the Picture. The tenth fhews where the Interfecting and Vanifhing Points of all other Lines are to be found; and the eleventh determines how to find them. The twelfth and thirteenth contain all that relates to Lines not parallel to the Picture, in the moft concife, yet full and clear manner that I can conceive. Perhaps the Demonftrations of the laft may deter thofe, who are not Geometricians, from examining it with that attention it requires; let fuch remember, that, in order to practife Perfpective, it is not abfolutely neceffary to be a Geometrician; becaufe, I practifed it long before I underftood Geometry; excepting a little practical, and, a fuperficial knowledge of Lines, in general; which are certainly requifite; as the whole of practical Perfpective confifts in it.

For which end I have felected, all that is neceffary, not only for Perfpective, but, alfo for various mechanical ufes, into one Book (the firft part of the Royal Road to Geometry) lately publifhed; perhaps, in the moft eafy and intelligent manner, ever yet done; to which I refer the Reader, for the geometrical Conftruction of all kinds of Figures here ufed; as well as to the Elements, for Demonftration, in the Theory.

If thofe Readers, who have neither time nor inclination to become acquainted with the Elements of Geometry, at leaft acquire fo much, as that Tract contains; I will vouch for the great advantage he will find to his ftudy of Perfpective. Indeed it is impoffible to underftand or practife Perfpective without it; for, being in itfelf, wholly geometrical, the very Language, and Grammar, of Perfpective is Geometry; let them take my word, for once, they will not find their time mifspent; but, on the contrary, fo much knowledge, as it teaches, will be purchafed at a very eafy rate.

They may, then, boldly venture on the Theory; omiting the Demonftration where they find they cannot comprehend it; which, in general, they will not find enveloped in myftery; having treated that part in the moft familiar manner poffible, by plain reafoning only, without other reference than to the Elements, for proof of what is advanced. Let them, at leaft, by the moft attentive perufal of the Theorem, endeavour, to conceive the Premifes of it; which, in fome of them, perhaps, may not be perceived, clearly, at the firft reading; the fecond will open fome Ideas, and the third perhaps compleat it; which, if it require a fourth, fometimes, will not be loft labour; for, when the Premifes of a Theorem are clearly underftood, the proof of it is not difficult, in many cafes. However, whether the proof of what is advanced be perceived or not, it is not effential to the Practice of Perfpective; the Reader may give me credit for it, I will be refponfible for the truth of it; he may be fully fatisfied that the thing is certainly fo, and make ufe of that knowledge accordingly.

The Theorem being underftood, let him, at leaft, go through one Example in each, as he proceeds; his Ideas will not only be ftrengthened, but he will, poffibly, fee the truth of the Premifes. Then, the Corollaries, Scholia, &c. are but fo many eafy, ufeful, and practical Leffons, deduced from the Theorems, which will be found of the greateft ufe in Practice; and, if the Theorem be clearly conceived, the Corollaries will be fo too.

The fourteenth Theorem has no affinity with, or dependance on the foregoing; it contains the whole Theory of practifing without Vanifhing Lines; an ingenious method, but feldom practifed; nor, indeed, is it fo generally applicable to practice.

The fifth Section contains fo much of the Theory of curvilinear Perfpective, as is really ufeful in Practice; or neceffary to be known, by any Artift, whatever.

The fixth is a refutation of feveral capital Errors and abfurd Opinions, which many Perfons entertain of Perfpective; and which are, there, clearly and fairly ftated, and fhewn to have no real exiftence.

The third Book is a copious Treatife on ufeful practical Perfpective, containing Leffons and Examples in every neceffary Cafe, that can occur in Practice; fhewing alfo its dependance on the Theory, in brief Demonftrations, where it is neceffary.

The Method, in which I have treated this ufeful part of Perfpective, is fuch, as I prefume will fufficiently recommend it. The Book is divided into twelve Sections, each treating on a diftinct fubject from the foregoing; begining with the moft fimple and firft Principles or Elements, and going on, from the moft fimple, to the moft complex Objects, in a regular, progreffive fucceffion.

After the neceffary Preliminaries and Elements, in the three firft Sections; the third fhewing alfo how to find Vanifhing Lines, and Vanifhing Points, in general, the firft requifite in Practice; in the fourth is contained all the practical elementary Problems in Brook Taylor's Effay, which are, of themfelves, a complete Syftem of practical Perfpective. Thefe Elements, of the following Work, being well underftood, will be found of great Utility; which has induced me to perfect fuch as the Author of them had left very imperfect; and fome, particularly his 21ft and 22d Figures, are greatly defective; we are neverthelefs infinitely obliged to him, for giving them to the World, fuch as they are.

To underftand this Section clearly, and be enabled to apply the Problems it contains, in all Cafes, practically (which, it muft be obferved, is the foundation of the whole) a tolerable fhare of geometrical knowledge is requifite; without which, it is impoffible to turn them to that general ufe, in Practice, in which they are fo univerfally applicable; inafmuch that, it may be truly faid, he who can comprehend them, perfectly, and apply them in all Cafes, generally, underftands Perfpective, thoroughly; but, being fuperficially looked over and not well digefted, they will be found of little ufe.

This confideration, perhaps, has induced fome Perfons to imagine, and affirm, that the whole of Perfpective may be comprifed in a very fmall Compafs. But, I muft make free to tell them, they are under a miftake; for, notwithftanding the Principles are few and general, yet, to know how to apply the Rules, in all Cafes that may occur in Practice, cannot be fo foon acquired; if fo, how comes it, that they are no greater Proficients? Or, can they imagine, that, amongft the number of able Men, who have wrote on the Subject, none fhould hit on that much wifhed-for Expedient? Either they muft acknowledge that their own faculties are not fo acute as others (which is paying a forry Compliment to their Underftandings) or they muft fuppofe that there is more in it than they at firft imagined. 'Tis true, the whole of Practical Perfpective is comprifed in this and the foregoing Sections; yet, how few would be able to apply thofe Problems, though clearly underftood, in delineating a regular piece of Architecture, is but too obvious to enlarge on; without previous, practical Leffons, diverfified varioufly, to familiarize them. For which reafon, it is neceffary to fhew their application, in various Examples, which I have not been

ſparing of. I have not, as others have done, ſtudied the moſt eaſy poſitions of Objects; but I have endeavoured to give the moſt pleaſing, pictureſque, and natural Repreſentations, and render the moſt difficult Poſitions eaſy.

Some Perſons know a great deal (in their own opinion) becauſe they know, perhaps, that all Right Lines, which are parallel, have the ſame Vaniſhing Point; who yet, know not how to fix and aſcertain any one, except the Center of the Picture, and that often very abſurdly (indeed they ſeldom, if ever, uſe any other) nor how to proportion a Line perſpectively; and further, to ſhew their conſummate knowledge in it, if you ſhew them a piece of Perſpective, in which there is not one Line tending to the Center, or Point of View, they will call that Point on the Picture, in which the principal Lines converge, the Point of View; ſo that, there may, by this criterion, be ſeveral Points of View in the ſame Picture; a circumſtance too palpably abſurd to need a refutation.

If this be to know Perſpective, it may indeed be very ſoon acquired; it may be taught to a Child. But, when they have drawn a Line to its proper Vaniſhing Point, how that indefinite Line is to be proportioned, in order to repreſent, certain finite parts of the Original, they are wholly ignorant, though the chief requiſite in Practice, and the principal buſineſs of the fourth Section of the third Book.

However, not to diſcourage them, from attempting to acquire ſo neceſſary a part of their Studies, they may be aſſured that a fixed reſolution and perſeverance, will ſoon ſurmount every apparent difficulty. Some are alarmed, and even frighted, at the ſight of ſo many Lines in a Diagram, for a Leſſon in Perſpective, imagining they can never comprehend them all. Certainly, they muſt, to a perſon wholly ignorant of the uſe and meaning of them all, appear a general confuſion; but when they have ſet about it, with a reſolution to know, and have well digeſted the Definitions of them, they will, at one glance, comprehend the uſe of ſeveral; and the reſt, by analizing them regularly, will ſoon become familiar to the Eye, and to the Underſtanding.

This, I believe, has induced ſeveral Perſons, who have wrote on Perſpective, to deal very unfairly in it, in not giving all the neceſſary Lines in their Diagrams. I have, in mine, given all, and more than are neceſſary to be drawn, at once; for let it be obſerved, that the Radials of Lines, for determining Vaniſhing Points, ſeldom need be drawn; and ſeveral of the operative Lines, or Viſual Rays, which are only uſed for cuting others, need not be drawn, at all; a Ruler being applied to the two Points (the Eye and the original Point) the indefinite Line may be cut, without drawing the whole Line, which could not anſwer, in a Diagram, ſo well as drawing the whole, in order to ſhew the direction of the Line; beſides, in Practice, one part is effected and the Lines rubbed out before others are drawn, which cannot be done in a Diagram, all muſt remain together.

Others, again, there are, who have almoſt determined to make an effort, only want to know ſo much of the Practice as to be able to project a plain Building, &c. and ſeem quite afraid of attempting the Theory; becauſe there is occaſion for Geometry, which, to many, is a terrible Bugbear; ſo that, rather than exert themſelves, they do without it, though the firſt eſſential, and chief requiſite perhaps of their Profeſſion. They want to know Perſpective, but are afraid to venture on the Theory. They deceive themſelves if they imagine it an unneceſſary Appendage; can it be unneceſſary to know how, and why, before we attempt any thing, practically? is not one, the means of effecting the other? I do aver, that the ſhorteſt way to acquire Perſpective is to underſtand it firſt, in Theory; the Practice of it will readily follow.

How different are the diſpoſitions of mankind. After I had (from the Jeſuits) acquired (as many would ſuppoſe) a proficiency in Practical Perſpective, I was far from being ſatisfied with all that could be obtained from that Author; and although I was not a Geometrician, at that time, and he not giving any Theory, I ſet about contriving means to have conviction of the truth of his Rules; by which means, I

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fell infenfibly into the method ufed by Vignola and Sirigatti; than which, nothing can be more convictive; and I was accordingly fatisfied, that the Rules, given, might be depended on.

But, having acquired fuch a proficiency in Geometry, as to enable me to comprehend Brook Taylor's Principles, how trifling, how limited appeared all I had learned of it before; I was even afhamed of my own productions, fome of which had, undefervedly, been admired: I found, indeed, I fcarce knew what was meant by Perfpective, till then. The greater progrefs I made in Geometry, the clearer, and more comprehenfive Idea I had of his Principles; which, though founded on common obfervation of Objects (fupported by Geometry) are beyond every thing ever thought on, by any who had wrote on the Subject; and yet, it is fo fimple, when underftood, that 'tis ftrange it fhould not have been brought to light fooner.

Having, in the fourth Section of the third Book, difplayed the whole Elements of Practical Perfpective, the fifth begins the application of them to real ufe; the whole of which, confifts in finding the reprefentation of a Line or Point, in any Plane, any how fituated: for, Points are the extremes of Lines, and Lines of Surfaces; and, Solids, or Objects, are bounded by their Surfaces. A Plane, however fituated, it is manifeft, is the fame, in all its properties; its fituation, in refpect to the Horizon, is, therefore, of no confideration, in Perfpective; and, to delineate Plane Figures is all that can be done by Rule, with abfolute certainty; the apparent Contours of fome round Objects are fomewhat difficult to determine, yet they may be done fufficiently correct. In this Section, the Conftruction of all kinds of plane Figures, perfpectively, in Planes any how fituated, is comprifed; and, in the fixth is fhewn how to conftruct Solids, of Planes only; by which, any Plane Building may be projected.

The feventh Section teaches, fully, how to reprefent Mouldings in general, and to break the fame at the Angles of a Building, &c. internal or external, right-angled or otherwife, with other neceffary, and decorative, parts of Architecture, &c. as Triglyphs, Confoles, Modilions, &c. alfo, how to form a Pediment.

The eighth is wholly adapted to circular and round Objects of all kinds; as Arches, Columns, Tufcan and Doric Bafe and Capital, round Steps, Wheels, Vafes, and circular Mouldings; including alfo the Ionic and Corinthian Capital.

Being now furnifhed with all the materials of a Building, the ninth Section fhews how to compound them, and to form a Building; from the moft plain and fimple to the moft elegant and rich, decorated with the various Orders; alfo detached Buildings, Views, &c.

The tenth is for internal Subjects, as Rooms, infide views of Churches, Arcades, Staircafes, and Cieling-pieces, reprefenting Domes, and Cupolas; which, though fomewhat particular, is founded on the fame invariable Principles.

The eleventh is adapted for houfhold Furniture; as Tables, Chairs, Cabinets, Bookcafes, Beds, &c. alfo, for Machines, Coaches, &c.

The twelfth treats of inclined Pictures and Planes, in general, containing Leffons in the moft difficult cafes, from known data; in which, the excellency of the new Principles is exemplified.

Book the fourth treats on the Perfpective of Shadows, which is indeed a copious Subject, and much more might be faid of it; but, as I am fenfible, that very few take the neceffary pains to project their Shadows by Rule (general Effects being all that is ftudied, or regarded) it would therefore be to little purpofe to give Rules, which will feldom, if ever, be followed. Nor is it at all neceffary, or even practical, to project every Shadow, mathematically; it would be attended with great lofs of time, and perhaps, in fome Cafes, produce a bad effect, though truly projected. In this branch of Perfpective, licence may therefore be taken, juftifiably, provided they do not run into grofs and palpable abfurdities; fuch as projecting the Shadows of Objects, already immerfed in Shade; or, as I have feen in the works of eminent Mafters, Shadows caft both ways, in the fame Picture, to the right, and to the left.

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I have alſo ſeen, the Shadow of a curve Line repreſented by a Right Line, when the Luminary was not nearly in the Plane of the Curve; which, with others, leſs obvious, ought carefully to be avoided, and guarded againſt.

Neverthelefs, there are certain general Rules may be given, which ought ſtrictly to be adhered to; and never, on any account, departed from. I have, therefore, in this Work, given ſo much as I conceive neceſſary to be known; for, in order to be converſant in Shadows, it is abſolutely neceſſary to be acquainted with the invariable Law of Nature, in the projection of Shadows; although it may not always be neceſſary to follow her dictates, implicitly.

The effect of reflected Light on Objects, from other Bodies, in vicinity with them, is likewiſe treated of in this Book; alſo the effect of Diſtance, uſually underſtood by the term Keeping, properly, aerial Perſpective; both which, contribute greatly to the perfection of a Picture; inſomuch that, without due regard being had to both, a Picture, ever ſo well deſigned and delineated, or coloured, will be but a flat and ſpiritleſs performance.

And laſtly, the reflected Images of Objects on the ſurface of ſtill Water, or on poliſhed Mirrours, in any Poſition, are, in the laſt Section of this Book, treated on.

Having, now, given a brief account of the Work, I ſhall juſt make one more obſervation, in reſpect of the notions of ſome Artiſts; who being always accuſtomed to ſketch by ſight, only, and having not the leaſt conception of projecting Objects by any Rule, they vainly imagine that Perſpective will teach them how to repreſent Objects, exactly as they appear to the Eye, without knowing any thing of their Proportions or Situations, in reſpect of each other, or of the Picture, which is impoſſible; ſo that, when they find there is a neceſſity for taking and applying their real Meaſures, or Proportions, being ſo foreign to what they conceived of it, they imagine ſo many obſtacles to lie in the way, that, they ſoon deſiſt from the attempt; chooſing rather, to remain in ignorance, and proceed with uncertainty, than take the neceſſary pains to acquire, what, being acquired, they would deem invaluable.

He, therefore, who can hit on ſuch an Expedient, as to teach them how to delineate Objects truly, by Rule, without applying any Rule at all, will be likely to meet with great encouragement from thoſe indolent Artiſts. I acknowledge my incapacity for the taſk; and therefore, I ſhall abide by the Rules, as they are given in the following Work.

A TRUE

CASE,

BETWEEN THE AUTHOR,

OF A LATE PUBLICATION,

His PRINTERS, and PAPER MERCHANTS.

TRUTH WILL EVER PREVAIL.

OF all the calamitous accidents, which, of late years, have happened in this Metropolis, none has attracted the attention of the Public, more than the Fire, at Meſſrs. Cox and Bigg's, Printers, in the Savoy, on the evening of March the 2d, 1776; on account of the many valuable literary Productions which were deſtroyed there; ſeveral of which were completed, and ready for Publication. Amongſt the reſt, was the Compleat Treatiſe on Perſpective, in folio, publiſhing by Subſcription, by Thomas Malton, and intended to be publiſhed in March. The following is a true Caſe between that Author and Meſſrs. Cox and Bigg; alſo, between him and Meſſrs. Wright and Gill, Paper-Merchants, in Abchurch-Lane, Lombard-ſtreet.

Thoſe Gentlemen were alſo ſufferers, by that Fire, to the amount of 140 Pounds, for Paper, credited Meſſrs. Cox and Bigg. Who being lately removed from the Strand, into the Savoy; and having taken in another Partner, * they had been making Conveniences, for carrying on a more extenſive Buſineſs. In conſequence of which, they had neglected to inſure, till all was completed; their former Policy being forfeited, or of little value, now. They likewiſe intended to inſure in another Office (the Union) every thing (if I am truely informed) was either ready to be executed or ordered to be ready. Perhaps the Money might not be ready; their neglect of ſo important and neceſſary an affair may, on that ſcore, admit of ſome vindication, in their favour; nevertheleſs, it was an unpardonable negligence, having ſo much of others Property on the Premiſſes, and in their poſſeſſion.

Although the Author of the Treatiſe on Perſpective was, by no means the greateſt ſufferer, by that accident; reſpecting the quantity or value of what he loſt; yet, all circumſtances conſidered, his Caſe was ſingularly hard; and, the treatment, he afterwards experienced, from them, and others, fellow-ſufferers,

* A Mr. Nettleton; who, if I am not miſinformed, was but nominally ſo, in truſt for Mr. Wells, Mercer, on Ludgate-hill. Be that as it may, it is certain, Mr. Wells was the perſon who advanced the Money for Nettleton; and, there is not the leaſt doubt, but that he was to be a ſharer in the profits of the Buſineſs, as a joint Partner.

cruel and inhuman; ſuch as no Perſon, who had not taken ſome pains to diſſolve every ſocial tie, nay, to diveſt himſelf of humanity, could have determined to carry into execution, at the time; it ſeems as if they were determined to cruſh him, if poſſible, by their premeditated, and, in ſome meaſure, their united efforts. As the diſintereſted Public are often miſled, in forming a judgement of ſuch matters, as they are no way concerned in, I was induced to lay the real and true ſtate of the Caſe before them, in the moſt candid and impartial manner.

This Treatiſe on Perſpective was the produce of many years ſtudy, the reſult of my leiſure hours, when I followed the profeſſion of a Cabinet-maker, with little ſucceſs; and which I quited ſeven Years ago, encumbered with as many Children, ſince increaſed one. Let any one picture to himſelf, the ſituation of a Perſon, who had quited a Buſineſs, not worth following, burthened with ſuch a Family; having no other means to provide for their ſupport, than the produce of his Studies, advanced almoſt to the meridian of Life.

I began printing the Work about three years ago; but, as all ſuch Works are uſually longer in hand than is conſiſtent with the Author's convenience what between the Engravers, Printers, Paper-merchants, and the encumbrance of eight Children, I found myſelf, when in the middle of my work, not in the moſt eligible ſituation; the encouragement, at firſt, not being adequate to ſuch an arduous and expenſive Undertaking. Notwithſtanding, I kept joging on, with reſolution and perſeverance, though but ſlowly; for Engravers are a hungry ſort of Gentlemen, who will not dig without eating, and well too; when, or before, they have done their Work, they immediately want their Money, and frequently much more than they deſerve; many of them being as mere Mechanics as any whatever.

Being thus circumſtanced, it is no wonder the Work went ſo ſlowly on; the Engravers work'd for me rather than be idle, expecting to be paid ſome time; the Printers had a very convenient jobb to ſet their Men on, when they had nothing elſe to do; and thus, we went gently on for ſome time, each laying the blame on the other. As time and perſeverance will ſurmount great obſtacles, the Royal Road to Geometry was firſt completed and publiſhed, in December 1774, which was well received by the Public. When both were finiſhed, I deſired to have them ſent Home, altogether, to ſave the trouble and expence in ſending often; but, as there are reaſons for every thing, they alledged ſome plauſible excuſe, to let them abide in their Warehouſe, and ſend for them as I wanted; which, judging the true reaſon to be ſecurity for Payment, I ſupinely acquieſced in.

When they moved from the Strand into the Savoy, they moved my Work, without giving me the leaſt intimation of it, although I had requeſted them to ſend it Home long before; and ſurely, as it was to be moved, it was as eaſy to have ſent it home to me, as to carry it farther from me. Not long after (for reaſons beſt known to them) they refuſed to deliver any more to my Orders, repeated ſundry times, to my great hindrance in delivering the Work to the Subſcribers, and conſequently to my loſs (as it happened) in not delivering ſo many as I might have done, or in making them ready for ſale; near a hundred Subſcribers wanted Books when the Fire happened. At the ſame time, they required Security for payment of the remainder of the expence in Printing, Paper, &c. which they had ſupplied. *Query*, If a perſon retains my Property, againſt my Will, as ſecurity for payment, is he not accountable for it, if the Property be deſtroyed? At length, by continual duning, obtaining Notes or accepting Drafts, they were over-paid, before the Fire. *Query*, 2*d*. Whether a perſon has a lawful right to be paid for Work done, which was never delivered, although

although requested? and whether, the Property being destroyed, the Money paid should not be refunded, in proportion to the Property lost? There requires, I presume, no great sagacity, or deep knowledge in Law, or, at least, in Equity, to determine.

A few weeks before the Fire, Mr. Bigg, as usual, came to me, and desired me to give him a Note of hand, or two, for the balance of their Account, as they were in great want of Money, that day; having in his hand a slip of paper, on which he had taken (as he said) the sum total from the Book; I think, about twenty seven Pounds. Now, as I never yet had a regular account of the whole, but only for the book of Geometry completed, and part of the Perspective; and as there had been some dispute in respect of the Money paid, which is perfectly clear by the Receipts given; by which, and the accounts I had, I was fully satisfied that very little was due to them; and consequently, I objected to the settling an Account I had never seen; for which, every one who knows any thing of the World, must have condemned me for a Fool. But, he being very urgent and importunate, I agreed, merely to serve their present emergency, as I thought, to give Notes for twenty pounds fifteen shillings, provided they kept back another of ten pounds, due the week after, for two months longer; for which, I gave him another, then, to raise money, which was sent me to pay the first. Mr. Wells, at that time, being concerned with them, those Notes were deposited in his hands; and the first was paid, after the Fire, as it was properly due before; the other, I was fully determined not to pay, unless compelled by Law; as they were scarce legally obtained, and for which I never received value in any shape; but, at the time I gave them, I was then going on with an Appendix to the book of Perspective, which induced me to it; as I concluded, that before the Notes became due, I shonld be as much in their Accounts; nor, did I doubt their probity, when we should come to settle them.

Being thus situated with the Printers, I shall take my leave of them for a while, and introduce to the Readers acquaintance, two very worthy Gentlemen, Messrs. Wright and Gill; with whom I had contracted for Paper, to the amount of 85l. of which I paid them 32l. with interest, and gave two Notes for 28l. each, bearing Interest. One of these Notes was paid, with Interest, and fifteen Guineas of the other; so that there remained, due to them, twelve pounds and five shillings when the Fire happened.

It is much easier to conceive than describe an Author's situation; who, having gone through the fatigue and expence of such a laborious Work, and completed it, ready for publication, expecting to reap the harvest of his labours; when, lo! a fatal Bar to his hopes, deprives him, in one Hour, of that resource he had so long and so ardently panted after. No redress, no remedy but Patience; the labour, and expence of printing it must be repeated, before he sits down to enjoy the fruits of his past labours, in quiet; ungrateful task; Patience, surely, never had a more ample subject, for exerting her dominion and influence over the Mind.

Such was my situation, on the third day of March, 1776. It is not easy to paint the Prospect which at first presented itself; the Harvest of my labours destroyed, when just ready for the Sickle. But, calling Reason to my aid, I formed the plan of my future proceeding; and comforted myself with the reflection, that the Engravers, Printers, and Paper Merchants were so near paid; there remaining, to the last, the trifling sum of 12l. five shillings only, as above; which, considering that they were very eminent, that is, very rich Men, and great traders, I never imagined they would be urgent for such a trifle, to them, at such a time, being reputed to be worth 40 or 50 thousand Pounds, each. What

pitiful

pitiful Ideas muſt thoſe have, who never felt the ſublime pleaſure of heaping up Money. There is not, perhaps, a pleaſure the World affords that can be compared with it; ſo exquiſitely ſublime, that all inferior pleaſures are wholly abſorbed in it. Thoſe gentlemen having loſt 140 Pounds, by the aforeſaid Fire, it was a terrible blow to their repoſe; but they, with others of the ſame caſt, took care to ſecure, to themſelves, the fragments of all that was left on the Eſtate, in Debts, &c. wholly excluding all ſuch claimants as myſelf, from having any ſhare in the ſpoil. Conſequently, having loſt my preſent dependance, it was judged a proper time to require payment, of my Debt to them, having ſtood long in their Books; which ſingle circumſtance is, to a lover of Money, a ſufficient plea for levying it by diſtreſs; the ſolid argument, that the culprit is not in a ſituation to pay, with convenience to himſelf, is an argument that has no weight; the moſt ſolid and convincing is, you muſt and ſhall pay. Sound, counting-houſe, reaſoning; ſuch as, I make no doubt, every retailer or manufacturer of Goods, for which he has obtained credit of ſome hungry, rapacious, over-bearing, wholeſale Dealer, has experienced one time or other.

In conſequence of the above reaſoning, I received a Letter from Wright and Gill, requeſting immediate payment, as they were really in want of Money. I doubt not the truth of their wanting Money, for that they will to the end of their days. Such penurious Money-rakers make no diſtinction between the Wants of others, who want the neceſſaries of Life, and theirs; which is, merely to accumulate, and heap money on money. This Letter was duly anſwered, requeſting them to add a little more to their common ſtock of Patience, on account of the misfortune which had happened, in which we were common ſufferers; and that I would wait on them in a few Days. Accordingly I attended their Levee, when I ſaw both Wright and Gill: the following is the ſubſtance of what paſſed between us, on that Interview.

Well Mr. Malton (ſays Mr. Wright) I hope you are come to ſettle that account, now; it has ſtood a long time in our Books, and it is very diſagreeable to us, to keep Accounts ſo long open. Yes Sir, ſays I, if you pleaſe we will ſettle it, but it is not convenient to pay you; as you know the loſs I have ſuſtained, you cannot imagine that I am in a ſituation to pay money yet, having loſt the means I had of obtaining it. Why, we are informed that your loſs is trifling, a mere ſhuffling pretence to delay payment. Sir, I ſcorn the appellation of a Shuffler; * my loſs Sir, is greater than yours, which I can juſtify; but, let that be as it may, I have loſt near half the Paper I had from you, with the additional expence of printing, annexed to it. Well, but I knew it had ſtood long, and ought to have been paid before the Fire happened; that he knew I had got a valuable Work, and that I might pay them if I would. What ſound, logical reaſoning; how futile muſt ſuch arguments appear, to an impartial and diſintereſted perſon. That it ſhould have been paid long ago was, now, but little to the purpoſe; it was not paid; and they knew I had loſt that, with which I hoped to have paid them and others, ere now; and the very identical Property I had of them, inſtead of making Money of it; which, had I done, they would be juſtifiable in the ſtep they took to obtain it. All the arguments I could make uſe of availed little or nothing; they had loſt too much already, by Cox and Bigg, who, he (Mr. Wright) ſaid, deſerved to be hanged; and, I make no doubt, they were his real Sentiments. Good God! is it poſſible any human Being could ſo far diſgrace human Nature? as to ſay a perſon deſerved to be hanged; for a piece of negligence, in which, they had not loſt,

* An opprobious term, much uſed by wholeſale Dealers; when an honeſt induſtrious Manufacturer, cannot make his Payments in due time.

on

on the whole, above a hundred pounds (who were reputed worth a hundred thoufand) the other had loft their Whole, divefted of all they had, in the World. And I am alfo of opinion, that if, in order to pay them the paltry fum of 12 or 13 pounds, I had fold my Copy-right for half, or a third part of its value, they would have highly applauded me, for my Honefty. I fhould look on fuch a perfon in the fame light, as one who had rob'd me of fifty pounds value, when he knew he could not make ten fhillings, to himfelf, by his villainy. No, had they never been paid, I know the value of the Work; I have grappled with every difficulty to preferve it my own Property, entire, and it muft be fome more dire Calamity, to me, that fhall wreft it from me.

When all I could fay availed nothing, I propofed, that, if they would deduct the Intereft I had paid (as furely, in fuch a cafe, they could not be hurt if they got the whole Debt, without Intereft) I would endeavour to pay them in a certain, fhort time, mentioned. I muft allow them to think and judge for themfelves, they faid; and generoufly propofed, if I paid them in the time, to charge no Intereft from that time; but they wondered how I could expect what I propofed, after fo long credit. I then propofed to give them a Book for the Intereft, and put them down as Subfcribers; ofwhich, to fell it, they might make two pounds, at leaft. No, the Book was ufelefs to them. Wretches, who had fubfcribed ten Guineas, juft before, to make a found in print, or for fome other finifter view, yet refufe to encourage a Work, for which they had furnifhed Paper, to the amount of 85 pounds; and the Subfcription to be paid by the intereft of the money; and, after I had loft, by fire, near half the Paper I had of them. What pity, that any, who either write or print, fhould contribute to enrich fuch felfifh mercenaries, by confuming their Paper. I then told them, that they were the beft judges what they had to do, and muft purfue fuch meafures as appeared to them the moft elegible; I could not pay them yet. They, then, affured me they would not wait much longer; in which they fpoke the truth; as all fuch like religioufly obferve their Word, in paying or in being paid; who frequently make their boaft, that they are always punctual in their payments; by way of reproach, to thofe who are not in a fituation to pay them. As if there was any merit in being punctual, to thofe who have it always in their power; or a reproach to others, who have not; provided they do not unneceffarily confume and embezzle their getings, and are induftrious to get.

Being, as I thought, in perfect fecurity, on their account, notwithftanding what had paffed; thinking, as being Men, they could not, and as Tradefmen (for their credit) they durft not put their menaces into execution. But I reckoned without my Hoft; for, on Saturday the 8th of June, a gentleman Officer (one Mr. Armftrong, of Carey-ftreet, Lincoln's-Inn) waited on me, from them, with a fcrap of Parchment, and flyly told me I was his Prifoner. On whofe account, Sir, fays I? let me fee your Warrant. I read its contents, defired him to walk up ftairs with me, and I would fettle the Account; having, fortunately, received as much that morning, which I had then in my pocket. What the confequence would have been, had I not been fo lucky, I will not fay; but, as it was, it fupplied me with courage to fupport it, with becoming fortitude.

Thofe who never had the honour to be acquainted with fuch like Gentlemen, executors of the Law, cannot have the leaft idea of the delicacy, the humanity, the tender feelings of fome of thofe Harpies in office; who are dead to every fenfe of feeling, where their Intereft is concerned. My Wife (ever anticipating woe) fufpecting his bufinefs, followed clofe in the rear; when there followed, perhaps, the moft entertaining Scene that can be imagined; of which, I will endeavour to give a true Picture.

There were, in my Study, three of my Children, Boys, who were juft come home, from a School in Yorkfhire, the oldeft juft turned of twelve years. Hav-

ing been rather a heavy Tax on me, for three years and a half, of thirty Guineas a year, I had now got them Home; not merely to ſave that expence, * or ſome part, but rather to inſtruct them myſelf, now; having, as I thought, finiſhed the work I had in hand. As ſoon as my Wife entered the room, as if ſhe followed a Spectre; I hope my dear (ſays ſhe) you need not go with the Gentleman? No, my dear, I don't intend it, ſays I; indeed but you muſt, ſays he; Sir, I ſhall not go out of the Houſe (replied I) I ſhall pay the Demand you have on me. On which, he ſtarted up, and collared me; and puting on one of his fierceſt magiſterial looks, Sir, ſays he, do you know who I am? Do you know, that I repreſent the Sheriff of Middleſex? He has a worthy Repreſentative, ſays I; but if you repreſent the Devil † I care not; I can pay your demand on me, and therefore, will not go with you. Immediately he calls up his Hell-hound; who, ere this, had gained entrance, and, like a Ruffian, attempted to drag me down ſtairs. The whole Houſe was alarmed; my Wife, ſtriving to ſcreen me from their violence, was handled ſomewhat roughly; the Boys ſet up their ſhrileſt Pipes, to a fine old tune; up runs a Daughter and the Maid; a Lady, on the firſt floor, was frighted almoſt to death; in ſhort, it was the moſt diverting ſcene that can be imagined; what pity, the Authors of it were not preſent, to enjoy it, and glut themſelves with their juſt Vengeance §. Reſiſtance being vain, I enquired where his habitation was; being informed, in Carey-ſtreet, a Coach was called, and away we drove, taking a Son with me, who happened to come in. He had to trudge into Abchurch-Lane, to the Attorney (Mr. Conſtable) to get a Bill of debt and coſts. If he had happened to be gone to his Country houſe, I perhaps, have taken up my reſidence in Carey-ſtreet, till Monday; it being a cuſtom with thoſe Gentlemen, to ſeize their prey on a Saturday, in order to detain them the next Day, if matters cannot be ſettled ſoon. The Bill was making out, when a meſſenger was ſent, to order twenty ſhillings to be tacked to it, for Intereſt; which, with the expences of one pound ſixteen ſhillings, brought it up to fifteen pounds, and one ſhilling. That buſineſs being done, the Office ſearched, and the Money paid, I regained my Liberty, and returned home, to comfort my diſconſolate Family.

I ſhould be glad (for the ſake of thoſe, whoſe misfortunes may ſubject them to the inſults of ſuch Wretches) that ſome humane Perſon, of the Law, would, in public print, let ſuch know, how far thoſe Gentlemen have authority to tyrannize over thoſe, who are already too much oppreſſed; and how they may be puniſhed, for over acting their part. What! is an unfortunate Man, in credit, to be treated, by ſuch Reptiles, like a Cutpurſe, or a Felon? If I was under a miſtake in point of Law, he ſhould have ſet me right, in a proper manner. But why ſhould a Perſon be obliged to go with them (the idea of which is terrible to a Perſon in Credit) when he can anſwer the demand againſt him? Is it to ſqueze the laſt Shilling from him, by all means poſſible? A Shilling or two for a Coach; a Shilling for being lock'd up; a Bottle or two of Wine; and what they can extort for ſearching the Office, (though they have no right to demand any thing), 'tis their own Buſineſs; and at their peril they detain a Perſon, if there be nothing more againſt him. It would be a real Bleſſing to the unfortunate, if every Sheriff, entering on his Office, would make it his firſt buſineſs to enquire, ſtrictly, how thoſe, to whom they delegate ſuch Power, have acted, in the execution of their Office; and either ſuſpend for awhile, or expel them entirely, if they be found, as too many might be found, to deſerve it.

* Let none be ſurprized at the mention of ſaving. Having three, at once, it was agreed for ten Guineas each, otherwiſe it is twelve Pounds a Year; out of which, I will maintain it, they clear five or ſix, for teaching; then where three are ſo many in a Family, with œconomy, there may be a ſaving.

† *Query*, Whether it was not a more juſt, and ſtriking Repreſentation?

§ To give them a better Idea, they may ſee a faint reſemblance, delineated in a Print, in the firſt Number of a Publication, five or ſix Years ago, called the Gentleman's Muſeum.

paltry

I ſhall, now, take my leave of thoſe generous and truly humane Gentlemen, thoſe fair and upright Traders; who, by means of their great wealth, have the power to monopolize a moſt neceſſary Article, for many miles around the Metropolis; who could have the barbarity to arreſt an induſtrious Perſon, for the paltry Sum of twelve Pounds five Shillings, the remains of near a Hundred; after having loſt near half the Paper he had of them (as related above) although he paid them Intereſt for their Credit; amounting, on the whole, to four Pounds, twelve Shillings, and ten Pence.

Would it not have redounded more to their Reputation, to have ſent for me, and addreſſed me thus? Well, Mr. Malton, we are ſorry that you are, alſo, a ſufferer with us, in that unfortunate Accident; which, to us, is trifling, becauſe we are able to bear it. But for you, who having juſt completed ſuch a truly laborious Work; burthened, as you are, with ſo large a Family, to be thus fruſtrated in reaping the emolument of your indefatigable Labour and Induſtry, we are really concerned. Having ſeen and experienced your Probity, we freely offer to credit you Paper, for reprinting the Work; which, we are glad to hear, merits the approbation of the Public; as we ſhall be happy in being inſtrumental to the rewarding of ſo much Merit. There was a ſmall Balance left unpaid; had not our loſs been already conſiderable, we would ſtrike it out of our Books; we cannot ſuppoſe you are in a ſituation now to pay; but if you approve of it, we will add it to the next Account; you may, hereafter, be in a better ſituation; we will endeavour to contribute towards it; we will each of us be a Subſcriber to your Work: as for the Intereſt you have paid, it ſhall be remited, 'tis enough that we get the Principal now. I muſt obſerve, here, that when Mr. Wright ſaid, he knew I had got a valuable Work, I aſked him what better I was for it, at preſent? would they let me have Paper, for reprinting it? The anſwer was, yes, if I would give them ſecurity for it. You aſked for none before, ſaid I, nor will I attempt to give you any now; ſurely, now the merits of the Work is eſtabliſhed, I am not in a worſe ſituation than when I began it. But I deſire not your credit, I can have it elſewhere, and be ſerved at a lower price.

Having thus diſcharged a duty to the Public, in reſpect of ſuch Blood ſuckers; I have only further to add, that I am fully perſuaded, they were inſtrumental to the following affair. Being fortunate, in geting their Money, I muſt, in courſe have it in my power to pay the other, alſo; but if I had, I was far from having the inclination, ſeeing I never had value for the Money, delivered to me; but, on the contrary, detained from me, by their Caprice, till it was deſtroyed. With what confidence could they think of compelling me to pay, for Work done, when they were conſcious of being the ſole cauſe of my loſing it? I ſhall find means (doubt not) to do myſelf juſtice.

On Wedneſday the 26th of June, I received a Letter from an Attorney on Mr. Wells's account; giving me to know, that Mr. Wells had put into their hands, the two Notes, for twenty Pounds fifteen Shillings, with order to arreſt me, if they were not paid the next Day, or on Friday. I waited on Mr. Wells the next Day, and let him know how I was ſituated; and that, if I muſt pay the Notes, they muſt allow me Time; but that, Cox and Bigg were the Perſons properly accountable for them. I knew not, then, that he was a party concerned, otherwiſe, than having advanced, as he ſaid, a thouſand Pounds, which he had loſt; he deſerved to loſe it, for his negligence, in not ſeeing them inſured; by which, others ſuffered ſo much. But, as ſuch, generally, are better acquainted with the affairs of thoſe who are indebted to them, than themſelves, he knew I could pay them, then, if I would. Perhaps I could; but I had twenty more important, and neceſſary demands for all the Money I could get.

I wrote to him, the next Day, and ſtated the Caſe in the moſt candid and impartial manner; laid the true ſituation of my affairs open to him; with the burthen of ſuch a Family, as I had to provide for; and that, in three Weeks time, I would uſe my utmoſt endeavours to pay one of the Notes. But, what is moſt extraordinary, although I had requeſted it, over and over, of Mr. Cox, and he had promiſed I ſhould, yet, I could never be indulged with an Account of what there was againſt me; to have, at leaſt, the ſatisfaction that I ever was ſo much indebted to them.

Notwithſtanding all I had ſaid or wrote to Mr. Wells, I was arreſted in a fortnight after, on Saturday (as uſual) the 13th of July. As I never imagined that they durſt, or that any Perſon, who was not a Jew or a Barbarian, could have proceeded to compulſion, in ſuch an affair, I was determined not to pay it, till I had advice concerning it; and, having the long Vacation before me, I conſequently gave Bail. But, finding I had no Plea to ſupport a Trial, as I could not give direct and poſitive proof, that he was a Partner in the Buſineſs; nor, indeed, did I know it then; and, being at a diſtance from Home, I ſent Orders for paying it, before the Term commenced. I muſt, however, do Mr. Wells this juſtice; he took no more than the Principal; I paid no expences, nor did he charge Intereſt on the Notes; like the other hungry, rapacious Wretches, Wright and Gill, who charged to the laſt Shilling of Intereſt.

Thus the affair ſtands, at preſent, between me and the Printers. I have waited till now, in hopes they might be in a ſituation, ere this, to reprint the Work; and, by that means, make me ſome reparation, without inconvenience to themſelves. But, I am never the nearer; the Partnerſhip is diſſolved, and therefore, I have but little to expect on that ſcore; nor could I longer delay puting the Work to Preſs. The Public may now be aſſured, that it will be regularly publiſhed in Numbers, or Parcels, every Month.

I cannot ſuppoſe that Meſſrs. Cox and Bigg are inſenſible to the peculiar hardſhip of my Caſe, as they are conſcious that I ſuffered, entirely, through their caprice; but I think, had they repreſented the Caſe properly, and as it really was, they might have alleviated it; and, at leaſt, have acquited me of thoſe Notes, I had, imprudently, but with good intention to them, laſt given; eſpecially, as they were not gainers by the payment. I have only, now, to wait with Patience, to ſee what will be the reſult of the Brief now impending, and collecting from Houſe to Houſe, throughout every Pariſh in England, &c. obtained through the Policy and Intereſt of Mr. Wells; who, although he would diſdain to have his Name appear in it, will not, I preſume refuſe to accept the ſhare of it, which may be allotted to Mr. Nettleton; for that, the World may be aſſured, is the ſole motive that induced him to intereſt himſelf ſo much in it.

I hope for my own, as well as for their Emolument, that the Public will be liberal; they never had a more intereſting occaſion. Although they have endeavoured to make my loſs appear trivial (as it certainly is, compared, in bulk, with ſome other ſufferers) I do here affirm, that I loſt 350 Books of Geometry, entirely, and near two hundred (out of five) of the Perſpective, which entirely fruſtrated all my hopes of Emolument, from the publication of that Work for the preſent.

BOOK I.

OPTICS or VISION.

SECTION I.

Of LIGHT and COLOUR.

TO treat on any Science, properly, it is neceſſary to begin with the Elements of it, or firſt Principles; which is the reaſon, as I ſuppoſe, that ſome writers, on Perſpective, have prefaced their Works with a chapter on Optics and Viſion. Some of whom, not daring to advance any thing of their own, on the ſubject, have favoured the World with extracts from Sir Iſaac Newton's and Smith's Optics; which, indeed, is very little to the purpoſe, as the much greater part is no way ſubſervient to Perſpective; conſidering it not merely optical, but purely a mathematical Science, ſupported on the firm Baſis of Geometry.

The ingenious Mr. Hamilton, in his Stereography (a work which does great credit to the Author) to which I acknowledge myſelf indebted, has given ſome judicious obſervations on the ſubject of Viſion; and ſome, which are rather exceptionable. But, the Theory of Colours (if that can be called a Theory, which is ſo little known, and nothing in it demonſtrable) he, very wiſely, declines enlarging on; as not being ſubſervient to Perſpective, nor indeed to Painting: I mean the Theory of the Priſm, which, Sir Iſaac Newton and others have ſo copiouſly expatiated on.

I would by no means have it thought that I intend, or have the leaſt deſire, to derogate from the character of ſo great a Man, whoſe Name I highly revere; but I am perſuaded, that the purſuits of the greateſt Men are, ſometimes, in themſelves, trivial and merely amuſing; when the character of a great Man is eſtabliſhed, it gives a ſanction of conſequence to his moſt inſignificant amuſements; for, why may not the moſt profound reaſoners relax from their intenſe ſtudies, and amuſe themſelves, ſometimes, with trifling matters? by which means, things of great utility have been brought to light; as muſt have been the caſe in reſpect of the Priſm: nor do I find that he has perfected what he had in view, but left his obſervations for others to improve on, having (in his own words) not inclination to take it up again, and purſue it further, as he had intended.

Had the Theory of Colours, as deduced from the Priſm, been amongſt the firſt and chief of that great Man's purſuits, I am much in doubt, if the reputation he has acquired had ever been eſtabliſhed, at leaſt on that Baſis; things of infinitely more importance to the Community, fixed his credit (moſt deſervedly) on the higheſt pinnacle of Fame; for, what uſeful and neceſſary knowledge has been communicated to mankind, by this acquiſition to the Science of Optics? which (with ſuch, apparently wondrous, ſagacity and penetration) he has explored and given to the World.

To define, with any degree of precifion and perfpicuity, what Light is, is not poffible; feeing we cannot comprehend, and enter into an accurate difquifition of Fire; which is the only caufe we can conceive of Light. From the different and obftinate refrangibility of the different Colours produced by a Prifm, when applied to the Sun's Beams, it is concluded, that Light is, in its nature, heterogeneous; that it is compofed of Particles, of different qualities, which produce very different effects on Objects. I cannot, from any thing I have yet feen or read on the Subject, give my affent to that opinion; for, I believe, that what we call Light (the vivid glow occafioned by any luminous body) is perfectly homogenial; and that it is not compofed of Particles; which implies that it is material, a Body; for, how elfe can it be a compofition of Particles? which, by means of the Prifm, are fuppofed to be diffevered, and feparated from each other.

But, what reafon can be affigned, that a Prifm fhould have this moft extraordinary power of feparating them; owing to its form only *, not the matter of which it is compofed: But it is not owing to either, as it is manifeft by Experiment. Allowing the rays of Light to be thus fifted or feparated, what ufe is made of it, or what further knowledge is deduced from it?

'Tis obvious, that the fame Object, having its Surfaces differently difpofed, or fituated, exhibits different Colours, according as its Surfaces are fituated to the Light; that is, to that quarter, from which any thing luminous caufes an illumination of them; and thofe parts, which are moft directly oppofed to it, are moft intenfe in Colour; whilft the oppofite Faces (the Light being obftructed by the Object) are almoft deprived of Colour; which they would be entirely, but for other Objects in vicinity with and oppofed to them: which other Objects being ftrongly illumined, are faid to reflect Light to them. Hence, a conclufion is drawn, that Objects have, in themfelves, no inherent Colour; but that, all the fenfe we have of different Colours, in the fame or in different Objects, is entirely owing to their different qualities and modifications (or, rather, to the conftruction of their Surfaces, fimply) by which, they are fitted and difpofed to reflect different coloured Rays, fome more copioufly than others. But, it does not neceffarily follow, that they have no inherent Colour; for although Objects, when oppofed to the Sun (that is, when nothing opake intervenes) exhibit very different, and more brilliant colours than otherwife; yet, if its Rays are obftructed, inftantaneoufly (the Light around them being then fuppofed ftagnated), they ftill exhibit Colour, and of the fame kind, or hue, though lefs vivid, and greatly inferior in the degree of it.

The chief Argument advanced in favour, and the only reafon, of weight, affigned, in fupport of their Hypothefis, is, that Objects exhibit no Colour, when they are entirely deprived of Light; which argument cannot be denied; becaufe, no Object, in fuch Cafe, is perceptible by means of Vifion; without Light our Optics are of no ufe at all. It is the fame if we clofe our Eyes, when Objects are fully illumined; but, certainly, the Colour, as well as the Object, remains, whether, by fhuting our Eyes, we perceive it or not. I look on this as a fpecious, and fubtle, not to call it a fophiftical way of reafoning; fince, without Light, there can be no Vifion; and confequently, the fenfe of Colour, more than of the Figure, Magnitude, or Situation of Objects, cannot be communicated. But, if Light be confidered as a Medium exifting of itfelf, without the Luminary, how can there be a privation of it, if it fills the whole furrounding fpace, as Air does? which, feeing that, Air is alfo confidered as a material Body, is fomewhat repugnant to the eftablifhed and univerfally received Maxim, that, two Bodies cannot fill or occupy the fame Space, at the fame Time. And, if Light be a Medium, of what ufe is it, without the Luminary? or, how can there be a perpetual emiffion, from the Luminary, the whole Space being, every where, filled with Light? and alfo, if it be material, what becomes of it? To fuppofe that it is abforbed, by the Earth and other Planets, would be ridiculous in the higheft degree; becaufe, the whole Surfaces of all the Planets bear no fenfible proportion to the immenfe Space between

* A Prifm, as defined by Euclid (Def. 13. 11.) is a Solid, contained by Planes, of which, two oppofite are equal, fimilar, and parallel; all the reft are Parallelograms; fo that, every Parallelopiped is a Prifm, by which, being right-angled, no effect of Colour is produced. Now, as it depends, folely, on the inclination of one Plane to another, an acute angled Pyramid (in which all the Planes are Triangles) or fruftum of a Pyramid, is the fame, in refpect of Colour, as a Prifm.

them, in which there is no obftruction to the progreffion of Light, flowing from the Luminary in all directions.

That Light fhould exift in darknefs (notwithftanding the Scriptures fay, that Light was created before the Sun) is not only a direct contradiction in terms, but to reafon and common fenfe, and the nature of things. Alfo, that Objects exhibit no Colour, when deprived of Light, is certain: becaufe they are no longer vifible; but, fince every other quality remains, it is moft probable (Colour being perceptible by fight only) were they objects of Vifion, in total Darknefs, Colour would remain likewife: For, although Colour cannot be perceived without Light, yet it is not the efficient caufe of different Colours, in Objects compofed of different materials.

In treating on Colours, the learned Boyle gives us a wonderful account of a blind Man's diftinguifhing Colours by the touch; which can only be by the afperity or roughnefs of the furface of the coloured Body, and which is conftrued as favouring their Opinion. It may be poffible, that a difference in Colours, artificially laid on the furfaces of Bodies, may be felt; or, that the qualities of Dies or Stains may alfo affect Bodies fo, as to make them fenfible to the touch. But, will any perfon venture to fay, that if Bodies, ftained or otherwife coloured, were polifhed or made equally fmooth, the difference, in Colour, could be perceived by feeling; or, that the natural Colours of Wood, Metals, or Stones, nearly of the fame qualities, except Colour, when equally polifhed, could be diftinguifhed by that mode of Perception? I affirm they could not; nor can I give any credit to fuch affertions. Then, fince that, Bodies, equally hard and equally polifhed, exhibit different Colours, I fee no reafon for fuppofing that the caufe of their differing in Colour is owing to the different texture or conftruction of their Surfaces; or, that it is poffible to diftinguifh them by their afperity; for, Bodies of equal hardnefs, and being equally polifhed, muft have Surfaces alike, except in Figure and Colour.

Light, however propagated, has, I prefume, the fame properties and influence; and, confequently, acts the fame on Bodies. I would afk then, what is the reafon, why Blue and Green are fcarce diftinguifhable by the light of Fire? If the action and reaction of Light, on the furfaces of Bodies, be uniform, by a certain law, according as they are fitted and difpofed to reflect any particular fpecies of colour-making Rays, whence arifes the difference in this cafe? I fhould be glad to hear or read an ingenious difquifition of that point; as, I do not doubt, there are Men of fagacity and penetration fufficient to fet it in the cleareft light, and make it as evident, to the underftanding, as any common Cafe in any Science whatever.

To return to the Prifm. It is certainly true (and it is a curious and entertaining Experiment) that, through an acute angled Prifm, we perceive Objects very differently coloured from what they really are by Nature; alfo, by applying the Prifm, properly, to the Sun's Beams, there is exhibited a curious and moft extraordinary Phænomenon, *viz.* a diverfity of Colours, the moft lively and beautiful that can be conceived; and, the more uncoloured the Surface is, on which they fall, the more vivid they appear.

Now it is certain, that here is a perception of Colour, on the furfaces of Bodies, which is not inherent; and fo is there when the Sun's Beams, only, fall on them: yet, we cannot from thence conclude, that they have no inherent Colour. The Colours which the Prifm exhibits are Red, Orange, Yellow, Green, Blue, Indico, and Violet or Purple. But why is there, here, made a diftinction of feven Colours, when, in reality, there are not above three or four fimple Colours in Nature? (unlefs Black and White may be called Colours) *viz.* Red, Yellow, Blue*; and Green; which may alfo be compounded of Blue and Yellow. Indico is Blue, Orange and Purple are Compounds, or mixed Colours. Now, if any Philofopher, or Artift whatever, can, from thefe three or four fimple and brilliant Colours, produce all the variety which we fee in Nature; or, did the Prifm exhibit di-

* It is faid in Emerfon's Optics (Cor. 3. Prop. 9.). There are as many fimple Colours as there are degrees of Refrangibility; and therefore an infinite number. And, hence (in Cor. 5.) it is plain, if the Sun's Light confifted but of one fort of Rays, there would be but one Colour in the World. In one place, he fays, Objects have no Colour naturally; and in another, they become coloured, by reflecting the Light of their own Colour, more plentifully than others.

 ftinctly

ſtinctly, all the ſimple Colours which are in Nature, only, without mixture, I ſhould then be better diſpoſed to give credit to their Theory: or, when I can have conviction, that a mixture of all theſe Colours, together, in any ratio, will produce a perfect White (as Snow) I may then be a perfect Proſelyte; till then, I am perſuaded that I muſt diſſent from their Opinions.

I ſhall, in the next Place, enquire, how a Priſm, particularly, has the wonderful property of ſeparating the different coloured Rays of Light. The Priſm is a body of Glaſs, or it may be of Cryſtal, or other pellucid, uncoloured Stone; is the property then in Glaſs, or Stone? No; in the Figure that is made of it, only; amazing! that the diſpoſal of the ſame Matter into different Forms ſhould produce ſuch very different Effects. Glaſs, diſpoſed into a portion of a convex Sphere, will make Objects, ſeen through it, appear magnified; and, being oppoſed to the Sun's Beams, will collect real Fire; aſtoniſhing indeed! If the Surfaces be concave, Objects, ſeen through them, appear leſs than they really are; in the forms of Priſms it has various effects; but we do not ſee the effect of exhibiting Colours through a right-angled Priſm; I mean, a four-ſided one, whoſe oppoſite faces are parallel, and the Angles right ones; neither has it that effect through the right Angle, of a right-angled triangular Priſm, but only through the acute Angles. The cauſe, then, of exhibiting Colour is not in the Matter itſelf, or Figure, nor in the Surfaces, ſimply, but in the inclination of the Surfaces to each other; for, being parallel, or at right angles, or nearly ſo, they do not produce that effect. We may as well aſk, why convex Surfaces collect Fire (but, that is well known) rather than plane or other parallel Surfaces, as, why a Priſm, whoſe Planes are inclined to each other, ſhould have the property of exhibiting the different Colours I have mentioned, rather than one that is right-angled; and I am perſuaded, that, with all the ſagacity and penetration Man is endowed with, he will never be able to account, truly, for either.

Yet, I do not condemn all enquiries into the cauſes of the various effects which we perceive in Nature, but think the purſuit rational, and truly commendable, when it is founded on certain Data, and real Hypotheſis; and the reſult of our reſearches productive of real utility, deducible from it, as in ſome other branches of Optics. It is aſſerted, by ſome, that the perfection of Teleſcopes is owing to the Theory of the priſmatic Colours. As I am not converſant in the mechanical conſtruction of Lenſes and their application to Teleſcopes, I cannot affirm that it has not been ſubſervient thereto; but, in my opinion, it has not the leaſt apparent tendency to benefit Mankind accruing from it. Certain I am, that it will never be of uſe to a Painter, to compound his Colours and form a Theory thereof; by means of which, he may ſooner, and with certainty, arrive at perfection, in his Art: and I muſt needs ſay, that the attempt made by Brook Taylor, in the Appendix to his ſecond part, does not ſhew the Doctor's judgment, in that, to be of a piece with the reſt of the Work.

The Sun's Beams paſſing through Glaſs, whoſe Surfaces are neither parallel nor perpendicular to each other, or nearly ſo, exhibit the various Colours, ſpoken of above, in ſome degree, according to the inclination of the oppoſite Surfaces, and purity or clearneſs of the Glaſs; through the thick part, near the knob, in the middle of a table of Crown Glaſs, I have often obſerved the Phænomenon, in a Window of thoſe Squares: but, it is alſo obſervable, that the effect ceaſes when the Sun ceaſes to ſhine on them. It is alſo certain, that Objects appear coloured otherwiſe than what Nature aſſigned them, when we look through Mediums denſer than Air; whoſe Surfaces, through which we look, are inclined to each other in certain Angles; and conſequently, there will be the effect of Colour produced, in ſome degree, which is not natural to Objects, in looking through Object-Glaſſes, or Lenſes, with which Teleſcopes are conſtructed; ſeeing that, their ſurfaces are inclined, at the edges. The buſineſs, therefore, of an Optician, in this reſpect, is, ſo to contrive and diſpoſe his Glaſſes, as to diveſt the Object of the borrowed Colours, which do not belong to them, naturally; and I could almoſt affirm, that the perfection, ſpoken of, has been found out by repeated trials; not from any certain eſtabliſhed Law, deduced from the Theory of the priſmatic

Colours: for, if a certain Theory had been eſtabliſhed, what hindered the immediate perfection of them, as ſoon as the cauſe of the imperfection was known, and certain Laws eſtabliſhed, whereby the unnatural effect of Colour might be removed from the Field of View?

From what has been obſerved, reſpecting the effect of Colours produced by the Priſm (unleſs it can be proved that the particles of Air, through which, Light muſt neceſſarily paſs, are Priſms) to infer, that Nature has given no inherent Colour to Objects, is bold and aſſuming; it is alſo groundleſs, ſeeing that the Colour, which is natural to each, remains when the Sun does not ſhine on them, though it differs greatly in the degree of it. We may with as much reaſon conclude, that Objects are much larger than we perceive them to be with the naked Eye, becauſe they appear ſo, when view'd through Glaſs whoſe ſurfaces are convex; and, we may as well enquire how Vegetables, as Flowers, Fruit, &c. ſpring out of the Earth, and are adorned with all that beautiful variety of Colours which we ſee in Nature; by what means they are continually varying in their Colours, from their firſt formation, in embryo, to maturity; as why, the Priſm exhibits the various Colours of the Rainbow, to which they are, in a great meaſure, ſimilar.

Colours produced by alkaline Salts, &c. and mixtures of different Fluids, with all ſuch like chymical operations, I look on, equally, as entertaining Experiments; but no way productive of any advantage, accruing from it, to the Theory of Colours neceſſary for a Painter to know, in order to reduce it to real uſe, in practice.

I ſhall not, therefore, treſpaſs any longer on the Reader's Time, as it is entirely foreign to the purpoſe of Perſpective; but confine my further obſervations to that part of Optics relative to Viſion, only. In reſpect of direct Viſion, it is certainly of uſe and ſubſervient to Perſpective; and eſſentially neceſſary to give a clear Idea of the Principles on which it exiſts, both in Theory and Practice.

Colour, whether inherent or incident to Bodies, is not material towards a perſpective Deſcription or Delineation; 'tis the Figure of the Object, only, it contemplates.

All ſolid Bodies, whatever, become objects of Perception from their Figure or Colour. Figure is more particularly inſeparable from our Ideas of Matter, than Colour; it being impoſſible, in the nature of things, to have any notion of Extention, abſtracted from the Idea of Figure, which limits the Space Bodies occupy or fill. It is the ſame with or without Light, and may be perceived by the ſenſe of Feeling only. From the known external Figure, and ſituation of an Object, may be delineated, by mathematical rules, a true perſpective Repreſentation, on a Plane; which, by the help of Light and Shade, will raiſe a perfect Idea in the Mind, of the figure of the Object, and the different forms and poſitions of its Surfaces, in reſpect of each other, without the aſſiſtance of Colour; but, to exihibit a true and natural Picture of Objects, and create a perfect Idea in every circumſtance, Colour is abſolutely neceſſary; and is the laſt degree of perfection, with which the omniſcient Creator has embelliſhed Nature, that can be given to a Picture.

Light, conſidered as a Medium, by which Viſion is convey'd to the Eye, is of too refined a nature for my ſpeculations. But, as the great Author of Nature has given a portion of Reaſon to every human Being, we have certainly a right to make uſe of that Reaſon; and, if we will exert it, properly, it is poſſible, nay certain, that one Perſon may penetrate as deeply into the myſteries of Nature as another, though not bleſs'd with quite ſo much Learning.

We are told by ſeveral great and profound Philoſophers (to be a Mathematician is not neceſſary) that we perceive Objects, only by means of Rays of Light, reflected from every point in their Surfaces to the Eye; which enter there, and form an Image, or Picture of the Objects perceived, on the Retina, or fine Membrane which ſurrounds all the back part of the Eye, internally. The Retina is ſaid to proceed directly from the Optic Nerve, which dilates or ſpreads itſelf as before-mentioned; and, the impreſſion being made thereon, we are further told, is conveyed, by the Optic Nerve, to the Brain, or ſeat of Perception.

But, why do they ſtop here? I expected. and ſhould be glad to be conveyed into the inmoſt receſſes of the Brain, and be ſhewn or told, how the Image of the Object, on the Retina; is there perceived; for I muſt own it is aſtoniſhing! that, from what is perceived or felt within, we ſhould have a true Idea of the Figure, Colour, and Magnitude, Situation and Diſtance of Objects, which are external, or ſituate without the Eye:

Now, after all this parade, and pompous diſplay of great ſagacity and deep penetration, what does the Sum total amount to? Why, that the Object is perceived, i.e. the Mind is ſenſible of the exiſtence of ſuch Objects as it perceives; and that, the Viſion of them is conveyed in right or direct Lines from the Object to the Eye, or from that wondrous Organ to the Object; but how, or in what manner, remains as much a myſtery as before. So that, after all which has been ſaid on the Subject, and, allowing the Image, formed on the Retina (inverted or otherwiſe) as perfect as they pleaſe, what nearer are we? where is the Perception of that Image or Picture of the Object? How the Mind, or Soul, perceives the Image on the Retina, any more than the real Object, we are juſt as much at a loſs to account for as ever.

In vain, therefore, does Man torment himſelf, in endeavouring to explore the hidden myſteries of Nature, which are for ever hid from our reſearches. Let us then purſue what is within our reach, and not an airy Phantom, which will ever elude the purſuit. In mathematical Sciences we have ſome certain Data, whereon to frame Hypotheſes; and although we muſt not expect perfection, except in Theory, yet, we can diſcover ſo much of Truth as gives us ſufficient encouragement to purſue it, and makes ample amends for the imperfection of human nature.

I do not intend, nor ſhall I attempt to explain the nature of Viſion, or enquire into the true cauſe of it; being fully convinced of the inſufficiency of our reaſoning faculties for ſuch a diſquiſition. It is ſufficient, for the ſcience of Perſpective, to know, or even to ſuppoſe that it is conveyed to the Eye, the ſeat of Sight, in Right Lines. But, whether it be by means of Rays * of Light, reflected from all parts of Objects to the Eye (as is the general Opinion) or whether the Allwiſe former of the Eye has given a power to that truly wonderful and amazing Organ, to convey to the Mind, by Viſion, the perception of Objects by other means, I do not mean to make the ſubject of my enquiry. Yet, I muſt acknowledge, that I have ſtrong objections againſt the general Opinion, as it is now received and almoſt univerſally aſſented to; *viz.* that the perception we have of external Objects, from Viſion, is by means of Rays of Light, reflected from all parts of their Surfaces to the Eye; and that thoſe Rays are material, or compoſed of Matter.

But, as it is of no conſequence, in Perſpective, by what means Viſion is performed, ſo it be conveyed in Right Lines, which may paſs for an Axiom; ſeeing that, the refraction of the Rays (if there be any) in the Air which ſurrounds us, is ſo very little, in Objects which are at an immenſe diſtance beyond the whole Atmoſphere, it cannot be ſenſible at any diſtance we can perceive Objects, within it, ſuppoſing the refraction uniform or regularly curved, between the Object and the Eye. But, I rather ſuppoſe the refraction of the Rays to be at the common Surface of two Mediums, only; at entering any other Medium, denſer or rarer, than that through which they firſt paſs, and thence proceed again in Right Lines; conſequently, there is no Refraction at all, within the Medium; either in Air or Water, I ſuppoſe Viſion to be convey'd in Right Lines, if the Object and the Eye are both in the ſame Medium.

At the ſame time, I would not have it thought, that I ſuppoſe any Rays to paſs, by means of Reflection, from opake Bodies; and in luminous ones, the Light, which proceeds from them, filling a concave Sphere, as far as they can be ſeen, I do not ſuppoſe to proceed in Rays, nor in Planes (as a modern Author has ſuppoſed); the Hypotheſis is abſurd; ſeeing that, the whole Space is illumined in every part, it is impoſſible, that from every Point in the ſurface of the luminous Body, Rays can proceed to an immenſe diſtance, filling all the Space between.

* I make uſe of this Term, though I have not the leaſt conception of a Line, or thread of Light, called a Ray; much leſs of a Pencil, or bundle of Rays.

Suppoſe

Suppose every Point in the surface of the Sun to emit a Ray of Light, proceeding in Right Lines from the Center; it is evident, that every Ray, at the distance of the semi-diameter, will fill a Space four-fold of its first dimensions, *viz.* in a duplicate ratio of the Distance; at the distance of the whole Diameter, it must fill a Space nine times as large; i. e. it increases in proportion to the Squares of the Distance; what, then, will be the magnitude of a single Ray of Light, proceeding from the Sun to Jupiter or Saturn, before it reaches those Planets*? or how can the whole of the Space it occupies be filled (as it certainly is) with Light, which proceeded from a single Point, in the Luminary only? Can one individual Ray be shattered into thousands, into millions, lying close to each other? what direction do they proceed in, from the Luminary? not in the direction of a Right Line from the Center, I presume.

I know it may and will be alledged that it expands as it proceeds, still filling the whole Space. I presume then, it cannot proceed in Rays, or Right Lines from the Center of the Luminary, as it is imagined. Air may be rarified and expanded to a very great degree; also, Water and other Fluids are expanded, as in steam, by Fire, or, in Mists and Vapours, by Exhalation, till it floats on Air. But can one drop, or the minutest particle of Water be expanded infinitely? as a Ray of Light, emited from one single Point in the Luminary, must, in this Case, necessarily be; else, how could we see those Stars, whose distances are, to all sense, infinite?

These things, which may amuse and pass for orthodox with the generality of mankind, cannot be satisfactory to a Person who dare think for himself, and is furnished with the means to do so. Therefore, since all which can be said about it is, at best, but an ingenious conjecture, and since it is in no wise subservient to Perspective, I shall at present take my leave of the Subject, to pursue that part or branch of Optics, which is essentially so, respecting direct Vision; and after that, in a separate Chapter, or Section, I shall give my objections to the received Opinion of the cause of Vision.

* I find by calculation (from the Tables in Harris's Use of the Globes, which I suppose are as much to be depended on as any other) that the distance of the Earth, from the Sun's Center, is 212 semi-diameters of the Sun, nearly. Wherefore, if, instead of a Point, we suppose a certain quantity of Light to flow from one square Inch in the surface of the Sun; each side of that square Inch will, at that distance, be increased to 212 inches, or 17 feet 8 inches; the square of which is 44944 square inches. Consequently, a quadrangular Pyramid, whose Base is 17 feet 8 inches square, and its altitude, the distance of the Sun from the Earth, is filled with Light, from one square Inch, only; and consequently, each single Ray, proceeding from every Point in its Surface, will, at that distance, be multiplied, or magnified 44944 times.

At the distance of Saturn (by the same Tables) a square Inch, and consequently a single Ray, will be encreased or multiplied 4,149,369 times; its distance being 2037 semi-diameters of the Sun, nearly; consequently, a quadrangular Pyramid, whose Base is 169 feet 9 inches, on each side, and its altitude 777 millions of Miles, is filled with Light, flowing from every square Inch on the surface of the Sun; an amazing circumstance indeed! if it is supposed, or considered to be a material Body. But, how far Light may proceed, from the Sun, beyond the realms of Saturn, is not yet, I presume, determined.

SECTION II.

Of the structure of the EYE, its parts described, and a short INTRODUCTION to the nature of VISION.

THE construction of an Eye, that most wonderful organ of Sight, by means of which, we have intelligence of the Objects which surround us, is various; yet has nearly the same Parts, in all the animal Creation. The Parts are few, and apparently simple, but astonishing in their mechanism; as, indeed, are all the animal Functions; but, what is there in Nature which is not so? The Figure of an entire human Eye, within its Socket is globular, of somewhat more than an Inch in Diameter; the Furniture of which are as follow.

 Let

Plate I. Fig. I. Let HIKL be a vertical Section through the center of the Eye, which, in this Figure, is confiderably larger than Life; in order, that the feveral parts may be defcribed with greater accuracy.

The external part, from H to I, is called the Cornea, or Horny Coat. It is perfectly tranfparent, and is fomewhat more convex than the part which is within the Socket, by means of a fine, clear, aqueous or watry Humour, between the Cornea and the Iris (the coloured Circle within). Immediately behind the Iris, (ghik,) through which there is in Aperture, in the middle, to let in Light (or Vifion by means of Light) is a whitifh Subftance (MN) like a ftrong Jelly, or cold Glue, of a moderate confiftence. It is as clear and pellucid as Cryftal; from which, it is called the Cryftalline Humour. It is in every refpect of the fame nature and ufe as a double convex, microfcopic Lens; and is faid to be more convex on one Side (the inward) than the other. Its ufe is fuppofed to contract the Rays of Light, by which Vifion is convey'd, to a Focus, beyond this Humour, in the Center of the Eye, at E; fromwhence, they become diverging, and fall on the bottom or back part of the Eye, at *acb*.

The Cryftalline Humour (MN) is faid, by thofe who are acquainted with and accuftomed to diffections of the Eye, to be furnifhed with mufcular Fibres; by means of which, it is made either more or lefs convex; or, by contracting them, is brought forward or otherwife as occafion requires; in order, to render the Images, or Pictures of Objects perfect, in the bottom of the Eye, as the Objects are nearer to or further from the Eye; all whch feems confonant to Reafon.

The Aperture or Pupil (hi) is alfo, by fome fuch means, expanded or contracted, as there is occafion for more or lefs Light to enter. I have obferved, in the Eye of a Child, in a Room juft light enough to difcover it, the Pupil enlarged to three tenths of an Inch in diameter; and immediately, on bringing into a full Light, it was contracted to one tenth, or lefs; amazing Structure! and this is performed involuntarily, whether we will or no. There can fcarce be any Perfon who has not made the experiment; on going into a darkifh Room, he can fcarce, at firft, diftinguifh any thing; when prefently, he will, after the Aperture in the Iris is open'd to a proper dimenfion, difcern Objects diftinctly; and again, on going into a full Light, he cannot bear to look on any illumined Object, till the Pupil is contracted; it is even painful to the Eye, at firft, going out of a dark Room into a ftrong Light, efpecially if the Sun fhines out.

All the back part of the Eye, within, is lined with a fine Membrane of a moft curious and delicate texture; called the Retina, from its refemblance of net-work. The large cavity, VV, which is bounded by the Retina, backward, and the Cryftalline Humour, before (which is contained within it) is filled with a glutinous Fluid like the white of an Egg; being perfectly pellucid and uncoloured, it is called the vitreous or glaffy Humour; in which the Rays converge, at E, the center of the Eye; and, croffing each other, in that point of contraction, they fall diverging on the Retina, and form the Images of Objects thereon; the fenfation, of which, is fuppofed to be communicated, by the Optick Nerve (which is connected with the Retina) to the Senforium, in the Brain.

As the manner in which the Rays of Light are refracted, in paffing through the various humours of the Eye, is entirely conjectural; feeing that, through a fmall pin-hole, applied clofe to the Eye, at q, there may be perfect Vifion, of an Object of any magnitude; in which Cafe, the whole Syftem of Rays fall on the Comea in one Point; how, then, are they refracted? As, this Effay is not intended as a Treatife on Optics, I fhall not infift on any thing; but, fhall only fuppofe the Vifual Rays (when the Eye is naked) to converge and crofs each other in the center of the Eye, at E (without confidering their Refractions) from whence, I fuppofe, they diverge in equal Angles, falling on the Retina, at *a* c *b*, in the back part of the Eye.

DEFINITIONS.

1. The AXE of the EYE is a Right Line paffing through the Center of the Eye and the Center of the Aperture or Pupil. As c E C.

The Point c, where the Axe cuts the Retina, is the place where Vifion is perfect; which point is varied as the Eye is moved, or its Axe directed to different Objects, or to different parts of an Object.

2. VISUAL RAY. If A B be fuppofed an Object of Sight; the Right Lines A E, B E &c. from all parts of the Object to the Eye, or to its Center (E) under which, or by the means of which, the Object is feen, or fuppofed to be feen, are called Vifual Rays.

3. OPTIC ANGLE. By Optic Angle may be underftood, either a plane or folid Angle.

If the Object A B be confidered as having no dimenfions but length, only; i. e. if it be a Right Line, fimply; the two Vifual Rays E A, E B, from the Eye to each extreme, form a Plane Angle, A E B; which is the Optic Angle, under which, the Line A B is feen. And, A E D or D E B is the Optic Angle of the Segment A D or D B.

4. CONE, or PYRAMID of RAYS. If the Object of Sight be either globular, or circular; as A B G; the Vifual Rays, E A, E B, &c. from the Eye to every part of the extremes of its Surface, being towards the Eye, form a Figure refembling a Cone, which is called the Optic Cone. Fig. 2.

But, when the Object is either a right lined Solid, or Plane Figure, as C D; the Vifual Rays E F, E C, E H, &c. form a folid Angle, compofed of feveral plane Angles, F E C, C E H, &c. which, being of a pyramidal form, is called a Pyramid of Rays.

The Vifual Rays croffing each other, in E, and, from thence, become diverging to the Retina, each oppofite plane Angle being equal, † there is formed a fimilar † 2. 1. El. Pyramid, c E d, which is called the oppofite Pyramid; and E is their common Vertex.

The Figure, a b or c d, which is the Bafe of the oppofite Cone or Pyramid of Rays, is the Image or Picture of the Object, A B or C D, on the Retina.

The correfponding Characters fhew in what manner the Image is inverted.

Let A B be fuppofed an Object, direct before the Eye, that is, perpendicular to its Axe (E C). It is fuppofed to be feen, by means of the Vifual Rays E A, E B, E C, &c. from every part or point in the Object, towards the Eye, whether it be Line, Surface, or Solid; which, from the point of contraction (E), proceed diverging, to the back part of the Eye, where they fall on the Retina, and form the Picture, *a* c *b*. of the Object A B.

It may be alledged, that it is impoffible for the Points A and B to be feen, at the fame time; feeing, the Rays, from thofe Points, do not enter the Pupil. I grant they cannot; nor F and G, diftinctly; nor indeed, little more than a Point, without moving the Eye; we have but a confufed intimation of the furrounding Objects, at a little diftance from that Point, to which the Axe of the Eye is directed; of which, every Perfon's own Experience muft foon convince him. But, the quick tranfition of the Eye, or the direction of its Axe, from one part of an Object to another, is fo little attended to, that, many imagine the field of View is much larger than it really is. As for the refraction of the Rays, through the various Humours of the Eye, I fhall leave for the fpeculation of thofe, who are blefs'd with a more fertile imagination than I have any pretenfion to.

Plate I. Fig. 1. The Image of the extreme point A, by means of the Viſual Ray A E, falls on the Retina at *a*, and the other extreme, B, is repreſented at *b*; and C E, the Axe of the Eye, conveys the Image of the point C to c, in the center of the Retina. From which it is evident, that the Picture of an Object, formed on the Retina, is neceſſarily inverted.

Nor is there any thing ſurprizing in this, as ſome may imagine. For, ſuppoſe A the top, and B, the bottom of the Object. It may be imagined, from the inverted poſition of the Image, *a* c *b*, that the Object muſt neceſſarilly appear upſide down. But, if we conſider, that the ſenſation of the Point A, being perceived or felt at *a*, is by the direction of the Viſual Ray A E, which determines its place in the Object; the Mind, by long experience, having acquired a habit of determining that part of an Obeject, perceived at *a*, to be above, and that at *b*, below; by the ſame reaſoning it is manifeſt, that the points of Object on the right hand are pictured on the left, and thoſe to the left, on the right; and which, is no matter of ſurprize at all; no more than, that, by the ſame experience, we have acquired an Idea of the diſtance of any Object perceived.

If a Perſon, who was born blind, could, when grown to maturity, be made to ſee, he would have no Idea at all of Diſtance, or the ſituations of Objects; which way was up, or which down; and would as ſoon attempt to lay hold of the moſt diſtant Object, which he perceived by the ſenſe of ſeeing, only, as others which were near him; it being impoſſible to diſtinguiſh, merely by Sight, whether the Point perceived at c, on the Retina, be at *C* or C, or at any diſtance beyond *C*, in the direction of E C. But, by Experience and a familiarity with Objects, according to their known magnitudes or diſtinctneſs of parts; or, according to the length of Ground, which we imagine to lie between us and the Objects, we judge them to be at ſuch or ſuch a Diſtance. Whereas, when we look at the Heavens, one Star appears as far off as an other; nor can we form the leaſt Idea of their diſtance from ſight, ſimply; for, a Star may as well be ten thouſand millions of Miles off as ten Miles, its Magnitude being in Proportion to its Diſtance. (See Fig. 7. A, B and C.)

The Art of Seeing is acquired, regularly and progreſſively, as all other Arts or Knowledge whatever. Do not we ſee a Child attempt to catch the Moon or other ſtriking Object, though at an immenſe Diſtance? but, growing up in a familiarity with them, by common and frequent experience, they become ſo intimately connected with our Ideas, that we form a judgement of their Magnitudes and Diſtances, inſtantaneouſly, at firſt ſight; and alſo of their Situations, in reſpect of themſelves and of each other.

It may, to ſome Perſons, appear ſtrange to call Seeing an Art; but that it is ſo is certain, although Nature has furniſhed us with that moſt wonderful Organ of Sight, the Eye; which, being perfect, is not poſſible to be open (with ſufficient Light for the purpoſe) but we immediately ſee what is before the Eye. A Child ſees as ſoon as it is born; but it ſees, without judgement, or diſcerning what it ſees. So would a Perſon born blind, and continued ſo, till he was mature in Judgment, in every other kind of Perception. Let us ſuppoſe ſuch a Perſon, immediately made to ſee. What Ideas he would have of what he merely ſaw, at firſt opening his Eyes 'tis impoſſible to ſay; but, of this I am well aſſured, that he could not diſtinguiſh the moſt familiar Objects, without Experience; as Figure and Colour are things of which he had formed no conception.

There are not many more oppoſite, regular Objects, than a Cube and a Sphere, or Globe; nor any, more eaſily diſtinguiſhed, by feeling; yet, even theſe could not, with certainty be known, merely by ſeeing them, which was one and which the other; becauſe, Experience had not taught, in what manner ſuch Objects affect the Eye. In reſpect of Colour, 'tis impoſſible he could have any Ideas, by which he could diſtinguiſh them; they would be, to him, as if never exiſting before. Of Diſtance, he could have no Conception; for being wholly unacquainted with the real Magnitudes of Objects and the effects which Diſtance has on them, their relative and apparent Magnitudes could not be diſcerned, ſo as to know that one was farther off than another.

This

This may appear strange, to those who have not considered it with attention; but I maintain it to be a truth, founded on sound reason. For, as he knows nothing of the difference between real Magnitudes, and the apparent Magnitudes of Objects, according to their several Distances; he could never imagine that a Man, or a Horse, which he saw at a Distance, was of the same kind as those he saw near him; till Experience had shewn, that the difference in apparent Magnitudes is, merely, the effect of Distance. So, of any other Object, as a House, for instance, into which, as he finds he can freely enter, being near, he could never imagine that to be a House, which by means of its Distance, appeared not so big as his Head. Consequently, since he could not imagine that a Man, or a House, at a Distance, was a Man or a House, of the same Magnitude as others which were near him, he could form no Idea of Distance, by sight; besides, the difference in Effect and Colour, occasioned thereby, are things entirely new; of which, he had not, till now, any Ideas.

Now, the judgement we form of Distance, &c. being acquired early, and grown up with us, is so little attended to, that at first thought of these things, we suppose it to be merely from Sight; than which, no Sense is more delusive and uncertian. The deceptions of Vision are many and frequent; as, when we look at Objects through refracting Mediums, i. e. through any transparent Medium whatever, solid or fluid; according to the different density of the Medium, or to its Figure, &c. we frequently imagine an Object to be nearer or farther off, larger or smaller than it really is; or to be where it really is not; nay, it is possible, by Light and Shade, judiciously disposed, and the assistance of Perspective, to deceive both the Eye and Judgment.

Of all the Senses, that of Sight is by much the noblest; it is, nevertheless, the most deceptive. He who can most deceive the Eye in the art of Delineating and Colouring is esteemed the greatest Genius. It is possible, to describe the representations of many Objects so very accurately on a plane Surface, that (as I have already observed) with the assistance of Colour, and a just distribution of Light and Shade, they would, in the true Point of view, deceive a judicious observer, and appoar to be real Objects. As a proof of the deception, in Vision, in respect of Magnitude, simply, I shall give a most striking instance; of which, though it has fallen within the cognizance of Thousands, few, or none, that I have conversed with on the subject, have formed a just Idea, or right judgment. I have asked several Persons, what were their Ideas of the apparent magnitude of the Moon? most have concluded, that it appears, when full, as large as the crown of their Hat; some, as a common Plate, or small Dish. I should be glad to know, by what standard such a judgment could be formed? for, unless some Distance, of the Object to be compared with, be determined, no comparison can be made; as, it may appear as large as the Dome of St. Paul's; but not, I am persuaded, at any distance we can see that Object, by the Light of that Luminary *. Now if we suppose the Plate laid on a Table, before us; or the Hat in one Hand, when we make the comparison, 'tis, perhaps, as random a supposition as can be conceived; and, without some determined Distance, it is vague and undeterminate. I know it will surprize many; but I do affirm, that, at the greatest distance we can hold any thing to compare with the Moon, it does not appear larger than a Silver Penny.

Now, if Seeing be not an Art, to be acquired by Experience, how comes it, that some Persons see with more judgment than others? Since as I am persuaded, that, all human Eyes (being perfect) see alike by Nature; and consequently, all would judge alike, of what they saw; but, it is certain we do not; of which, striking instances might be given. A Picture, or an Object, it is manifest, must strike every Eye alike, in the same Point of view; how is it then, that the most judicious and accurate representation, of a well-known Object, does not communicate, to every beholder, the same Idea of the Original?

* There was a Moon-Light Piece (by a capital Artist) in the Strand Exhibition, last, in which (if I remember well) the Moon appeared near that Object, and almost of equal Magnitude.

It would ſcarce gain credit, to affirm, that, on aſking a Perſon, who was admiring a large, and well-executed, Print of St. Paul's Cathedral (one who had lived, ſome Years, near that well-known Object) what it repreſented? When, after a ſtrict attention to it, I was anſwered, with an Interrogation, is it Weſtminſter Bridge? 'Tis true, it was a Female; but, ſhe had Eyes, and could diſtinguiſh between a Cow and a Horſe, at firſt ſight, as well as any other Perſon.

It is, to me, almoſt impoſſible to account for this; as I have known many Children, of thre or four Years old, ſpeak with certainty of very paltry Repreſentations. I think it can ariſe, only, from their inattention to the Objects they ſee; as it is impoſſible, that a Child can have had more Experience than a Perſon grown to maturity. Nevertheleſs, it is certain, that ſome have much more Judgment of what they ſee, with leſs Experience; or, at leaſt, they acquire it much leſs time.

That Viſion is conveyed in Right Lines to the Eye, I preſume no perſon will attempt to diſpute (I would be underſtood to mean, of ſuch Objects as are ſituate on or near the Earth; and when no other Medium, but Air, is between the Object and the Eye). I ſhall, therefore, give it as a general Axiom; which, nevertheleſs, may be illuſtrated or ocularly demonſtrated, by ſtraining a fine ſilken thread, to a Right Line, from any Point, as *F*, of an Object *(DF)* to the Eye.

Then, if any other Object, as Q R, be interpoſed, as ſoon as it touches the Thread E *F*, they will appear to be in contact; and if it be ſo interpoſed, as to hide the Point *F*, the Right Line or Thread, which may repreſent a Viſual Ray, will be refracted, or broken into two Right Lines E R, R *F* forming an Angle, ER*F*.

Hence it is evident, that the Points F and *F*; alſo, the Point C or *C* being in the ſame Right Line with *C*2, will perfectly coincide with each other, to the Eye, at E.

SECTION III.

Containing a brief THEORY of DIRECT VISION.

THEOREM I.

OBJECTS appear to have that Proportion to each other, reſpectively, as the Angles under which they are ſeen.

Let *a* c *b* repreſent a ſection of the Retina, a portion of a concave Sphere; and, let A B be an object direct before the Eye; divided, in any Ratio, in D and F.

Fig. 1. DEM. It is evident that the Object A B, by means of the Viſual Rays E A, E B, is ſeen under the Angle AEB; and the ſeveral Parts of A B, as A F, F D, and D B, are ſeen under the ſeveral Angles A E F, &c.

But, the oppoſite Angles *a* E f, f E d and d E *b*, are, reſpectively equal to the Angles A E F, F E D, and D E B — — — 2. 1. El.

And, the Images *a* f, f d and d *b*, on the Retina, of the Originals, A F, F D, and D B, are the meaſures of thoſe Angles, reſpectively (Th. P. Ang.*) the Retina being ſuppoſed a portion of a Sphere; each part is, therefore, equally diſtant from the Center.

But, the Ark *a* f is to the Angle *a* E f, as the Ark f d is to the Angle f E d, and as d *b* to dE*b*. — — — — — 19. 6. El.

* (Th. P. Ang.) refers to the Theory of Plane Angles, in my Treatiſe of Geometry.

I Therefore,

Therefore, the Images or Pictures on the Retina, and confequently, the apparent magnitudes of Objects are in the fame ratio, or proportion, as the Angles they fubtend at the Eye. Q. E D.

Nor is there occafion to have recourfe to the Images on the Retina; but, allowing the Vifual Rays E A, E B, &c. to be Right Lines, and an ark of a Circle, of any radius, being drawn, cuting thofe Rays; the portions of the Ark, l m n o, intercepted between the Vifual Rays, as l m, m n, &c. meafure the feveral Angles A E F, F E D, &c. refpectively; and confequently, the parts A F, F D, &c. of the Object A B, will appear to the Eye, at E, in proportion to the Arks l m, m n, &c. which are equally diftant from the Eye.

All remote Objects appear equally diftant from the Eye; as the Planets, Stars, &c. appear as if they were all in the fame concave Sphere; of which every Eye is the Center. So likewife, when we ftand in the line of direction of two, three, or more Objects, they appear to touch each other; nor could we judge, merely from fight, which was fartheft off, or which the largeft.

For, fuppofe the Vifual Rays E F, E D, E B, produced to *F*, *D* and *B*; any other Objects, as *D F* and *B D*, being in the fame direction with F D, and D B, and touching the fame Vifual Rays (E F, E B) will coincide with F D and D B; the lefs will hide the greater from Sight, and appear to be of equal magnitude.

2. I am fomewhat furprized at what Mr. Hamilton has advanced, in Art. 6, Sect. 1. where he fuppofes, that Objects do not appear in proportion to the Angles they fubtend, but in proportion to the Tangent of half the Angle; and imagines the Retina to be, nearly, a Plane, in the center of the feat of Vifion.

How Mr. Hamilton could conceive and endeavour to account for fuch a ftrange and unwarrantable Opinion I cannot imagine. If, as he has truly afferted, the fpace on the Retina, fo far as there is diftinct Vifion, does not exceed an Angle of one or two Degrees, on every fide of the Axe; which is as much as to fay, it does not exceed an Angle of two or four degrees; and I am fully convinced, that it does not exceed one Degree; what difference can it make, whether that fmall portion of the Retina be a Plane or a portion of a Sphere? he certainly could not fuppofe a large portion of the Retina to affume that Form.

Now, if the Retina does affume the form of a Plane, of the fpace *a b*, then it is certain that the Images, *a f*, *f d*, and *d b*, on that Plane, (of which, the Right Line *a b* may be fuppofed a Section) are not in proportion to the Angles A E F, F E D, and D E B, equal *a* E *f*, *f* E *d*, and *d* E *b*, refpectively.

DEM. For, the Tangent c f or c d is not in proportion to c a or c b as the Angle c E f is to c E a; becaufe, c f is equal f a (AF being equal FC) therefore, the Angle c E f is not equal f E a, or c E a double c E f. — — 3. 6. El.

But, *a b* is parallel to a b; therefore, *a f*, *f c* and *c b* are in the fame Ratio, as a f, f c and c b. — — — — — Cor. to 6. 6. El.

Thus it is clear, if the Retina does take the form of a Plane, that Objects do not appear in proportion to the Angles they fubtend; but, as it is the only inftance in which I ever knew it contefted, fo, I am perfuaded, it will never be univerfally adopted, on that Hypothefis. However, as it is merely conjectural, it feems to me more rational, to fuppofe the Retina to retain its original fpherical Form, which the all-wife Creator has given it, than, that it fhould, continually, at every motion of the Eye, be changing to that of a Plane.

Wherefore, on the prefumption that the internal Eye is truely fpherical, it is very reafonable to fuppofe, that, Objects appear, exactly, in proportion to the Angles under which they are feen; and not in proportion to the Tangent of half the Angle, as Mr. Hamilton afferts, and has endeavoured to demonftrate, in 7 and 8 of the fame Section: which cannot be as himfelf has ftated it, but on the fuppofition of the Retina affuming the form of a Plane, which is abfurd to fuppofe. Or

Plate I. Fig. 1. granting it really fo, the portion of the Retina, which the compafs of an Angle of one or two Degrees, at moft, takes up, is fo very fmall, that it is impoffible to diftinguifh it, in a Sphere of an inch in Diameter, from a Plane. As the 360th part of the circumference of a Circle, of a fmall radius, is not diftinguifhable from a Right Line; confequently, there could be no difference in the apparent magnitude of an Object, in either Cafe, which is feen under fo fmall an Angle. As, the fmall portion at d, on the Retina, fubtending an Angle, at E, of about five Degrees, is not diftinguifhable from a Right Line.

3. It is alfo evident, if the Vifual Rays are refracted in paffing through the aqueous Humour, before they enter the Pupil (unlefs that Refraction be rectified again at the Cryftalline Humour or elfewhere, and, by that means, proceed in the fame direction to the point of convergency, in the center of the Eye) that Objects cannot appear exactly in proportion to the Angles they fubtend. Nor have I any conception how a Vifual Ray, AE, falling remote on the Cornea, can be refracted fo as to pafs in the fame direction through the Aperture, h i, to the point of convergency, at E, and from thence to the Retina.

For, fuppofe a Vifual Ray, A *e*, from any Point A (feen very oblique) falls perpendicularly on the Cornea, at *e*; if it pafs at all through the Pupil, h i, whilft the Axe is directed to C, it muft pafs in the direction *eg*; where it falls on the Cryftalline, at *g*, and where, it is poffible it may again be refracted, in the direction *gh*, to the Retina; making an equal Angle with the Axe, (E C) as the original Ray, A e; in which Cafe, the Image of the Point A is feen at *h*, in its proper pofition.

But, it is manifeft, that, whilft the Axe remains in the direction E C, the point A cannot be feen, at all, nor even the point F, diftinctly; the intimation we have of Objects, at a lefs diftance from C, is fo very confufed, that, without moving the Eye, we fcarce perceive their exiftence; of which, every Perfon's own experience will foon convince him. Unlefs the Axe be directed to any Point (as E) no perfect Image of that Point is formed in the Mind, or on the Retina. Confequently, the Rays go direct through the various humours of the Eye; as when we look through a Pinhole; for, how can it be otherwife? feeing, it is manifeft they muft all pafs through the fame Point; (q is then their common point of convergency) the Image formed on the Retina is, in every refpect, the fame (except in Hue, for want of fufficient Light) and of the fame Magnitude as it appears to the naked Eye. Which fingle circumftance explodes, in my opinion, the notion of Refraction at the Cornea and through the various Humours, in the paffage of the Rays to the Retina. For it is manifeft, that, if the Point of convergency be at q, without the Eye, the Image will occupy more fpace on the Retina than at E, or any other Point within the Eye. I fhall therefore fuppofe, that, when the Eye is naked, the Point of convergency of the Rays is in its Center; and, on that fuppofition, do infer, that Objects do appear in proportion to the Angles under which they are feen. Of which I will give an eafy and familiar Example.

Fig. 3. 4. Let AB, CD, and FG be three Globes, at different Diftances from the Eye, at E, and of different Magnitudes; feen under the Angles AEB, CED, and FEG refpectively *.

Now, if the Diftance of the Globe CD be equal to twice the Diftance of AB, and FG be double of CD, four times the Diftance of AB; then will the Angles AEB, CED, and FEG, which they fubtend at the Eye, if the Diameters of the Globes are in the fame proportion as their Diftances, be equal; and confequently, their apparent Magnitudes, to the Eye, at E, are equal.

Draw E S, to the Center of the Globe F G; draw C *D* perpendicular to E S, and equal to the diameter of the Globe CD; alfo, draw B *A* perpendicular to E S, and equal to the Diameter of the Globe AB.

* The full Diameter (FG) of a Globe cannot be feen; for, the Vifual Rays being Tangents to the Globe, from the fame Point E, as E*f*, E*g*, are all equal (Cor. 2. 16. 3. El.) and, by reafon of the convexity, towards the Eye, they cannot touch the two extremes of the fame Diameter; unlefs the Diftance be infinite; and then, the Vifual Rays are confidered as being parallel; *fg* is, therefore, the apparent Diameter of the Globe FG.

DEM.

Dem. It is evident, that the Visual Rays F E and G E, from each extreme of the Diameter of the Globe F G, will pass through the extremes of C *D* and B *A*.
For, in the similar Triangles FEG, *D*EC and BE*A*; FG : SE :: *D*C (equal CD) : KE, and, as B *A* (equal A B) : L E. — — 6. 6. El.
Then, since they touch the same Visual Rays F E and G E ; consequently, they are all seen under the same Angle, F E G. — — by Theorem
And, the Diameters, A B, C D and F G, being in proportion to their central Distances, are, therefore, all seen under equal Angles, AEB, CED and FEG.
Consequently, the Images, a b, b c, and c d, on the Retina, subtending equal Angles, are equal ; and therefore, the appearances of the three Globes A B, C D, and F G are equal. Q.E.D.

If the Globe C D be supposed to be moved to M, its central Distance, E M, being equal E S ; and, being, in Diameter, equal half F G ; it will subtend an Angle G E H, i. e. C E I, half C E D, equal FEG ; as the Arks 1 2 3 4, which measure those Angles, exemplify.

Consequently, the Image, (h c) of the Globle G H, at that Distance, is, in magnitude, but one fourth part of the Image (c d) of the Globe C D, of the same Magnitude, and seen at half the Distance ; for, they are in the duplicate ratio of the portions of the Ark 3 0 to 3 4. — — Cor. 1. 13. 6. El.

5. Yet it is manifest, that two or more unequal Objects, at an equal Distance from the Eye, and seen direct, do not appear exactly of the proportion of their respective Magnitudes ; likewise, equal parts of the same Object, on the same side of that Point where it is cut by the Axe of the Eye, do not appear equal.

Let AB be an Object, of length simply, bisected by the Axe EC, to which it is perpendicular, in the Point C ; and suppose A C and C B again bisected, in F and G. Fig. 1.

Then, the parts AF, F C, &c. are equal, and AB is double F G ; but, they do not appear of that proportion, at E.

Dem. Now because A F is equal to F C, the Angles A E F, F E C, under which they are seen, are unequal — — — — 3. 6. El.
For, if the Angle A E C be bisected, the Right Line E O, bisecting it, will cut A C, in the Point O, in the ratio of E A to E C.
But, E A is greater than EC ; for, E C is perpendicular to A C.—Cor. to 12. 1. El.
Wherefore, A O is greater than O C, and the Angles A E O, O E C, under which they are seen, are equal ; consequently, the equal parts A F, F C, are seen under uneqnal Angles ; therefore, they appear unequal, by the Theorem.
And, because the Angle C E F is greater than F E A, the equal part C F will appear greater than F A,
After the same manner it may be proved, that the equal part C G appears greater than G B. Consequently, C G + C F, i. e. F G, appears greater than half A B. Q. E. D.

Cor. Hence, it is evident, that if A B be produced, and the equal divisions, C G, G B, B P are continued, they will appear continually less, being seen under less Angles. For the Angle B E P is less than B E G, which, is less than G E C ; as the measures, p o, o r, and r *C*, of those Angles, sufficiently evince.

N. B. The farther any two or more Objects are from the Eye, the nearer they will appear in the proportion of their respective Magnitudes.
Because they are seen under less Angles ; and, the Tangents of small Angles d viate less from the ratio of the Angles, than the Tangents of greater Angles.

6. As Objects of different Magnitudes, at equal Distances, do not appear in the proportion of their respective Magnitudes, so neither do Objects of equal Magnitude, seen at different Distances, appear exactly in the ratio of their several Distances.

Let

Plate I. Fig. 4. Let A B be an Object at any Diſtance (E F) from the Eye, at E; and, let C D be another Object, parallel to A B, of equal Magnitude, as to length ſimply; alſo let C D be at twice the diſtance of A B (E G double E F). And let them be ſo ſituated, that, right lines, A C and B D, joining their extremes, are parallel to E G i. e. perpendicular to A B and C D.

By means of the Viſual Rays E A, E B, &c. thoſe Objects are ſeen under the Angles A E B and C E D. I ſay, the Angle A E B is not double of the Angle C E D.

Dem. Becauſe A B is parallel to C D, E F : E G :: E c : E C, as c F : C G. 6. 6. El.

But E F is equal to ½ E G (Hyp.) wherefore, c F = ½ C G, equal A F; that is, A F is biſected, in the point c.

Now becauſe A c is equal to c F, the Angle A E c is leſs than c E F 3. 6. El. for, if they were equal; A c would be to c F as E A to E F.

But, E F is perpend. to A B, therefore, E F is leſs than E A. Cor. 3. 12. 1. El.

But A c and c F appear (to the Eye at E) in proportion to the Angle under which they are ſeen; i. e. as A E c to c E F; (by Theo.) and c F appears equal to C G, being ſeen under the ſame Angle; conſequently C G appears greater than half A F.

By the ſame reaſoning, G D appears greater than half F B.

Therefore, C D appears greater than half A B; i. e. the Angle C E D is greater than half A E B; although C D is equal to A B, and the diſtance double.

Let I K be another Object, equal and parallel to A B. and at three times the Diſtance, from the Eye, at E.
I ſay, that I K will appear greater than one third of A B.

Produce E G to H, and I K to L, making K L equal to H K; and draw E K and E L. Then will A F be triſected, in k and l.

Dem. Becauſe A F is parallel H K, k F : K H :: E F : E H } — 6. 6. El.
And, becauſe K L is equal to K H, k l is equal to k F. }

But, E F is equal to one third of E H; conſequently, F k and k l are, each, equal to one third of A F, equal K H.

Then, becauſe F k, k l, and l A are equal, the Angles F E k, k E l, and l E A are unequal; for, if the Angle F E k, was equal to k E l, F k would be to k l as E F to E l. — — — 3. 6. El.

But E F is perpendicular to A B; wherefore E F is leſs than E k; conſequently, F k would be leſs than k l.
And, for the ſame reaſon, if the Angles k E l and l E A were equal, k l would be leſs than l A; becauſe E k is leſs than E A.

But F k, k l, and l A are equal; therefore, the Angle F E k is greater than k E l, and k E l than l E A.
Therefore, F k, i. e. H K, appears greater than one third of A F.

By the ſame reaſoning, it is manifeſt, that H I appears greater than one third of F B, equal H I;
Therefore, I K, appears greater than one third of A B. Q. E. D.

Thus, I have made it appear, to Demonſtration, that Objects do not appear in that proportion to each other, as their reſpective Diſtances; in which I have been more particular, becauſe, Mr. Kirby has expreſly ſaid, Page 13, that theſe Pictures (i. e. the appearances of Objects) will be to each other, as their ſeveral Diſtances are to each other*. Rohault ſays, nearly ſo; Page 243.

* The Appearances, i. e. the Angles under which Objects are ſeen, being in proportion to their Diſtances is, literally, abſurd; becauſe, as the diſtance of an Object is increaſed, the Angle, under which it is ſeen, decreaſes; which kind of reciprocal Proportion is always to be underſtood in this Caſe.

7. If the Eye be moved from E to O, making O F equal to F H, and O I, O K be drawn, E B will be cut in the fame Points, c and d, as by the Vifual Rays E C, E D.

But, I K will not appear of the fame proportion to A B, at the Point O, as C D, at E, the Objects A B, C D, and I K being equal.

Dem. Becaufe A B, C D, and I K are parallel, the Triangles E c F, E C G, alfo, O c F, and O K H are fimilar; wherefore E F : E G : : c F : C G, and, O F : O H : : c F : K H. — — — 4. 6. El.
But, E F is equal to half E G, and O F is equal to half O H;
confequently, c F is equal to half C G, and alfo to half H K; equal C G.
By the fame reafoning, F d may be proved equal to half G D or H I.
Therefore, A B is cut in the fame Points c and d; as it was affirmed.

2. C D appears to the Eye, at E, in proportion to A B, as the Angle C E D to A E B; and I K appears, at O, as the Angle I O K to A O B.

Now, the ratio of the Angle C E G to A E G is greater than K O H to A O H.
For the Angle A E G is greater than A O F, equal C E G;
wherefore, the ratio of c E F to A E F is greater than c O F to A O F.
For, let E m be drawn parallel to O K; the Angle F E m is equal F O c
confequently, the ratio of F E m to F E c is equal to that, of F O c to F O A, equal F E c.
But F c is equal to c A (proved) wherefore, F m is equal to m c.
And, equal Parts of a Right Line fubtend lefs Angles, the farther they are from a perpendicular to the Line, from the Eye; proved.
Therefore, the Angle F E m to F E c, i. e. F O K to F O A, has a lefs ratio than F E c to F E A, i. e. than G E C to G E A, being nearer equality.
Confequently, H K appears lefs, in proportion to A F, at O, than C D at E.

After the fame manner, H I may be proved to appear lefs, in proportion to F B, at O, than G D at E.
Therefore, I K appears lefs, in proportion to A B, at O, than C D at E. Q.E.D.

Hence it is manifeft, that, notwithftanding the fections of the Vifual Rays E C and O K, E D and O I, with A B, are the fame; the Diftance of the Eye on one fide of A B, and the Object on the other being proportional; yet, the appearances of the Objects, to each other, are varied, confiderably, as the Diftance is greater or lefs.

For, two Objects of equal height, at a great Diftance from each other, and parallel between themfelves, and the Eye at the fame Diftance from one of them, and in a Right Line with both, will appear, in proportion to each other, nearly as their Diftances; i. e. the fartheft off will appear half the height and half the width of the neareft, being at twice the Diftance, and, one third in height and width at three times the Diftance, nearly, i. e. fomewhat more.

As I K appears, at O, fomewhat more than half A B. But, C D (equal I K) appears much greater than half A B, at E. Confequently, if the Eye be at a greater diftance from A B, than at O, and the Object at an equal diftance on the other fide, they would appear nearer in proportion to their Diftances, reciprocally; and therefore, when the diftance of both is fo great, that the Vifual Rays, O I, O K, approach nearly to parallelifm (which, it is obvious muft be infinite, before they can be perfectly fo) they will appear nearly as their Diftances. Notwithftanding, on a Picture parallel to the Objects, they muft ever be reprefented in the ratio of their refpective Diftances, and Magnitudes.

THEOREM II.

Parallel Right Lines, however ſituated, being produced, appear to approach towards each other; and, if produced infinitely, they will appear to meet in a Point, at an infinite Diſtance *.

Plate I. Fig. 5. Let I F and H G be two parallel Lines; and, E, an Eye, ſituated any how between them. Let K, B, D, and G repreſent various Diſtances in the Line H G, and let the Lines I K, A B, &c. be drawn perpendicular to H G, cuting the other Line (alſo perpendicularly) in the points I, A, C, and F. Alſo, let the Viſual Rays E A, E B, &c. be drawn.

DEM. The ſpace between the Lines, I F and K G, at the ſeveral Diſtances E S, E O, &c. appear, to the Eye at E, under the ſeveral Angles I E K, A E B, &c. each of which, as the Diſtance is farther from the Eye, is leſs. C. 14. 1. El. For, the Space between the parallel Lines is, every where, equal. Def. 7. Geo.

The Space, at I K, is ſeen under the Angle I E K; and the ſpace between them, at A B, under the Angle A E B; which is leſs than IE K, nearly in proportion to the Diſtances, E S to E O; C D ſubtends the Angle C E D, and F G the Angle F E G; each of which is leſs than the former; for their Subtenſes, A B, C D, &c. are equal; and the Lines E A, E B; E C, E D, &c. forming or containing the Angle, are continually longer; wherefore, the Angle F E G is leſs than C E D, which is leſs than A E B †, &c. — Cor. to 14. 1. El.

From which it is manifeſt, that at a greater Diſtance the Angle will be ſtill leſs; conſequently, it will, at laſt, become inſenſible, the interval, between the Lines, is loſt to Sight, and, the Lines appear to meet each other. Q. E. D.

Hence, the Lines, I F and H G, are ſaid to vaniſh; and conſequently, the ſpace between them vaniſhes alſo.

2. Let E R be drawn, from the Eye, parallel to I F and H G; thoſe Lines will appear to approach, continually, towards E R; and being produced, infinitely, they will all appear to meet in the ſame Point; however E R may be nearer to one than the other; for, the farther any parallel Line is from E R (which may be called its Radial) the more ſudden is its apparent approach to that Radial,

The Space I A, of the one, appears to advance from *h* to *i*; an equal Space K B of the other, advances from *p* to *o*; which is leſs than the other, in proportion to the Angle K E B to I E A; i. e. as the Ark *po* to *hi*. The Space from A to C appears to advance from *i* to *k*; an equal Space, B D, in the other, from *o* to *n*. From C to F it advances from *k* to *l*, an equal Space, DG, from *n* to *m*; in the proportion of the Arks *kl* to *mn*.

It is evident, that if thoſe Lines were produced longer, they would ſtill appear to approach nearer to the point e, as E L, E M, evince. And, being infinitely produced, they would at laſt appear to terminate in the Point e, and be loſt to ſight.

For, it is manifeſt, that the farther any Points, L and M (conſidered as being in the parallel Lines I F and G H) are diſtant, the nearer the Viſual Rays, E L, E M, are to a coincidence with E R; which, it is evident, muſt be infinite before they can coincide perfectly; in which caſe, E L, E M, and E R will be the ſame; for they will all appear to unite.

* It is ſaid in Smith's Optics (Vol. I. Art. 156, Page 58) that parallel Lines, ſeen obliquely, appear to converge more and more as they are farther extended from the Eye; which, will ever be the caſe however the Eye is ſituated, in reſpect of the parallel Lines. For, being ſeen obliquely, means nothing more than when we look towards either of their extremes; as they are ſeen direct, only where a Line, from the Eye, cuts them both at Right Angles; on either ſide of the Perpendicular they are neceſſarily ſeen oblique.

† When the Angle, AEB or CED, under which any Object, as AB, is ſeen, does not exceed 20 or 30 Degrees, at the moſt, and that Object is removed to the ſeveral Diſtances, from E O, to E P, E Q &c. the difference between the Tangents O A, O C, &c. deviates but little from the Arks O C *q*, O C, &c. and that ſtill leſs as the Angle is leſs; as P C, P *F* 2, &c.

3. Hence

3. Hence, it is easy to account for the appearance of a horizontal Plane, or continued level Surface; which, being below the Eye, will gradually appear to ascend; and, being above the Eye, it appears to descend; and, if they were produced infinitely, they would appear to meet in a Right Line, on a level with the Eye.

Let the Right Line H G be a section of a horizontal Plane, below the Eye, at E; and let I F be a section of one above the Eye. Also, let I K be a section of a Plane direct before the Eye * (at E) perpendicular to the other two.

Now if the Visual Rays, E A, E B, E C, &c. be drawn, they must necessarily cut the Plane of which I K is a Section; in a, b, c, &c.

It is evident that the Space KB will appear to rise, towards S, on the Plane I K, from K, its intersection, to b, where the Visual Ray E B cuts that Plane; the space K D will rise to d, and KG to g.

For the same reason, the space I A will appear to descend from I to a; I C, from I to c; and I F, from I to f; each Plane appearing gradually to approach towards S, where a Perpendicular, E S, to that Plane, cuts it.

And, if the Planes were produced infinitely, they would at last vanish, or be lost to sight, in a Right Line passing through the Point S, parallel to the intersection of the other two Planes, with the Plane I K.

And this is evidently the case however a Plane be situated; whether horizontal, vertical, or inclined, it is the same; for, a Plane is still a Plane, in all positions, and has no properties peculiar to any position, in respect of the Horizon.

Therefore, there may be drawn this Conclusion, that every Plane, in which the Eye is not situate, will appear to approach towards, and at length to meet, another Plane, passing through the Eye, parallel to the former.

4. Let A B, C D, and F G represent three Objects of equal height, at the several Distances, E O, E P, and E Q, from the Eye, at E.

The Visual Rays E A, E B, E C, &c. being drawn, those Objects will appear in proportion to the Angles A E B, C E D, and F E G.

For, the portions of an ark of a Circle, whose Center is E, intercepted between the Visual Rays, measure those Angles, respectively, and determine the apparent proportions of the Objects A B, C D, and F G to each other; seeing that, the Angles which they subtend are in the same Ratio. — — 19. 6. El.

Wherefore, since, in Perspective, the Representation is always on a Plane, and not on a spherical Surface; suppose I K a section of a Plane, in a direct position; the Spaces, a b, c d, and f g, between the Points where the Visual Rays cut and pass through that Plane, considered as a Picture, are the proportions of the Representations of the Objects, A B, C D, and F G; but their apparent Magnitudes are the portions *i o*, *k n*, and *l m* of the Ark *h e p*.

Let H G represent a section of the level Ground, or any other horizontal Plane; and A B, C D, and F G, Objects perpendicular thereto.

If I K be supposed a section of a vertical Plane, the foot or seat B, of the Object A B, will appear, on that Plane, at b; and consequently, K b will represent the space of Ground, K B, between the Picture and the Object A B; i. e. it will appear to rise so high on the Picture; K d will represent the Space of K D, and K g of K G. So likewise, a, c, and f shew how much the tops of those Objects appear to descend, on the Picture.

5. Now, if the position of the Picture be inclined to the Horizon, as *a* K, the Objects and the Eye remaining as before; the Representations *a b*, *c d*, and *f g*, on that Plane, are very different from the proportions of ab, cd, &c. on the Plane I K.

* By being direct before the Eye, I would be understood to mean, in a vertical Position; and when a Right Line, from the Eye, falling in the middle, between its Bounds, cuts the Plane perpendicularly.

For, if the Eye be not in the Plane, or in a continuation of it, the Eye, being considered as a Point, can have but one position, either to a Plane or to a Right Line, being produced; however the Plane may be situated, in respect of the Horizon.

But,

Plate I. Fig. 5. But, the apparent proportions, of both, are the ſame, at E; *viz.* the portions on the Ark *bep*, which meaſure the Angles AEB, CED, and FEG.

Nor, is it poſſible to be otherwiſe; ſeeing that, the Objects, AB, CD, &c. remain the ſame, in reſpect of each other; and the ſituation of the Eye, is not varied; conſequently, the Angles AEB, CED, &c. are not altered, and the Objects AB, CD, &c. the Subtenſes of thoſe Angles, muſt, neceſſarily, appear the ſame, repreſented on any Surface whatever, and in any Poſition, cuting the Viſual Rays EA, EB, &c.

N. B. The Proportions *CD*, and *FG*, &c. on AB, are the ſame, as cd and fg on IK (AB being parallel to IK); wherefore, the Triangles AEB, aEb, alſo CED, *CED*, and cEd, &c. are ſimilar; and therefore, as fg : cd, or to ab, :: *FG* : *CD*, or to AB. — — 4. 6. El.

Conſequently, Viſual Rays, being cut by parallel Planes, in any poſition, are not only themſelves cut in equal Ratio, but alſo the Sections, or the Projections on the Planes, by their ſections with the Rays. — — — — — — 1. 8. El.

And, their Proportions are as their Diſtances ES, EO, EP; i. e. as fg : ES :: *FG* : EO as *F2 G2* : EP, and, as FG : EQ; (6. 6. El.) and ſo of any other.

Hence, it is manifeſt, that the Sections of any ſyſtem of Rays, by parallel Planes, are ſimilar.

PROBLEM.

Fig. 6. A Right Line (AB) being obliquely ſituated in reſpect of the Eye, at E, to determine the Point D, to which if the Axe of the Eye be directed, the two Parts, AD and DB, ſhall be ſeen under equal Angles.

Without drawing the Viſual Rays, or biſecting the Angle made by the Rays.

Make AD to AB, as the diſtance of the Point A, from the Eye (E) is to the diſtance of the ſame Point A, added to the diſtance of B. — Pr. 32. El.
Then, AD : AB :: EA : EA + EB; and, AD : DB :: EA : EB.

Dem. The Viſual Rays EA, ED, EB being drawn, the Angle AEB is biſected by ED; wherefore, AD and DB are ſeen under equal Angles, at the Point E, For, if the Optic Angle AEB be biſected by the Right Line ED; then is AD : DB :: EA : EB. — — — 3. 6. El.

2. If the Axe of the Eye be directed to C, the middle Point of AB, then will the Point B be more diſtinctly ſeen; becauſe its Image falls nearer to the Image of C than the Point A. And, if the Ark *cf* be made equal *ca*, and E*f* be drawn, meeting AB produced, in F; AC will then be to CF as EA to EF; and, notwithſtanding the difference between AC and CF, their Images on the Retina, and conſequently their apparent Proportions, are equal, being ſeen under equal Angles.

Now, if thoſe Viſual Rays are cut by a Plane in the poſition IK; the part ac, the greater portion of af, will repreſent AC, the leſſer Segment of AF; and cf, the leſſer portion, repreſents CF, the greater Segment;
For, ſince the Optic Angle, AEF, is biſected, by EC; ac : cf :: Ea : Ef; 3. 6. alſo, as AC : CF :: EA : EF; conſequently, ac, cf repreſent AC and CF.

3. If the Right Line AB be produced, at the extreme A, cuting the Plane IK, at I; the whole indefinite repreſentation of that Line, on the Plane IK, is the ſection IK, of a Plane paſſing through the Line AB and the Eye, and terminated by a parallel Line from the Eye (EK) to that Plane.

For, becauſe EK is parallel to AB, they will appear to meet in a Point at an infinite Diſtance; † which is repreſented by the Point K, on the Plane IK; for, ſeeing that the Line EK paſſes through the Eye, its whole appearance is but a Point; conſequently, the Point K repreſents the whole Line EK, produced infinitely; and AB, infinitely produced, will appear to meet EK in that Point.

† Theo. 2.

If the Line A B be produced on the other Side of the Plane I K, and any Point (G) be aſſumed, between the Eye and the Plane; its repreſentation, on that Plane, is the Point g, in which the Right Line E G, produced, cuts K I produced; the part I g, beyond the Interſection, I, repreſents I G, the portion of the Line, A B, lying on the ſide of the Plane, I K, towards the Eye; as I a or I b repreſents the part I A or I B, on the other Side; all which is ſo very obvious, from inſpection of the Diagram, it is needleſs to ſay more about it.

4. After the ſame manner as the apparent magnitudes of Objects are determined, ſo are their apparent Diſtances, or Bearings, in reſpect of each other; *viz.* by the Degrees on the Ark of a Circle, which meaſure the Angles they make at the Eye.

Let C, D, and F, be three Objects, and E the place of the Eye. Fig. 7.
Draw the Viſual Rays C E, D E, F E; then, the Angles D E C, C E F are the Optic Angles of their apparent Diſtances, or Bearings at the Station E.
Notwithſtanding their real Diſtances from each other, C F is nearly double that of C D, yet, the Angle C E F, is much leſs than the Angle D E C; as it is evident, from the portions, c f and c d, of the Ark d f.
If *f d* be ſuppoſed the ſection of a Plane, the appearances, or places of thoſe Objects thereon, are at *f*, *c*, and *d*.

5. Suppoſe C, D, and F to be Stars, in the unbounded Expanſe of the Heavens, at an immenſe diſtance from each other and from the Eye, at E.
It is impoſſible to form an Idea of their real Diſtances or Situations, in reſpect of each other; for, if the Star C was at B, and F at G, or any where elſe in the direction E G, their apparent places are ſtill at c and f, in an imaginary Arch of the Heavens, as it appears to us, equally diſtant in every part.
Now, ſince the whole Diameter of the Earth's Orbit is not ſufficient to make any ſenſible difference in their Bearings, and, conſequently, of their apparent Places, in the Arch d f, there cannot be any poſitive Idea formed of their real Diſtances; for the portions f c and c d, of the Arch f d, are the meaſures of their apparent Diſtances, only; i. e. of the Angles d E c, c E f, or the Originals of theſe Angles D E C, C E F under which they are ſeen.

Hence is conſtructed the Celeſtial Globe or Sphere, which is a true Picture of the Heavens, of Stars, &c. divided into various Figures called Conſtellations.

If d c f be ſuppoſed a portion of a Sphere; or, let H I be an entire Globe.
It is evident, that from its Center, E, which is alſo the Center of the Arch d f (E being ſuppoſed the Eye of a Spectator) the Star C will appear on the Globe at o, and D at s, &c. in the ſame poſition, ſituation, and diſtance, in proportion to the Radius, E H or E o, to E c, as in the imaginary Arch of the Heavens, d f. So that, whether the real Star be at A, B, or C, or any where elſe in the Right Line E C; its apparent or repreſentative Place, on the Globe H I, will ſtill be at o; and conſequently, can make no difference, in its true Place thereon; but its apparent Magnitude will be in proportion to its Diſtance, nearly.

6. Let AB, DF, and GH be three Objects, promiſcuouſly ſituated in reſpect of Fig. 8. the Eye, at E. It is manifeſt, that, whilſt the Eye remains at E, the three Objects muſt neceſſarily appear the ſame, upon any Surface whatever, cuting the Viſual Rays EA, EB, &c.
Suppoſe a h a ſection of the Rays, made by a Plane, and E C a Perpendicular from the Eye to the Plane, cuting it at C. Then, the Viſual Rays E A, E B, which are neareſt to the Point C, cut that Plane leſs oblique than E F; and E G, E H are ſtill more oblique, as they are more remote from C. Whereas, on a ſpherical Surface of which a *d f b* may be ſuppoſed a Section, being an ark of a Circle, deſcribed on the center E, the Rays EA, EF, EH, &c. are all perpendicular

Plate I. Fig. 8. dicular to its Surface, being perpendicular to the Ark C*fh*. Wherefore, ſince E a, E*d*, E*g*, &c. are equal, for, they are all Radii; conſequently, the ſurface of a Sphere is equidiſtant from its Center, in every part; and, the Repreſentations, a b, *df*, and *g h*, of the Objects AB, DF and GH, on its Surface, are alſo their true Appearances, from the Point of View E.

But, on the Plane, or its Section, a h, the Object A B, has its Repreſentation, a b, nearly equal to a b on the Curve; whereas, d f, it is obvious and demonſtrable, is larger than *df*; and g h ſtill larger than *g h*, as it is more remote from the Point C; nevertheleſs, its true appearance is *g h* on the Curve, or ſpherical Surface; for, they both repreſent the ſame Object, from the ſame Point of View, which cannot vary in its Appearance.

Therefore, the Repreſentations of Objects, on various Surfaces, and in various Situations, although they may differ greatly in Figure, will, if truly repreſented, appear the ſame in the true Point of View.

The three Objects A B, D F and G H, though of various dimenſions, appear equal; not owing to their Diſtances, but to their Poſitions in reſpect of the Eye, being ſeen under equal Angles, A E B, D E F and G E H; as the portions *a b*, *df*, and *g h*, of the Ark C*fh*, indicate. Whereas, their Repreſentations on the Plane, of which a h is ſuppoſed a Section, are unequal; for, a b, the repreſentation of A B, being neareſt to the Center (C) is the leaſt; d f is greater than a b, and g h than d f.

Yet, it is manifeſt that the Repreſentations, a b, d f, and g h, have the ſame appearance, to the Eye at E, as their correſponding Images, a b, *df* and *g h*, on the ſpherical Surface. For, the Eye being in the true point of View (at E) they are both ſeen by the ſame Viſual Rays as the original Objects themſelves; and conſequently, under the ſame Optic Angles. Wherefore, their Appearances are the ſame on either Surface; by Theorem 1ſt.

From what has been advanced in this Section, it is evident that there is a manifeſt difference betwen the Repreſentation of an Object, on a Plane, and its true Appearance; which difference is the greater, as the Eye is nearer to the Object. For, ſince the Viſual Rays muſt neceſſarily cut any Plane, on which they fall, more and more oblique, the farther their Interſections are from that Point, in which a perpendicular Line from the Eye would cut the Plane; conſequently, the Repreſentations of Objects, on a Plane, cannot be in proportion to their true Appearances, but deviate continually, more and more, as they fall farther from the Point deſcribed. Whereas, on a ſpherical Surface, which is every where equally diſtant from its Center, every Right Line, and in every direction, which tends to its Center, is perpendicular to its Surface, and none of them can be ſaid to cut it oblique; conſequently, the true Repreſentations, and alſo the true Appearances of Objects, can only be depictured on the Surface of a Sphere; that is, the Repreſentation on its Surface and the true Appearance of the Object, from the Center, as the Point of View, are the ſame.

SECTION IV.

Containing ſome objections to the received Opinion of the Cauſe of Vision.

IN the laſt Section I have explained, demonſtrated, and briefly illuſtrated the whole Theory of Direct Viſion; or ſo much, at leaſt, as is eſſential, to give a clear and juſt Idea of Perſpective. To dwell longer on it, in this place, is unneceſſary; ſeeing that, Perſpective itſelf, in Theory, is nothing more than a continuation of the ſame Science. But, before I enter upon the ſubject of Perſpective, I ſhall, in this Section, give my objections to the received Opinion of the cauſe of Viſion

It

It is not an eafy matter to remove Prejudices, which are deep rooted; efpecially if they are ftrengthened with the fanction of Men of known abilities. I am at a lofs to conceive why it is, that the generality of mankind fo willingly impofe on their own judgments, or are fo ready to be impofed on by others; eager of being amufed with deep and profound Difcourfes, pretending to penetrate into the myfteries of Nature; which, neither themfelves nor their Authors know any thing of. In fact, we fee and know nothing further than the furfaces of Bodies; the formation of them, or their conftituent parts, we are, and ever fhall be, totally ignorant of. We daily fee the effects of one Body acting on another, and, by repeated Experiments, we find their full force; that they are regularly and uniformly the fame; but, the true caufes of thofe effects are to us unknown; we cannot form an adequate Idea, how, and in what manner they act, and produce fuch wondrous effects, although fo fimple in their Nature.

We have not the leaft Idea how a Solid can become a Fluid and a Fluid become again a Solid, by different degrees of Heat; what Fire is we know not, nor how it acts on Bodies, fo as to diffolve fome, to change the nature of others, and, in a great meafure to annihilate them. We know as little of the Element of the Air we breathe in; or, by what means it is often fo violently agitated in one place (as in high Winds) whilft it remains at reft, or has a contrary motion in another. We know not how a Solid is diffolved by a Fluid, and the Solid become perfectly liquidated; how two different Fluids mix and incorporate, fo as to be beyond the power or art of Man to feparate again, if they have an equal fpecific Gravity; nay two Solids, as Metals, &c. may be fo incorporated as to be quite another kind of Body; and the conftituent parts, of each, beyond the power of Art to diffever, or to diftinguifh them apart. Some again may be feparated, if they have not the fame fpecific gravity, as Gold or Silver, from any other bafer Metal. Some Fluids can never incorporate; as Oil or Spirits with Water, &c.

How a Fluid is compofed, or what form its minuteft particles are of, we are totally ignorant. Is it not ridiculous to fuppofe Particles, of any Figure? It is the common Opinion that they are globular; becaufe, the leaft drop of Water or other Fluid affumes that form, than which, nothing is more abfurd; becaufe, perfect Globes may be piled into a Pyramid, as firm and permanent, almoft, as Cubes, with very little lateral preffure; confequently, a Fluid might be fufpended in the fame Figure, if its conftituent Particles were folid Globes. For, on the fuppofition of Particles, they muft neceffarily be folid, or each Particle muft, again, be compofed of Particles.

To compare a Fluid with fine Sand is truely ridiculous. Sand will fupport itfelf in a pyramidal form; and, if the grains were perfectly globular, it would always affume one, of the figure of its Bafe, terminating in a fingle grain at the Vertex. It is not fo with Fluids; a Fluid cannot reft till it recovers an equilibrium in every part, prefently acquiring a perfectly level Surface. Befides, if its Particles were globular, there muft be interftices between them filled with Air, which cannot poffibly be incorporated with Water, as, in that cafe, it would be, to all fenfe. Alfo, if Metals, &c. when in a fluid ftate, by means of Fire, were compofed of globular Particles, it is reafonable to fuppofe they would remain fo when fixed; confequently, they would be extremely porous, and admit Air to pafs freely through, which is repugnant to common experience. No, fay thofe acute reafoners, it is by reafon of the Particles changing their Figure that they become fixed. Amazing truely! that the leaft particles of Matter, which I fhould fuppofe (on their Hypothefis) indivifible, confequently impenetrable and immutable, by any means, can be varied in their Shape or Figure. It is alfo amazing, that, by being globular, they are fo eafily feparated, when in a Fluid ftate, and, when fixed, are fo extremely difficult to diffever.

If the Particles of folid Bodies are kept together only by cohefion, why does not the fame power act on them in their fluid ftate? or why, when the particles of a folid

ſolid Body are, by any means, ſeparated, do they not cohere together again? or, what reaſon can be given, why they ſhould cohere more ſtrongly one way than another; as in Wood of all kinds? It is downright Sophiſtry, all; a language without meaning. That Bodies may be ſeparated into Parts or Particles is undoubted; but, that they are compoſed of Particles, is what I cannot give them credit for. Every Body, whether Solid or Fluid, before any of its parts are ſeparated, is one entire Maſs of Matter.

Again, if Fluids are compoſed of Particles, and allowing them to be tranſparent in their nature, their globular form would render them, in ſome degree, opake; ſo that, a Fluid would not admit of diſtinct Viſion through it. Now, I am clearly of opinion that all the parts of a Fluid lie perfectly cloſe to each other, without any Cavities interſperſed; it is impoſſible there can be any; conſequently, there is not the leaſt particle of Air contained in Fluids. If any diſſoluble Body be immerſed, as Sugar, &c. in Water; does not the Air, which is contained in its cavities, immediately aſcend to the Surface, as ſoon as it is at liberty, by the diſſolution of the immerſed Body?

The cauſe of Tranſparency, either in Fluids, or, more particularly, in ſolid Bodies, is, to us, an impenetrable myſtery. It is aſtoniſhing that there ſhould be diſtinct and perfect Viſion through Glaſs, or pellucid Stones; and, with all the knowledge and penetration human wiſdom can pretend to, we cannot form an Idea tending to comprehend it. Yet, this aſpiring, preſumptuous, all-ſufficient Man, would pretend to account for the cauſe of Viſion, itſelf, and for all the other Senſes; nay (if we will take their Words for it) for every operation in Nature. But I ſhall make free to tell them, plainly, that they cannot account for one. Firſt, let them account for their own Exiſtence; can they tell where the firſt ſpring of ſelf-motion begins? no, not ſo much as for the motion of a Finger.

As for the generality of Mankind, they neither do nor are capable of judging for themſelves, in many caſes; and only give their aſſent, or diſſent as they are influenced, or depending on the judgment of others, whom they either know or believe to have Judgment. That this is the caſe, in general, is certain; and is the reaſon why the opinions of ſome great Men are, though ever ſo arbitrary, adopted as orthodox; becauſe, there are but few who dare venture to contradict them, and think or judge for themſelves. And, becauſe thoſe Perſons have advanced ſome received and eſtabliſhed truths, theſe therefore conclude, that all which they preſume to advance muſt neceſſarily be ſo.

Thoſe profound Gentlemen knowing their own importance, and the deference which the World pays to their Opinions, are but too apt to be intoxicated with it, and often take too much Conſequence to themſelves; imagining, that they are really endowed with a penetration, into the myſteries of Nature, beyond the reſt of Mankind. Or, from the knowledge they have of Mankind, the great deference they pay to their own ſuperiority over them, and the implicit aſſent which is given to all their productions, make them forget, that they, as well as the reſt, are fallible.

Reſpecting Viſion. It is ſaid in Smith's Optices, Page the firſt, and that from the Authority of Sir Iſaac Newton, "that we ſhall find it difficult to conceive, "Light to be any thing elſe but very ſmall and diſtinct particles of Matter, inceſ- "ſantly thrown out from ſhining Subſtances; and every way diſperſed by Reflec- "tion from all others." I freely own, that I am at a loſs to conceive how it can be Matter, and that for ſeveral reaſons which may be given.

Firſt; it is an indiſputable Maxim, univerſally allowed by all Men of common underſtandings, that two Bodies cannot occupy or fill the ſame Space at the ſame Time; even Air cannot be where there is already any other Body. Steam, or Water expanded, almoſt, if not totally, excludes Air; conſequently, no other Body can occupy the ſame Space where there is Air. Now, ſince the whole of Space, within our Atmoſphere, at leaſt, is filled with Air, how can it be alſo filled with Light, if Light be, alſo, a Body, compoſed of Matter? for, ſince it is manifeſt, to ocular

3 conviction,

conviction, that, in full Day Light, there is no part of the Space around us but what is wholly illumined; consequently, being wholly, and in every part, filled with Light, if it be a Body, Air must be totally excluded; agreeable to the general Maxim, that two Bodies cannot fill the same Space. But, I presume, we do not find that to be the Case; for, the Space is filled with Air, and cannot possibly be excluded by Light.

I expect it will be alledged, that Air, being a Fluid, is composed of globular Particles; and that, Light fills up the interstices between them. Of what shape and magnitude, then, are the particles of Light, compared with the particles of Air? If Light be an existing Medium it must also be a Fluid; consequently, its Particles are also globular. And, if it be supposed to proceed from the Luminary, progressively, in Right Lines, in all directions, I own I am at a loss to conceive how that can possibly be effected, through innumerable globular particles of Air; to say nothing of the inconceivable velocity of its motion, being supposed to proceed from the Sun to the Earth in seven minutes and a half. For which extremely nice calculation we are indepted to Roemer; who first observed, that the Eclipses of the Satellites of Jupiter happen, when in Opposition, that is, when the Earth is in a Right Line between the Sun and Jupiter, fifteen minutes sooner than when in Conjunction, or the Sun in a Right Line between the Earth and Jupiter*. The difference of the distance of Jupiter from the Earth, at these two periods, is the whole Diameter of the Earth's Orbit; computed at more than 160 millions of Miles; half of which is the Distance of the Sun from the Earth. From which curious observation it is conjectured, that, Light is projected, from the Sun to the Earth, in seven or eight minutes; that is, the Sun being supposed to be continually emiting Light, it fills a concave Sphere, equal in Diameter to the Earth's Orbit, in that Time. Quere; how is the Sun supplied? and, being Matter, what becomes of it?

Now, admiting this extraordinay observation to be truely calculated, it undoubtedly proves that Vision is not instantaneous, at any distance; which seems reasonable; but I cannot see that it proves the progression of Matter from Jupiter to us. Besides, the Light which proceeds from Jupiter, or his Satellites, is but reflected, at second Hand; perhaps, Light, coming immediately from the Sun, may be much swifter in its motion. I am almost of opinion, that if it were possible to screen the Sun from our sight, by the interposition of an opake Body, close to its Surface, being removed in an instant, that we should see the Luminary instantaneously; and, the whole Space between be instantly illumined.

The Law of reflected Bodies, that is, the velocity with which an elastic Body rebounds, according to the force by which it is first impelled, I am not acquainted with. But, of this I am well convinced, that the velocity of any impelled Body is not uniformly regular, but is continually varying, and the Body, in motion, tending to rest; consequently, it cannot acquire an additional force by striking against another Body, at rest, so as to rebound with the same velocity by which it was at first impelled; and, if Light, as a Body, acts on the same Principles, it must come, immediately from the Sun to us, with a greater velocity than when reflected from any other Body.

The Distance of Jupiter from the Sun is computed at 424 millions of Miles; and the Earth but 81 millions; which, is not one fifth part of the Distance of Jupiter. Consequently, the velocity, with which Light is first impelled, must be greatly diminished before it reaches Jupiter, and more so in being reflected back from Jupiter to the Earth. I shall endeavour to illustrate what I advance.

* It very rarely, if ever, happens, that they are in a Right Line, in a strict sense; because, the Plane of the Orbit of Jupiter is inclined to the Ecliptic, that is, to the Plane of the Earth's Orbit, in an Angle of one Degree and 20 Minutes. But, the Line of the Nodes, that is, the Line of intersection of the Planes of the two Orbits, is supposed to have a revolution (as the Moon's Orbit in 19 Years) but in what time is not, I believe, as yet determined. Consequently, they can never be perfectly in a Right Line, but only when the Line of the Nodes pasies through the Centers of all the three; viz. the Sun, the Earth, and Jupiter. Wherefore, at a Conjunction, Jupiter will appear either above or below the Sun, according as he is in the upper or lower part of this Orbit; and, at an Opposition, the Earth will be either above or below a Right Line passing through the Centers of the Sun and Jupiter.

Plate II. Fig. 9. Let S represent the Sun, in the center of our System, of Planets, &c. and CDEF the Earth's Orbit, whose semidiameter, SE, is 81,000,000, of Miles.

Let AB be a portion of the Orbit of Jupiter, and J the place of Jupiter therein; whose Distance, SJ, is above five times SE or SC. Let ab be the Orbit of one of Jupiter's Satellites, in which the Satellite is supposed to be, at a, emerging from behind Jupiter, after an Eclipse.

2. It is affirmed that we see Objects, only by means of material Rays of Light, reflected from them to the Eye; which impress on that Organ the perception of Vision.

Suppose the Satellite, at c, to be in a direct Line behind Jupiter; it is plain that it could not be seen at C or E, because it would be hid by the Body of Jupiter; but, as soon as it emerges, that is, as soon as it can appear, as at a, in the Right Line Ca or Ea, it becomes visible; not, merely, because there is no other Body between, but because it becomes illumined from the Sun, in the Right Line Sa; which, whilst behind Jupiter, from a to b was deprived of Light.

Let Sa represent a Ray of Light from the Sun to the Satellite at a; the Earth, at C, being directly between. The same individual particles of Light, which pass by the Sun to Jupiter, pass by the Earth to Jupiter, and back again, before either Jupiter or the Satellite can be seen; according to their Hypothesis.

It is reasonable to suppose, that the velocity with which those particles of Matter first set off, from the Sun, is greater than when they arrive at the Earth; how much, then, must it be decreased before they reach Jupiter? Which Opinion, Rohault (Chap. 27, Art. 34,) endeavours, to confirm, by a comparison with a conical Vessel filled with Water, being raised, at its Surface, by ejecting in more Water at the Vertex. The comparison has indeed some affinity, though a very gross one, because Light is supposed to expand as it spreads; by which means, its direct motion may be equible.

Suppose Light loses one fourth of its velocity before it reaches Jupiter, it can have lost but one twentieth part at the Earth; and it is reasonable to suppose, that the force with which it returns from Jupiter cannot be equal to what it came with. Suppose then it has lost a fourth part more of its force, before it reaches the Earth again, at C; which, considering how languid its Light is, compared with the Light of the Sun, is very moderate. According then to this supposition, Light comes from Jupiter to the Earth, with little more than half the velocity with which it comes from the Sun, directly.

C represents the place of the Earth, at an Opposition and E at the Conjunction; the difference in the Distance of those two situations, from Jupiter to the Earth, is CE, the Diameter of the Earth's Orbit.

Now the motion of a Particle of Light (for a Ray, or continued Right Line of Light, I have no conception of) in its passage from the Sun to the Earth, is, I presume, at is first and greatest velocity, always the same; but, at J, it has lost one fourth of its force; and, from J to C again, it has lost a fourth more; and consequently, in its passage from C to E, which is nearly half JC, it must necessarily have lost one eighth of the velocity it had, at its return to C; at which time the calculation is made.

Wherefore, the motion of Light, from C to E, has, at most, but half the velocity it has from S, to C or E, directly; and, it passes, according to Roemer's calculation, from C to E in 15 Minutes; consequently, it passes from S to E (with double velocity) in a fourth part of the time, that is in 3 minutes and 3 quarters.

But, if the velocity, to and again, be regular and uniform (which I believe is contrary to the laws of motion, of Bodies impelled and reflected again) being 15 minutes in passing from C to E, half of which, SE, must consequently take 7 minutes and a half. For a Satellite, emerging at a, when the Earth is at C, appears 7 minutes and a half sooner, and at E 7 minutes and a half later, than at D or F, where the distance of Jupiter from the Sun and the Earth is equal, according to the calculations made by the ablest Men. But since it is not possible, or, at

 least,

leaſt, probable that the Velocity of Light can be uniform and equal, at all Diſtances from the Sun (if it has any motion at all) the greateſt velocity muſt be at its firſt emiſſion from the Luminary; and conſequently, it takes leſs time in paſſing from the Sun to the Earth than in its return from Jupiter.

3. It is certain that no opake Body, I mean, ſuch as are not luminous, can be ſeen at a great Diſtance; unleſs its magnitude be ſufficient to obſtruct a great quantity of Light, when oppoſed to a luminous Body; or, when it is ſo ſituated, in reſpect of a luminous Body, that the Light, it receives from the Luminary, is reflected again *; of which, the various appearances of the Moon is ſufficient proof. Conſequently we could not diſcern either Jupiter or his Satellites (whoſe immenſe Diſtance is more than ſeventeen hundred times the Moon's Diſtance) if they were not illumined by the Sun.

Jupiter being a ſuperior Planet, that is, he moves in an Orbit beyond, yet concentric with the Earth's, he is, conſequently, always illumined on that Face towards the Earth; and conſequently (being of immenſe magnitude) may always be ſeen, when above the Horizon, by the naked Eye; except when in near vicinity with the Sun. But the Satellites are ſecondary Planets or Moons (of which Jupiter has four) accompanying the primary ones, as the Moon does the Earth; and are ſo very ſmall, in compariſon of their primaries, that they cannot be ſeen at that diſtance without a good Teleſcope.

Now, if thoſe Satellites cannot be ſeen, after they emerge from behind Jupiter, until the Light (which is always ready, at hand, to illumine them) is reflected, back again, from the Satellite to us; and, ſuppoſe this Light to be a material Body, I am firmly perſuaded that they would never be ſeen at all, by us; ſuch an immenſe Space, from a to E, above 500 millions of Miles, for Matter to be projected in about 45 minutes, is beyond the power of my reaſoning faculties to find credit for (with God nothing is impoſſible.) Beſides, they not only reflect the Light directly, but alſo obliquely, in all Directions, filling a Hemiſphere; which, I ſhould ſuppoſe, is too groſs an improbability (being Matter) to gain credit with a thinking rational Being; let thoſe, who can, find belief and give credit for it.

The whole of this Phænomenon I ſuppoſe is this; (for I have never ſeen the Experiment; as it muſt require extraordinary Teleſcopes, which magnify to a great degree, to diſcern the emerſion diſtinctly) either it is impoſſible to perfect Teleſcopes ſufficient to diſcern the emerſion, till after the Satellite is conſiderably advanced from behind Jupiter; and alſo, from the near vicinity of the Sun, when in Conjunction with the Earth, at E, whoſe ſuperior ſplendor muſt hinder the Sattellite from being ſeen, for ſome time; together with the ſo much greater Diſtance, than when at C; which, I know from my own obſervation, renders them ſcarcely viſible, with a common reflector; all which, may concur to render it inviſible for 15 minutes, after it might be viſible from C. For I make not the leaſt doubt, that their motions and revolutions are as regular and equal as the motion of the Earth itſelf; which, to us, is the only ſtandard meaſure of Time.

4. If we ſee Objects, only by means of material Rays of Light paſſing from the Object to the Eye; by what means are opake Objects, which are immerſed in Shade, ſeen at all? as they do not receive Light, immediately, from any luminous Body; nor, perhaps, from any other, oppoſed to them, by reflection. But, with ſuch reaſoners, who can give what properties they pleaſe to Matter, there is no arguing; ſeeing that, they can make Light reflect and rebound from one Object to another, at pleaſure. But, are not thoſe ideal properties of Light of their own creating, entirely? That Light is reflected from one opake Object to another is beyond a doubt; but, that real Matter is projected, to and again, in every direction, I cannot acquieſce in.

* By Light being reflected, I would not be underſtood to mean, that there is any kind of real Matter projected from the reflecting Body; but only, by being illumined, itſelf, it becomes, in ſome degree, luminous, ſo as to ſhine with its borrowed luſtre, and illumine others.

Let

Plate II. Fig. 10. Let A B reprefent, what is called, a Ray of Light, from fome luminous Object, falling on any Surface, as CD.

I fhall fuppofe this Ray of Light to be a phyfical Right Line, the leaft in its dimenfions of breadth and thicknefs, that can be conceived; confequently, this phyfical Ray of Light can ftrike any Surface, on which it falls, in a phyfical Point only, the fmalleft that can be conceived.

Now by the laws of Reflection, according to all the writers on Optics, whenever a Ray of Light ftrikes or impinges on any Body, it is reflected again from that Body, making an equal Angle with a Right Line (B F) perpendicular to the Surface, at the Point in which it ftrikes the Surface *. Or, if it be a Plane Surface, it makes equal Angles with the Plane (A B I=E B K) both, the Original Ray, A B, which is called the incident Ray, and the reflected Ray, B E, being in a Plane, which is vertical to the other Plane.

It is reafonable to conclude, if this Ray of Light, A B, be material, and it is by means of material Rays ftriking the organs of Sight, that Vifion is performed, or the fenfe of Seeing is effected, that, the Point B could only be feen by an Eye placed, any where, in the direction of B E, as at E; B E being confidered as the fame individual Ray, A B, reflected or broken at the Surface, in the Point B†. It is certain, that if the furface C D be Water, or a polifhed Mirrour, the Image of any Object, at A, will be feen at B, by the Eye at E, or fome where in the direction B E, only.

But experience convinces us, that the Point B, or any other Point in the Surface C D, may be feen as perfectly in a thoufand other Directions; as at F, G, H, &c. Confequently, if Eyes were placed all around, within the compafs of a Hemifphere, of which the Plane C D may be confidered as the Bafe, or being elevated but a few Degrees above the Plane C D, the Point B may be feen by them all, at the fame moment of Time.

Now, fince the Point B can receive but one Ray of Light from the luminous Body, how can that identical Ray be reflected in ten thoufand Directions, in all Directions above the Plane C D, and thofe Rays to be material? at the fame Time, the whole Space is filled with Air in every part. But, the fame Eyes can alfo fee every other phyfical Point in the whole Surface C D, as C, D, &c. confequently, every Point in the Surface muft alfo reflect Rays in all Directions, croffing and cutting the former in every Point; which, unlefs, not only two, but, an infinite number of Bodies may occupy the fame Space, and at the fame Time, cannot poffibly be, on the fuppofition that Light is a material Body.

5. Again. Since, as I have obferved before, it is afferted that Vifion is performed by means of thofe material Rays of Light, which enter the Aperture of the Eye, and, impinging on the Retina, affects the Optic Nerves with the perception

* It is advanced by Sir Ifaac Newton, (Prop. 8, Part 3. of the Second Book of his Optics) that Light is reflected from Bodies before it impinges on their Surfaces. But I muft own, that in all the feven reafons he gives for that Opinion, I cannot find one of weight, nor all of them together, fufficient to prove it. Which, Smith, from that authority, and from Sir Ifaac's firft and fourth Quere, endeavours to prove, is not reflected immediately in an Angle, but makes a regular parabolic kind of curve, at a very fmall diftance from the Body, which he calls the fpace of activity; and which fpace, he fays, is fo extremely fmall, that, confequently, in phyfical Experiments, the incurvation of a Ray of Light may be confidered as performed in a phyfical Point. Emerfon fays (in a Scholium to Prop. 1. of his Catoptrics) that the Curve, defcribed by a Ray of Light, is fo extremely fmall, that it may be looked upon as a fingle Point.

It is, to me, aftonifhing, how any Perfon dare prefume to advance fuch Opinions! and, for what purpofe they do advance them, except it be to make the World have a high veneration for their extraordinary fagacity and penetration. Can they, either by experiment or otherwife, prove the truth of their Hypothefis? or do they fuppofe mankind fo credulous as to give them credit for it, from the ridiculous experiment of a Hair, or of the Knives, and fringes of Colours produced by them? And yet it is certain, that fome (not to be behind them in penetration) either do or pretend to do; for I have heard the fame thing advanced, verbally, by a Perfon who has a tolerable fhare of mathematical knowledge.

† See Fig. 10. No. 2. A B is fuppofed to be an incident Ray, and C D the reflected Ray; making a Curve, B E C; before it touches the Surface G H; which Curve is performed in fo fmall a Space, that the Points, B and C, are fuppofed to coincide. Quere; how is the Curve to be afcertained, and determined? or, how fhall we have conviction that it does not touch the Surface, G H?

of the Objects from which they flow, how ſhall we account for Viſion through Glaſs, or much harder diaphanous Subſtances, as Cryſtal, or Adamant? which is impenetrable, by any tool made of the hardeſt Steel.

Let any object, as A B, be ſeen, by an Eye at E, by means of the Viſual Rays A E, B E, &c. and, let any tranſparent uncoloured Stone, as Cryſtal, &c. be interpoſed directly between the Eye and the Object, as C D, of any thickneſs; having its oppoſite Surfaces parallel Planes, and well poliſhed. The Object, A B, will be ſeen through it inſtantaneouſly, as repreſented at a b, with every circumſtance of Colour, &c.

Now, I ſhould be glad to know, how or by what means thoſe Rays of Light, tranſmited or reflected from the Object (A B) to the Eye (at E) paſs in an inſtant through this ſolid Body of Cryſtal or Adamant, if they are material? and firſt, I ſhould be glad to know, what is the cauſe of Tranſparency? or, in what the difference conſiſts, between tranſparent and opake Bodies?

The reaſons given by Sir Iſaac Newton, in the third part of the ſecond Book, and which, Smith, in his Optics (Ch. 8, P. 95) has very wiſely copied, word for word, concerning the cauſe of Tranſparency, Opacity, and Colours, in Bodies, is but little to the purpoſe; it does not convey, to me, the leaſt Idea of the cauſe of Tranſparency, and how Viſion is conveyed through tranſparent Mediums, in every direction, inſtantaneouſly.

The learned and reverend Dr. Harris, in his Lexicon, tells us (under the article Tranſparency) that, a diaphanous or tranſparent Body is one which, probably, has its Pores all Right or Direct, and nearly perpendicular to its Surface.

By the Pores being all right or direct is meant, I preſume, that they go directly through, in Right Lines, without any obſtruction; that they are perpendicular, or nearly ſo, to the Surface, I muſt needs ſay, he was extremely ſagacious who firſt found out that extraordinary quality. But how ſhall we interpret the Word, probably? is it not indeed a tacit acknowledgement, that it is all nothing more than Conjecture? I cannot ſay that it is even a probable Conjecture.

I am afraid that the Doctor was not very happy in his memory; for, in another place he tells us (under the article Diaphaneity) that the Pores of a diaphanous Body are ſo ranged and diſpoſed, that the Beams of Light can paſs freely through them, every way; which plainly contradicts the former aſſertion, that they are nearly perpendicular to its Surface; and which, in reality, is ſaying nothing, becauſe, a Surface may be made at pleaſure; and we cannot ſuppoſe that Pores can change their Poſition, as a Surface may be altered, at diſcretion; or how a Ray of Light, in paſſing through a triangular Priſm, can be nearly perpendicular to both Planes, I am at a loſs to deviſe.

For my part, I look on it as ridiculous and preſuming, to tell us that Glaſs and all tranſparent Bodies are full of Pores and minute Interſtices, through which the Rays of Light have a free paſſage; becauſe, theſe Pores muſt be in every part of it, and in all Directions; neither can there be any ſolid part, between one Pore and the contiguous or adjoining Pores; for, if there be, it muſt neceſſarily impede, or entirely ſtop the progreſs of ſome Rays of Light in their paſſage to the Eye, and, conſequently, will prevent the Viſion of the Object from being perfect. Wherefore, ſince the Pores are ſo very numerous, have no ſolid part between them, and that, in all Directions; for it is plain, that they are not confined to one poſition, but muſt lie in all directions, which is evident; for, turn the Glaſs or Cryſtal (C D) as you pleaſe, the perception of the Object is as diſtinct as before; from which it is clear, that the Pores muſt be the ſame in all Directions, and then, I would aſk, what becomes of the ſolid Body? for, in reality, it muſt be all Pores.

It is an inſult on our Underſtandings, on our Reaſon, and on common Senſe, to ſuppoſe, or for any perſon to attempt to perſuade us, that Glaſs, or Adamant, the moſt compact of Stones, is more pervious, or that the Pores are more direct than in ſoft Wood, or in any Wood; which every one, who has conſidered it,

Plate II. knows to be full of Pores (like Veins in the animal Body) through which the Juices paſs for its nutriment; and, in many kinds of Wood, they are perceivable by the naked Eye, and lie in direct Lines. Can any Perſon perſuade himſelf, that the Pores are more numerous, or more direct, or that they are more capacious, ſo as to admit a freer paſſage, through Adamant, for the moſt ſubtle Fluid whatever (which I will ſuppoſe Light to be) than through a thin piece of Wood; in which the Pores are obvious, and clearly viſible? Air, which is much denſer than Light, or Water, ſtill denſer than Air, has a free paſſage through; but not through Glaſs, I preſume. Yet, if you oppoſe it to a Candle (being cut very thin acroſs the Pores) you may perceive, indeed, a few ſcattered Rays of Light paſs through, but very far from having diſtinct Viſion of the Candle, or even the out Line of the Flame, only. The reaſon is very obvious; becauſe, the parts, between the viſible Pores, being more condenſed and compact, receive the Light which falls on them, and either abſorb or reflect it; which, therefore, does not paſs through to the Eye; conſequently, the parts of the Object, which are in a direct line to the Eye, cannot be ſeen; therefore, the Viſion of the Object is imperfect.

As this is evidently the caſe when it is oppoſed to a luminous Body, which is ſeen by its own Light; what will be the conſequence when it is oppoſed to an opake Body, which (as we are told) is ſeen only by means of reflected Light? Why, that there is not the leaſt appearance of it to be ſeen at all; not even through fine oiled Paper, which, I muſt needs ſuppoſe, is infinitely more porous than any Glaſs or Stone whatever.

Whereas, through perfectly pellucid Subſtances, every viſible Point, in any Object whether luminous or opake, is diſtinctly ſeen, without the leaſt impediment; conſequently, if Light be material, and a Ray is tranſmited from every Point, in a viſible Surface, to the Eye (for every phyſical point may be ſeen) and if thoſe Rays are conveyed through tranſparent ſolid Bodies, they muſt neceſſarily paſs through Pores, direct in all poſitions.

Now I am fully convinced, that the Pores in Glaſs, &c. are of thoſe ingenious Gentlemen's own creating; who, when they are at a loſs for proof of certain Hypotheſes (for want of better) they imagine Bodies to poſſeſs ſuch and ſuch qualities as may beſt anſwer their purpoſe. But, are thoſe Chimeras, of their own fertile imaginations, to paſs on the World for real exiſtencies? are the Concluſions drawn from ſuch Premiſes candid? by no means, they are very diſingenuous, inſomuch that, I deny it to be in the power of any Man, to give ocular or other demonſtrative proof that there are Pores in Glaſs or tranſparent Stones; and, I do believe that the moſt pellucid ſubſtances are the freeſt from Pores; for all porous Bodies are compreſſible into leſs compaſs, which neither Glaſs nor Stones can poſſibly be; nor Water, which is perfectly tranſparent.

Pores are, in my opinion, rather the cauſe of Opacity than of Tranſparency in Bodies, ſeeing that, they abſorb the Light in their receſſes. Yet, I do not ſuppoſe that all Bodies, as Wood or Metals, which are the freeſt of the kind from Pores, have any degeee of tranſparency, but when they are exceeding thin; as leaf Gold, &c. but, they approach nearer to tranſparency than the more groſs and porous kinds. The real cauſe of Tranſparency, and how Viſion is conveyed through tranſparent Bodies, are (I am firmly perſuaded) among the hidden myſteries of Nature, which is not given Man to explore.

6. It is, I believe, a Paradox, not eaſily accounted for; that, if two triangular Priſms, of Glaſs or other diaphanous Subſtance, having equal Angles, are ſo placed together (as A B C, B C D) that the outſide Faces (A B and C D) are parallel, we have direct Viſion through them both (from the Eye, at E, to an Object at F); whereas, if either of them (as B C D) be taken away, the Object is loſt to ſight from that Point of View, by reaſon of the ſuppoſed refraction of the Rays of Light, in paſſing through the Priſm A B C.

Fig. 12.

Now,

6

Now, if the Pores, through which the Rays E a, E b, &c. pafs, go directly through both Prifms (from a to c, and from b to d) how can they be varied by the removal of one of them? the taking away of one Prifm can certainly have no effect on the Pores of the other, fo as to alter their Direction; yet (whatever be the Caufe) it is certain, if either Prifm (as B C D) be removed, that the Vifion, of the Object F, is turned afide out of a Right Line, and totally loft to fight, from that Station; for, it will be feen by an Eye at *E*, and not at E, and appear to be in the direction *E F*. Or, the Object, at F, being removed to G; will appear to be at F, in the direction of E F; and F will be its apparent place.

Again; if the Pores go direct through Glafs, &c. the Rays of Light do not ftop at the Surface, and confequently, they cannot fuffer either Reflection or Refraction by it. For, I prefume, the true definition of a Pore is a fmall Cavity or Interftice, which admits a free paffage for Fluids. If, therefore, they do enter, and pafs freely through, how can the furface affect them? or how can the Rays of Light, if Light is a Body, be turned afide, within the Glafs, in any other direction than that of the Pores?

It is affirmed and allowed, that the Angle of Refraction is always to that of Incidence in a certain Ratio or Proportion; and fince the Angle of Incidence may be any Angle at pleafure, it neceffarily follows, that Light paffes freely through in all Directions; which (according to the eftablifhed Hypothefis, that it is corporeal) implies Pores in all Directions, according to one of the Doctor's Definitions; and, if it were poffible for Pores to go right or direct through, in all Directions (which is repugnant to reafon and abfurd to fuppofe) the whole Prifm muft be all Pores; which omniporous quality, being attributed to any kind of bodily Subftance, I am perfuaded, no Man, in his fenfes, will acquiefce in.

7. I fhall juft give one more objection to the materiality of Light, and conclude this Section, and Subject.

Suppofe the Eye, at E, viewing an Object, A B. There is fuppofed to be Rays of Light, A E, B E, &c. tranfmited from every point to the Eye, forming an Image of the Object on the Retina, which is generally allowed to be inverted; although a late writer has given fome reafons to the contrary. Be that as it may, it is certain, that if thofe Rays of Light enter the Eye, they muft pafs through the Aperture or Pupil, and converge to a Point (E) within, before they can diverge againto form an Image of the Object; at a c b. Fig. 13.

Now if the Eye be fo fituated (in refpect of Diftance) to the Object, that the extreme Rays, A E and B E, incline to each other in an Angle (A E B) not exceeding 40 Degrees; it is certain, that the Eye is capable of taking in the whole of that Object at one View; although, every part cannot be diftinctly feen at once. Every phyfical Point, in the furface of the Object, as C, D, F, &c. is fuppofed to tranfmit a Ray of Light to the Eye, as well as in all other Directions, at the fame moment of Time. Confequently, the whole fyftem of Rays generate a Cone or Pyramid of Rays, clofe wedged together in every part; which, all enter the Eye at the fame inftant, paffing through a Point of the fame dimenfion as one fingle Ray, at E, the Vertex of the Pryamid or Cone; which Circumftance is fo egregioufly abfurd, that it is fufficient, in my opinion, to refure all that can be alledged concerning the Rays of Light being material.

For, can a great quantity of Air (the rareft Fluid that we are acquainted with) pafs through a Pin-hole in an inftant? Will it not (like Water) require more or lefs time, according to the dimenfion of the Hole, to pafs through? can any force drive the fame quantity of Air through a Hole of half an inch in Diameter, in the fame Time as through one of an Inch? No; the ratio of the Time required, with the fame force, will ever be in proportion to the area of the Aperture; that is, the ratio of different Apertures, to each other, is equal to the ratio of the Time required, with equal force.

And

And yet, a Pyramid of Light, whofe Bafe is equal to St. Paul's Cathedral, nay, millions of times greater (for the Eye can take in, not only the Sun, but, a great part of the Hemifphere, at once) and its Altitude of any dimenfion, as far as the fixed Stars; yet, I fay, this prodigious Pyramid of Light, a material Body, a Fluid, can pafs (without any known impulfe) through ten thoufand imaginary Pores in the Cornea, in their paffage to an imaginary Point within the Eye; through which, the whole is conveyed in an inftant, to the feat of Vifion; and what becomes of it after? the Eye muft be very capacious to contain it all. Nay, more; the very fame Bafe, or Object, can fend forth millions of Pyramids of Light, to other Eyes all around, at the fame Time, and in all Directions. Impoffible! being Matter. Can any Perfon form an Idea, to comprehend how the fame Body, or Surface, fimply, can emit, or reflect what it receives, this inftant, from any other Body, in innumerable directions at the fame Inftant? Let thofe, who can, find belief and give credit for it.

Much more might be faid in fupport of this Argument; but, as it is not directly to the purpofe of Perfpective, I fhall not trefpafs any longer on the Readers time and patience, in this Digreffion from the Subject. I fhall only beg leave to draw from it this Conclufion; that Light is not an exifting Medium compofed of Particles; which, being reflected from Objects, in all Directions, and ftriking on the Organs of Sight, conveys the Vifion of them to the Mind, and occafions the fenfe of Seeing; intimating, by means of the different qualities of its heterogeneous Rays, not merely the exiftence of the furrounding Objects, but, they are alfo fuppofed to excite, in the Mind, the idea of Colours on their Surfaces; which, otherwife, have no exiftence: ftrange Doctrine!

But, with fubmiffion, I think, that the perpetual exiftence of fuch a Medium is repugnant to the notion of luminous Bodies emiting Light, inceffantly; which, proceeding from them, progreffively, in Right Lines, excites Vifion; not inftantaneoufly, but propagated in Time: which, opinion, is more confiftent with the notion of its being reflected from other Bodies. For, how a ftagnating Medium, a Fluid, can be fo actuated, as to be reflected, from Objects, in all Directions within itfelf, and confequently, in direct Oppofition to its firft, or incident motion, and with fuch amazing velocity, is beyond the reach of human reafon to conceive; much lefs to comprehend and explain.

It is the diftinguifhing Property of lucid or luminous Bodies to difpenfe Light all around them; but how, or in what manner, it is not my intention to enquire into; being well affured, that the attempt would be as fruitlefs as prefumptuous. Such Phænomena are, and ever will be, to Man, impenetrable and infcrutable; Myfteries not to be unfolded but by infinite Wifdom itfelf. To us, there is a large field of knowledge, open and in view, whereon to exercife our reafoning faculties, and which lies within our reach; let us not, then, ftep afide into intricate Mazes and Labyrinths, out of which it is impoffible to extricate ourfelves; in which, the farther we wander the more we are bewildered; till, wearied with the vain purfuit, we are, at laft, obliged to own, that all our boafted knowledge is but to know how little can be known.

SECT.

Plate I.

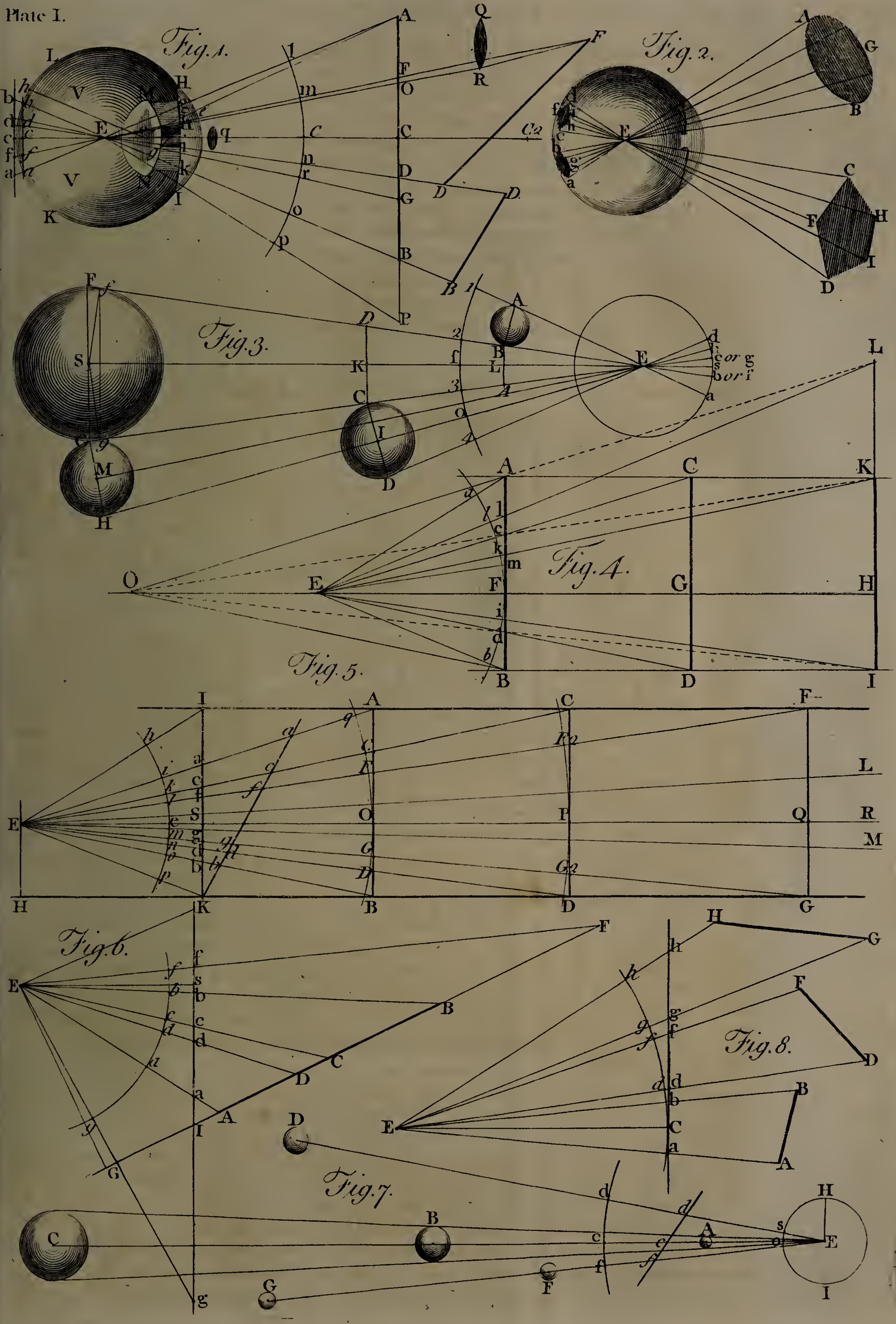

Plate II.

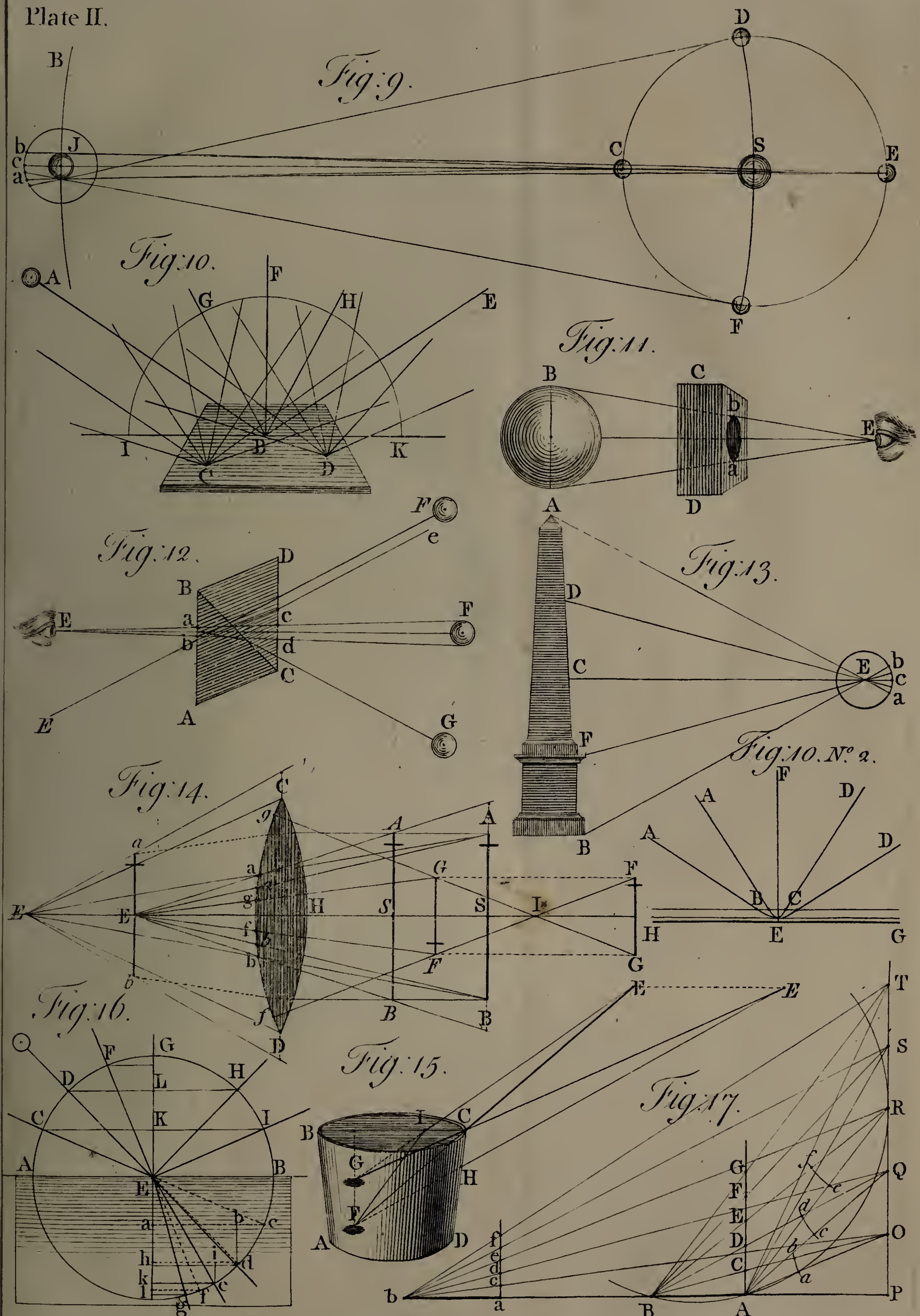
Fig:9.
Fig:10.
Fig:11.
Fig:12.
Fig:13.
Fig:10. No 2.
Fig:14.
Fig:15.
Fig:16.
Fig:17.

SECTION V.

Of Refracted Vision.

THE laſt Section touched, though very ſlightly, on that part of Optics called Catoptrics, or reflected Viſion. In this I ſhall briefly touch on Dioptrics or refracted Viſion; both which, I look on, in many reſpects, as Deceptions in Viſion. For it is evident, that, in the caſe of Reflection, on the ſurface of Water, &c. or poliſhed Mirrours, we do not ſee the Object, but only its Image or Appearance. So likewiſe, in the caſe of Refraction through tranſparent Mediums, ſolid or fluid, we do not ſee the real Object, but its Image or Repreſentation only; which is likewiſe a Deception; ſeeing that, the Object appears, in ſome caſes, larger, in others, ſmaller than it really is, and always appears to be where it really is not; although, we imagine that we are looking directly at the Object.

Perſpective Repreſentations, are alſo manifeſt Deceptions in Viſion. The buſineſs of Perſpective is to repreſent Objects on a Plane Surface, by the rules of Optics; which, in the true Point of View, will give the Idea of a real ſolid Object (being properly and judiciouſly ſhaded) having the appearance of projectures and recedings, of one part before or behind another; and having alſo the ſame hue or teint of Colour, the Deception becomes ſtronger; inſomuch, that it is poſſible ſo to deceive the Eye, as to imagine the Repreſentation to be real and ſubſtantial.

In looking at Objects through a Teleſcope, either reflecting or refracting; although we level it exactly to the Object, in a Right Line, and imagine we look directly at the Object, through the Tube; yet, it is plain that we do not ſee the Object. For, in a Reflector, we look directly at a concave Mirrour, which is placed direct between the Object and the Eye; conſequently, in that caſe, it is impoſſible to ſee the Object, but only its reflected Image on the Mirrour, which is firſt received on another Mirrour, at the other end through which we look, and reflected again to the former, oppoſite to the Eye, (a moſt curious and ingenious invention; which was firſt conſtructed by Dr. Gregory, and thence called the Gregorian Teleſcope.) The Image, of the Object received on the Mirrour, is magnified, by means of convex glaſſes in the ſmall Tube; ſo that, we do not ſee even the reflected Image on the Speculum, but the Image of that, only, magnified, to a great degree. In reſpect of the rafracting Teleſcope, the Object itſelf is magnified, in the ſame manner as the Image on the Mirrour, in the Reflector; ſo that, we do not, in either caſe, ſee the real Object, but only its magnified Image, between the Object Glaſſes and the Eye. I ſhall illuſtrate it by a ſingle magnifying Lens.

1. Suppoſe an Object, as A B, and a convex Lens, C D, placed between the Object and the Eye, at E. It is manifeſt, ſince Viſion is conveyed in Right Lines from the Object to the Eye, that, if we ſaw the real Object, by the Viſual Rays A E and B E &c. it would appear, in proportion to the Lens C D, of the magnitude *ab*, only; but we find, that it appears larger than its real magnitude, in proportion to the Lens.

For, if the Rays A a and B b fall perpendicularly on the convex Surface, C H D towards the Object, they will paſs directly to the other Surface, which is concave towards the Object, cuting it at a and b, and are thence refracted, to the Eye at E, as a E, b E; then, if A*A* and B*B* be drawn parallel to the Axe of the Lens (E I) and E a, E b are produced, cuting A *A* and B *B* in *A* and *B*, then, is *AB* the apparent place of AB, or its Image; which, being ſeen under a larger Angle *A* E *B* than the real Object A B, if the Lens was removed, or Plane, it will conſequently appear larger; by Theorem 1ſt.

Plate II. Fig. 14. Hence it is manifeft, that the real Object, A B, is not feen; for it is not poffible that the Point, A, can be feen under the refracted Lines A a, a E; and if E a, E b be produced, it is evident that the Object A B, in that place, muft be larger, to appear equal to a b, on the Lens.

This way of determining the Image or the apparent place of the Object, is according to Smith, P. 51. Art. 139. which, in fome refpects, feems right (He does not indeed determine the Refraction by drawing a right Line, from A or B, to the Center of either Surface; nor does he give any certain Rule to determine the Refraction. It is impoffible to afcertain the Point where any incident Ray, from an Object, cuts the Lens; feeing that, the inclination of its Surfaces is continually varying, (from the Center to its Extremes) but I find there are Cafes in which it is very exceptionable. For, according to this method of determining the apparent magnitude, or place of the Object, it can never appear larger than at the diftance of the Lens. But it is certain, that the Eye and the Object may be fo fituated, in refpect of the Lens, that notwithftanding, the Object, A B, is confiderably lefs than the Lens, it will appear larger; and confequently, its apparent place is on this Side (in the focus of the incident Rays) between the Eye and the Lens, as at *a* E *b* the Eye being removed to *E*; for *a b*, the Image of A B, appears, to the Eye at *E*, under a greater Angle (*a E b*) than the Lens C D; and therefore, A B, or its Image *a b*, appears, to the Eye at *E*, larger than the Lens C D, which it never could do, if its apparent place was on the other fide of the Lens.

In this Cafe, the Object A B appears erect; but, if another Object, F G, be placed beyond the Focus of the Lens, and Right Lines F *f*, G *g*, be drawn, through the Center, I, of the oppofite Surface, confequently perpendicular to it; and if F *G*, G *F* be drawn parallel to the Axe cuting F *f* and G *g* in *G* and *F*; then is *F G* the Image, or apparent place of F G, which, in this Cafe, is inverted; and confequently, the real Object F G is not feen; for the extreme F is feen at f, and G at g, on the furface of the Lens, in a contrary direction to their true places.

The contrary effect is produced through a concave Lens, in which, the Object always appears lefs than its real magnitude, according to its Diftance; which, being but the reverfe of the other, 'tis needlefs to illuftrate it.

It is evident, that Vifion is conveyed through any tranfparent Medium whatever, obliquely fituated, in refracted or broken Lines; confequently, in fuch Cafe, the Object never is where it appears to be. For, the Point A, which is fuppofed to be feen by the Eye at E, appears to be at *A* or a, and not at A where it really is; and the Point S, which is feen directly through the Center of the Lens, although it be feen in the Right Line E S (for there is no refraction perpendicularly thro' parallel Surfaces of any kind) yet, its apparent place is at *S*; and, if either the Eye or the Lens be removed, the apparent place of the Object is varied.

2. Lenfes of all kinds, excepting fuch as are plane on both Sides, or fuch Menifcus, (hollow on one Side and convex on the other) as being portions of concentric Spheres, confequently parallel Surfaces, partake of the nature and properties of Prifms, whofe Surfaces are inclined to each other. Wherefore, the Rays, in paffing through them, are more or lefs refracted, according as the Surfaces are more or lefs inclined; that is, the lefs the radius of the Sphere, to which the Lens is formed, the greater is the magnifying power; becaufe, the refracting Angle is greater; and, confequently, projects the Rays to a greater diftance from the Center.

Hence it becomes a Paradox, how a large Object (I mean one, which, according to its Diftance, occupies, or appears equal to the whole furface of the Lens) can be regularly magnified; for fince it is manifeft, that through parallel Surfaces there is no fenfible Refraction (except the Surfaces are confiderably diftant from each other and feen through obliquely, as *E* b c e, Fig. 12.) and the furfaces of every Lens are parallel at the Vertex, and nearly fo at a fmall Diftance from it; and, the greater the angle of Inclination, the Refraction is greater; wherefore (the furfaces of a convex Lens, being leaft inclined at its extremes, i. e. the refracting Angle is

the

the greateſt; and, ſeeing it is continually varying, from the Vertex to the Extremes) it ſeems reaſonable, to conclude, that the Object would be more diſtorted about the edges of the Lens than near the middle; which I do not find to be the Caſe. Query; by what means is the Object magnified equally?

3. There is another circumſtance, which I do not remember to have ſeen noticed, by thoſe who treat of the properties of Priſms, in reſpect of Refraction; and which, I do not ſee a reaſon for. Right Lines, ſeen through a triangular Priſm, or any two inclined plane Surfaces, being parallel to the interſecting Line of the two Planes, that is, to the refracting Angle of the Priſm, do not appear Right Lines, but, curved; and they are more or leſs curved, as the Angle of Refraction is greater or leſs; the Curve being concave towards the refracting Angle.

Now it is reaſonable to ſuppoſe, that the ſyſtem of Rays from any Right Line to the ſurface of the Priſm, which is a Plane, would alſo form a Plane; which, by its interſection with the ſurface of the Priſm, muſt neceſſarily generate a Right Line, in any poſition of the Priſm; which, would be the Repreſentation of the original Line; and, when the Angle of Refraction is parallel to the original Line, they will alſo cut the other Surface, towards the Eye, in a Right Line, parallel to the oppoſite; how is it then that it appears curved? The vertex of the Curve, whatever it be, is where it would be cut by a Plane, paſſing through the Eye, perpendicular to the Original Line and the Planes of the Priſm.

I have already deſcribed a Priſm, in the laſt Section, and the nature of Refraction by it, as much as is my deſign, in reſpect of the deception of Viſion through a Priſm. Thoſe who would be more acquainted with the properties of Priſms and Lenſes, in general, I refer to Sir Iſaac Newton's, or, where they are treated more at large, to Smith's Optics; where, if he has patience to go through it, he will find enough to exerciſe his patience on. I do not mean to diſparage the Work, for I believe it to be the beſt of the kind, in many reſpects; yet, I think it deficient, in ſome Caſes; but, in reſpect of Colour, &c. it is of a piece with the reſt, and, being dwelt too long on, becomes tedious, if not trifling.

4. As to the Phænomenon of Colours ſeen through a Priſm it is really ſurpriſing; but, it is only the Edges of Objects that are tinged or produce the Colours. A perfectly plane Surface is not at all varied in its Colour, when ſeen through a Priſm; except at the Edges, when it is joined or oppoſed to a darker Body; and thoſe Edges are moſt coloured when they are parallel to the refracting Angle; for, when they are perpendicular to it, and ſeen direct through, they have no Colour at all, but the natural Colour of the Body, whatever it be. Alſo, the Colours ſeen through the Priſm, do not always follow in the ſame order as when a Beam of the Sun's Rays paſſes through; if a ſmall Object be oppoſed to the Light (as, a Bar in a Saſh-Window) being parallel to the Priſm, the Object is wholly loſt in the Colours, between the Red and Blue; the Red being towards the refracting Angle; an intenſe Red always follows after any Line or Edge of an Object which is oppoſed to, or which obſtructs the Light in one poſition of the Priſm, the brighter Colours ſucceeding; but, being reverſed, there is a deep Purple, and Blue following; the other Colours are very faint. The Colours are more intenſe and vivid the brighter the Object, or the greater the oppoſition and obſtruction of the Light by an opake Object. If a ſlip of White Paper be laid on any dark Body, being ſeen through a Priſm, its Edges are bordered, one with Red, the other Blue, according to the refracting Angle; but being laid on another, of the ſame Colour, none is perceived, except it makes a Shade on the other, not lying cloſe, which Shade occaſions Colour. Alſo, if a ſtrong, black Line be drawn thereon, the Red and Blue proceed contraryways from the Line, which is quite loſt between them; the Colours being produced, on any Surface, only where there is an oppoſition with a darker Body. How, then, it is poſſible to deduce or to draw a concluſion, from ſuch Experiments, that all Objects, which we perceive, are by Nature of the ſame, or have no Colour at all, ſave what is effected by the Light reflected from their Surfaces?

Plate II. 5. As in Nature, there is neceffarily Refraction at the common Surface of any two different Mediums, whether folid or fluid, except when we look perpendicularly through the Surface, or Surfaces, at the Object, although the true reafon of it be to us unknown; fo it would be fuperfluous and foreign to my Defign to multiply Cafes wherein it happens. I fhall, therefore, only take notice of one common Cafe, which is fo very common, though but little attended to, as to be obvious to every Perfon who is bleffed with the faculty of Seeing.

Let any Perfon put a ftreight Stick into Water, flanting in any direction, and it immediately appears to be broken, at the furface of the Water; the part within, appearing to take a different direction to that which is out of the Water; and, the more it deviates from a Perpendicular, the more it is refracted or broken, till the Stick makes an Angle, with the furface of the Water, of about 45 Degrees; and then, the Refraction is continually lefs, which is evident; for, when the inclination of the Stick, to the Surface, is fuch, that the Angle it makes with the Surface is very fmall, confequently, the Refraction cannot be great; and being immerfed perpendicularly, there is no Refraction; which is the fame thing as looking directly into the Medium, at an Object in the Water, or through a Body of Glafs, very thick; in which Cafe, the Object appears to be confiderably nearer, but, it is in the fame Direction as we fee it. Alfo, when we look at the Heavens of Stars, &c. thofe which are in the Zenith are feen in a Right Line; and, the farther they are from that Point, the more the line of direction is refracted or broken, fo that, they are fuppofed not to be in that place in the Heavens where they appear to be; which Refraction, is faid to be owing to the Atmofphere of Air and Vapours, which furrounds the Earth, being denfer than the upper Regions.

6. It is a common, though curious and entertaining, Experiment, to immerfe a piece of Money or other Object in a Bafon full of Water; which will appear to be raifed confiderably higher than when the Veffel is empty.

Fig. 15. Let ABCD reprefent a Veffel, fuppofe of Glafs; and fuppofe E an Eye looking at an Object, at F, at the bottom of the Veffel, being empty. Whilft the Eye remains fixed, at E, fo that the Object at F, can juft be feen over the edge, at C, let there be Water poured gently into the Veffel; as the Veffel fills, the Object will appear, to rife, gradually, to G, fo as to be feen quite clear of the edge of the Veffel, in the direction EG.

Now it is certain, that the Object is at F, which appears to be at G; confequently, if the real Object be feen at all, it is feen in the refracted Lines FI, IE, where the Right Line EG, to its apparent place, cuts the furface of the Water; but, more probably, the Image of the Object, only, is feen at G. Confequently, a ftreight Stick or Wire being put into the Water, from E to F, will appear broken at the Surface (at C) and appear to go in the Direction CG.

Again. Let the Eye be removed to *E*, the Water remaining in the Veffel, fo that the Object is apparently feen over the edge, at C; and, whilft the Eye remains fixed, let the Water be drawn gently off, by means of a Cock, or otherwife, at the bottom; the Object, apparently at G, will gradually fink lower in the Veffel, and totally difappear from that Station.

Hence it is manifeft, that when the Water is in the Veffel, the real Object, at F, is not feen by an Eye at *E*; for, if it was poffible, it muft be feen through the Side of the Veffel, at H, which it is plain is not the Cafe; and, hence it is plain, that the Object being feen, apparently at G, is a manifeft Deception in Vifion. The Cafe is the very fame, in Objects feen oblique through Glafs, or any other pellucid Subftance; excepting fome fmall variations in the degree of Refraction.

7. Refraction in Water, according to all writers on Optics, is fubject to one invariable Law, without any fenfible error, but it is not to be demonftrated mathematically. I have made the Experiment myfelf, as accurately as it will admit of, and find it to be nearly as follows.

If AB be ſuppoſed the ſurface of Water, and CE, DE or FE an incident Ray of Light falling on it, at E; on which Point, ſuppoſe a Circle be deſcribed, and the Perpendicular, EG, drawn. Then, if CE be produced to c, and ac be drawn, parallel to AB; take ab equal to three fourths of ac, and draw bd perpendicular to ac, cuting the Circumference in d; the Ray CE is ſuppoſed to be refracted, at E, and go in the direction Ed, into the Water. By the ſame Rule, DE goes in the direction Ee, and FE in Eg. Fig. 16.

But, the ſame Rays CE, DE, &c. will alſo be reflected in equal Angles, GEH equal DEG, &c. Can the ſame Ray be both reflected and refracted? impoſſible; yet, an Object (❂) may be ſeen at H, by the reflected Ray EH, as well as at e, by the refracted Ray Ec. It is however certain, that if a Circle be deſcribed on the Plane AGBg, and Lines are drawn thereon, as in this Figure; being immerſed perpendicularly in Water, to the Diameter AB; CEd, DEe, and FEg will appear Right Lines; which is very ſurpriſing. But yet, I do not ſee which way this Experiment proves, that Rays of Light go in thoſe directions into the Water; the whole Maſs being illumined in every part. If the Eye, being immerſed in Water, at e, ſees the Object ❂ over the edge of the Veſſel, or any other obſtacle, at E, on the Surface (which, I preſume is the Caſe) it indicates, that the Object is not ſeen in a Right Line; or, rather, that its Image is ſeen at E, on the ſurface of the Water.

8. Before I conclude this Section and Subject, I cannot help taking notice of an extraordinary paſſage; which is in the Concluſion drawn from Prop. 8, Part 3, of the ſecond Book of Newton's Optics, Page 69; concerning the extraordinary poroſity of Water; which is 19 times lighter, and conſequently, he ſays, 19 times rarer than Gold; and Gold is ſo rare, that Water may be forced through its Pores. For, as he was informed, by an Eye witneſs, a Globe of Gold being filled with Water and ſodered up (but of what thickneſs he does not tell us; I have heard ſay above a quarter of an Inch) and, being preſſed with great force, the Water ſqueezed through its Pores; and ſtood, all over its Surface, in multitudes of ſmall drops, like Dew. From which, he concludes that Gold has more Pores than ſolid Parts; but how ſuch a concluſion can poſſibly be drawn I cannot conceive. I ſhould ſuppoſe, that, in ſuch Caſe, the Water would ſpout out in ſtreams, rather than ſtand on the Surface like Dew. Yet Gold will not admit either Air or Light through its Pores, though much rather Fluids than Water; which, he alſo concludes, from the ſame Experiment, has above forty times more Pores than Parts; and conſequently, Gold (the compacteſt Metal we know of) according to that Ratio, contains above twice as much empty Space as Matter.

It is no wonder that Water is pellucid, being ſo extremely porous, and admits Light ſo freely through its Pores; but, it is ſomewhat ſurprizing that it is not compreſſible, which all porous ſubſtances muſt be. And I think it alſo ſurprizing, that a Man of Sir Iſaac's ſagacity ſhould advance ſo much on the credit of any Experiment he never tried himſelf; which, he certainly had opportunity enough for, being Maſter of the Mint near thirty years.

If Water be ſo full of Pores, as Sir Iſaac imagines, it might be compreſſed into leſs Space; which, the ſame Experiment abſolutely proves that it cannot. If it be compoſed of globular Particles, they may certainly be ſqueezed ſo cloſe together, that (ſuppoſing every Globe inſcribed in a Cube) the difference between the quantity of Matter and of Space, would be no more than the difference between a Globe and a Cube, whoſe Diameter and Side are equal, which is nearly double; I mean, that the quantity of Matter contained in Cubes, which admit of no Space between them, is nearly double of that which is contained in the inſcribed Globes; and conſequently, the Space between Globes, lying in that poſition, is nearly equal to the Globes; for, the ſame Ratio muſt continue *ad infinitum*.

But it is manifeſt, that Globes will lie conſiderably cloſer together; as Circles leave only triangular ſpaces between them; ſo Globes of equal Diameters, will lie ſo together as to leave a kind of triangular pyramidal or priſmatic Space between,

Plate II. joining each other. The difference between the Globes and Space may be very nearly afcertained, by puting a quantity of equal Globes or Bullets into a hollow triangular Pyramid, which may eafily be made, the true figure of a pyramid of Globes (for it is a regular Tetrahedron) and, having placed them regularly, pour in Water juft to cover them, filling up the Interftices between; then take out the Globes and fill the Box equally with Water. I have tried the Experiment, and find the difference between the Globes and Space to be, as 2 to 1, as near as can be; in the former Cafe they are nearly equal. Query; where is the remaining 40 times to be found? are the leaft particles of Matter, which cannot be conceived of any dimenfions, to contain, in each, 40 times more Space than Matter? for, as I have obferved before, the ratio, of the Globes to the Space between them, will continue to infinity; and what other regular Figure can there be, to contain more Space between them than Globes?

The Queftion then is, whether we are to give credit to fuch Affertions, or are we to credit the evidence of our own Senfes and Reafon? I fhall truft the latter, before the bare affertion of the greateft Man that ever lived.

9. As I have had neither time nor inclination to go through the whole of Smith's Optics; I fhall juft take notice of one paffage in it, in which his geometrical reafoning is fomewhat erroneous; but, whether it be of Confequence, in the following part of the Work, I have not enquired; as the Subject is not to my prefent purpofe. I fhall, literally, quote his own words, as follow.

Fig. 17. In Vol. 1. Page 59. Art. 157, he fays, "The apparent magnitude of a given "line, A B, feen very obliquely at a given diftance, O A, increafes and decreafes "in proportion to the increafe and decreafe of O P, the perpendicular diftance of "the Eye from the line A B produced; provided the diftance A O be very large in "comparifon to A B. For let the Ray B O cut a line A C perpendicular to A B "in C; and while the Eye is raifed or depreffed in the perpendicular O P, the "line A C will increafe and decreafe as O P does; and fo will the Angle A O C, "fubtended by A C, and this Angle meafures the apparent magnitude of A B."

Firft, he fays, that the apparent magnitude of AB increafes and decreafes in proportion, as the Eye is raifed or depreffed, in the perpendicular OP; and the Angle, it fubtends, meafures the apparent magnitude of A B.

Let P O be produced to T; in which, take feveral Divifions, each equal to P O, at Q, R, S, and T; and draw Q A, Q B and R A, R B, &c.

Now, becaufe P Q is double P O, and P R is triple, the Angle B Q A, (according to him) will be double B O A, and B R A triple; for, A G being drawn perpendicular to A B, it is certain, that the Divifions A C, C D, &c. are equal, A G being parallel to P O †; confequently, A C increafes as O P does; and fo, he fays, does the Angle A O C, fubtended by A C; wherefore, the Angle at Q is double, and, at R triple, of the Angle A O B. But, the Perpendiculars A C, A D, and A E, are not the meafures of thofe Angles; for they are truly meafured by an Ark of a Circle, of the fame or an equal Radius, only.

† Cor. 6. 6. El.

According, then, to his Words, the Angle A O B increafes as P O increafes; which I deny, and will prove to the contrary.

It is obvious, that the Angle A Q B is confiderably larger than A O B, but it is not double, although P Q is double P O; but, A R B is very little larger than A Q B; and notwithftanding, P R is triple P O, yet the Angle A R B is not double A O B; as the Arks *a b*, *c d*, and *e f*, evince.

Now, by the fame reafon, the Angles A S B, A T B are continually greater than A R B; but, they are continually lefs. For, if P R be a mean Proportional between P A and P B, the Angle A R B is the greateft that can be made, by Right Lines drawn from the Extremes of A B, and touching the Perpendicular P T.

DEM. Defcribe a Circle through the three Points R, A, and B; by Prob. 40 Geo.
Then, becaufe P R is a mean Proportional between P A and P B (by Hyp.)
the fquare of PR is equal to a Rectangle under P A and P B; } Cor. 9. 6. El.
i. e. P B × PA = PR□ — — — — — —
Confequently, P R will touch, or be a Tangent to that Circle, drawn through
the Points R, A, and B — — — P. 16. 3. El.
But, the Angle A R B is at the Circumference of that Circle; conf. A S B and
A T B, which are beyond the Circumference, are lefs than A R B. — 20. 6.
Therefore, A R B is the greateft Angle, that can be, touching the Tangent
P T, fubtended by A B; and confequently, the Angle A S B is not encreafed,
as the Eye is raifed in the Perpendicular O P.

10. Again. He faies, in the following Paragraph, as deduced from the foregoing, that "the apparent magnitudes of equal parts, A B, a b, of a Line, P A a, feen very "obliquely, at great diftances from the Eye, are, reciprocally, in a duplicate propor- "tion of thofe diftances. For example; let O b be double of O B, and the angle "O B P will be double of O b P; and accordingly, fince A B, a b are equal, the "perpendicular A C will be double of a c; and, being feen twice as near as a c, will "appear four times bigger than a c. Again, if O b be triple of O B, the line A C "will be triple of a c, and being feen three times nearer than a c, will appear nine "times bigger than a c; and fo on.

Let O b be double O B; I fay, the Angle O B P is greater than twice O b P; Fig. 17.
and he expreffly faies that O B P is double O b P.

DEM. Becaufe O b is equal 2 O B, b B is greater than O B. — P. 13. 1. El.
wherefore, the Angle b O B is greater than O b P. — — 12. 1.
But, the Angle O B P, being external, in refpect of the Triangle b O B, is
equal to b O B + O b P; wherefore, it is greater than double O b P. 10. 1.
But, if the Angle a b c be equal half A B C, a c will not be half A C. 3. 6. El.
Confequently, fince the Angle A B C is more than double a b c (and A B is
equal to a b) A C is ftill more than double a c.
Therefore, the apparent magnitude of A C to a c is more than duplicate;
feeing it is more than half a c, and is alfo feen at lefs than half the diftance.

Now I am far from fuppofing, that this Author was fo deficient in Geometry as thefe Examples feem to indicate; nay, I am well convinced he was not, as is evident in the next Book; and his Provifos are fome extenuation of the errors; he ought, then, to have told us that they are nearly fo, by approximation, and not that they are fo, in exprefs Terms. But, in any Cafe whatever, there is not an equal ratio, or nearly, between the increafe and decreafe of the Perpendicular A C, and the Angle A O C.

As this and the preceding Article are the only paffages which any way tend to advance the Theory of Perfpective (having been quoted, by Mr. Kirby, for that, though but little, purpofe) I have, therefore, been more particular in my remarks on them. And, as what I have advanced in the two laft Sections, and part of the firft, is not directly to the purpofe of Perfpective, it will, I know, by fome, be deemed impertinent and foreign to the Defign of this Treatife; let them, if they pleafe, pafs it over, and proceed immediately to the Subject, which is not vitiated by it. As I do not intend to publifh a Treatife on Optics, and as Perfpective has a near affinity to that Science, what exceptions I have always had, to fundry paffages in the works of optical writers, I thought proper to give here, where I was treating on an effential part of the Theory of Perfpective, and a branch of the Science of Optics.

I hope I have not trefpaffed, too much, on the time and patience of the candid and unprejudiced Reader. I fhall now proceed to Book the fecond, which treats on the Theory of Perfpective; where, I fhall endeavour to make fome amends for his time fpent, I hope not loft, in perufing this Digreffion.

BOOK

BOOK II.

Of the Theory of Perſpective, rectilinear and curvilinear.

SECTION I,

Containing a general INTRODUCTION to PERSPECTIVE.

TO define the Terms of Art peculiar to any Science, with brevity and perſpicuity, I have always looked on as a particular excellence in the Work; but I have frequently found, that an affectation of brevity has left the Term, intended to be explained, rather obſcure, at leaſt doubtful; whilſt others, endeavouring to render it clear by a multiplicity of words, have, at laſt, involved it in perplexity.

Although my deſign is to be as brief as the nature of the Subject will admit of, yet I am afraid I ſhall rather be thought prolix, than otherwiſe, in ſome of the following Definitions or explanation of the Terms; but certain I am, that, the Time ſpent in acquiring a perfect knowledge of all the Terms I have defined, will not be loſt, as they contain many uſeful Leſſons to a young Student. I have endeavoured to explain every Term in the moſt familiar manner; not ſaying more than was neceſſary, yet, enough to be clearly underſtood. To be too brief is worſe than prolixity, ſo it be not tedious and trifling; the one leaves us doubtful, the other, probably, makes it clear at laſt.

I may, perhaps, be particular and ſingular in my opinions; but, I think it better to define each Term ſeparately, than to include ſeveral in one Definition, as is very frequent; and I always chooſe to name the Term, I mean to define, firſt, rather than end the Definition with it, or name it promiſcuouſly. My reaſon for which is, that 'tis much eaſier to find when refered to; or, when there is occaſion to look for any particular Definition, without reference, the Number not being known.

The Order in which the Terms are defined is, with me, a material circumſtance; begining at the Foundation, and going gradually on in a regular ſucceſſion; never uſing any Term, if poſſible to avoid it, in the explanation of another, which has not already been defined. In general, I find them promiſcuouſly jumbled together, without Order; going, as it were, from one end of the Science to the other, to and again; too haſtily introducing, perhaps, ſome new or favourite Term, before others which ſeem neceſſary to be firſt known, in order to prepare the way, by removing ſome impediment. How I have ſucceeded, muſt be left to the deciſion of the candid and impartial Reader.

I am no favourer or admirer of new and uncommon Terms; nor, indeed, do I think that every writer, on any Science, has a right to impoſe new Terms; unleſs he has found out ſome new Principles, on which it was not poſſible for him to expatiate, by the Terms already known and in uſe; as Dr. Brook Taylor has done in Perſpective; on which, I have ſpoke more largely in another place. It was impoſſible for him to convey the Ideas he intended to inculcate by the old Terms, and therefore, he was under the neceſſity of inventing and enforcing new ones; which are extremely expreſſive of the thing meant. But, if every writer, on that or any other Subject (who, becauſe he knows ſomething of it, imagines he knows more

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than any who have wrote before him) was to take the liberty to impofe new and unmeaning Terms, of his own, fuited only to his own trifling Ideas of the Subject, the Science would, by that means, become perplexed and intricate; each Perfon, who happened to receive his knowledge of it from different Books, would, confequently, underftand and call the fame Thing by different Names; than which, efpecially when they are abfurdly or falfly named, nothing tends more to perplex, and involve the Science in obfcurity.

I have already, in the Preface, given my reafons for omiting, in this Treatife, geometrical Definitions and Problems; becaufe, I fuppofe the Reader already tolerably verfed in Geometry; if not, I advife him firft to ftudy it, at leaft Practical Geometry; without which, it is ufelefs to attempt, and impoffible to fucceed in the Study of Perfpective: the better he is acquainted with Euclid, the greater progrefs will he make in Perfpective; of which, Geomety is the foundation. To affift him in it, I have compiled and compofed a Volume (mentioned in the Preface) which may be called an abridgment; it, neverthelefs, contains all that is effential. My chief aim, in that work, was to make it ufeful, the Study of it pleafant, and attainable by any tolerable Capacity, and applicable to various ufes in Life; particularly fubfervient to this Treatife of Perfpective, as I always refer to it for Demonftration: in which cafe, it may be deemed a part of this Work, and ought always go together. But, although I have, there, fully defined a Plane (Def. 6th.) yet, as it is fo very effential in Perfpective, it was by no means proper to omit it here; feeing that, on it the whole Theory of Perfpective is built.

A PLANE is a perfectly even, ftreight, and regular Surface, which is neither convex nor concave in any part; but agrees, in every part, with a Right Line or ftreight Ruler, applied, any how, to the Surface.

2. By the motion of a Right Line, a Plane may be conceived to be generated; either by a direct, lateral motion, or by fuppofing it whirled around (fo as not to generate a Cone) on any Point in it.

3. It is eafy to conceive, that, if the Eye of a Perfon be in a Plane, or in a continuation of it, the neareft extremes or limits, towards the Eye, hide all the reft of the Plane; for, the whole Plane vanifhes, and is loft to an Eye in the Plane; which appears but a Right Line, extended in length to the apparent dimenfions of the Plane, confidered as having only length and breadth: thicknefs or fubftance is the property of Solids, a Plane has none; 'tis the Surface, only, that is confidered.

N. B. The Picture (in Perfpective) is always underftood to be a Plane. Therefore, the Board, Canvas, or Paper, on which we draw, is the Plane of the Picture.

N. B. 2. A Plane may be of any Figure; and, in Perfpective, it is frequently confidered as being infinitely extended, without regard to Figure, or to its limits.

A CUBE is bounded by fix Planes, which are all Squares; as A, B, X. Fig. 1. Plate III. Every other Parallelopipid has alfo fix Planes, which are all Parallelograms; either right angled, as Fig. 2, or acute angled, as Fig. 3.

A PRISM is a Solid, whofe Bafe, A, and Top, B, are either fimilar Triangles*, Quadrangles†, or Poligons‡ of any Number of Sides. The other Planes or Surfaces are all Parallelograms. (As X, X.)

* Fig. 4.
† Fig. 1, 2, & 3.
‡ Fig. 5.

PYRAMIDS have their Bafes (A) of any Number of Sides. The other Planes, of the Sides, are all Triangles. (As Y, Y.) Fig. 6.

N. B. No Solid can be formed of Planes, having lefs than four; and that muft be a Pyramid.

PlateIII. Of PLANES and their POSITIONS, in GENERAL.

Fig. 7. 1. Horizontal is the firft and moft natural pofition of Planes; fuch, are all Planes which are parallel to the Horizon; confequently, all horizontal Planes are parallel amongft themfelves. As Z, H, H.

2. Vertical. All Planes which are perpendicular to, or which cut the Horizon at right Angles, are called vertical Planes. As V. Fig. 7, 8, and 9.

Fig. 8. Vertical Planes may be in all pofitions, in refpect of each other, *viz.* parallel, perpendicular or inclined; as may be conceived, by revolving a vertical Plane, on a Right Line, AB, perpendicular to the Plane of the Horizon; confequently, they will, if produced, all pafs through the Zenith and Nadir * of our Horizon.

3. Inclined. All Planes whatever, which are neither parallel nor perpendicular to the Horizon, are Inclined Planes.

For, if a Plane cut the Horizon, or would if produced, in an acute Angle, it is not vertical or perpendicular; confequently, it inclines to the Horizon on one fide more than the other; which, Inclination, is always meafured on that fide making the acute Angle; as, for Example.

Fig. 9. The Plane X inclines to the Horizon, in the Angle BCA; and it is alfo faid to incline to a vertical Plane, in the Angle BCE; which Angle, if they have the fame Interfection, CD, or parallel Interfections with the Horizon, is the Complement of it sinclination to the Horizon. See N. B. Def. 13th. Geo.

4. It may not be improper, here, to obferve (for it is neceffary to know and underftand well) that the Angle of Inclination, of one Plane to another, can be meafured, only in a Plane to which both the other are perpendicular; or, which is the fame thing, if a Line be drawn in each Plane, from the fame Point in their common Interfection, and perpendicular to it, an acute Angle made by thefe Lines is the Inclination of the Planes; for it is evident, that a Plane paffing through thofe two Lines will be perpendicular to the common Interfection † and confequently to both Planes. To illuftrate it.

† 2. 7. El.

Suppofe the horizontal Plane Z raifed up into the Pofition of X, inclined to the Horizon; the line CD, on which it was fuppofed to turn, may be confidered as the common Interfection of the two Planes, X and Z.

It is evident, that the point A will, in that motion, have defcribed the Ark AB; and, if the Angle ACD be a Right one (as it is fuppofed) BCD is ftill a Right Angle; wherefore, AC and CB are both perpendicular to CD; § and the Plane ABC, defcribed by the motion of CB, is alfo perpendicular to CD, † and confequently, to the two Planes X and Z.‡ Therefore, the Angle ACB, in the Plane ABC, is the Angle of Inclination of thofe two Planes.

§ Def. 11. Geom. † 2. 7. El. ‡ 9. 7.

Draw AD, at pleafure, cuting the common Interfection, CD, in D; and, from the fame point D, draw DB, in the inclined Plane X.

Dem. Now, if ACD be a Right Angle, ADC is acute — Cor. 3. 10. 1. El. wherefore, AD is longer than AC; and alfo, BD than BC — P. 12. 1. El. But, the Chord, or Subtenfe, AB, fubtends both the Angles, ACB and ADB; confequently, the angle ADB is lefs than ACB. — — Cor. to 14. 1. El.

Or, fuppofe BF perpendicular to the Plane Z; and let FD be drawn.

Then, a Plane DBF, paffing through BF, is perpendicular to the Plane Z, but not to X. And, becaufe FD is longer than FC, and BD than BC, it is manifeft, that the Angle BCF is greater than BDF.

But, the Plane CBF is perpendicular to both the Planes, X and Z (as before) and, confequently, to their Interfection CD.

* Imaginary Points in the Heavens, diametrically oppofite to each other; the one perpendicular over our Heads, the other under our Feet, in the lower Hemifphere.

From which it is clear, that the Angle made by a Plane cuting two other Planes, perpendicular to their common Interſection, is the Angle of Inclination of thoſe two Planes; ſeeing that, the Angle made by any other Plane, paſſing through A B or B F, will neceſſarily be leſs, the greater the Inclination of A D, or F D, to C D.

To ſet this matter in the cleareſt Light poſſible, it being ſo very eſſential in Perſpective, as well as in other Sciences and Arts, I have added the following Figure.

Let A B C and C B D be two rectangular Planes, cuting each other in B C, their common Interſection. Let E F and F H be two Right Lines, one in each Plane, perpendicular to their Interſection B C, at the ſame Point, F. Fig. 10.

Wherefore, the Plane E I F G, paſſing through thoſe Lines, is perpendicular to both Planes, A C and C D; § and, the Angle E F G, made by that Section, is the largeſt that can poſſibly be made by a Plane which is perpendicular to either of them. § 2. 7. El.

For, ſuppoſe the Line E H perpendicular to the Plane A C B, only; and, a Plane EIKL to paſs through that Line, it will be perpendicular to the Plane AC;† and becauſe the Plane E F G is perpendicular to both the Planes A C and C D, and paſſes through the ſame Point E, it will, alſo neceſſarily, paſs thro' the Line EH; wherefore, E H is the common Interſection of thoſe two Planes,* EIFG and EIKL produced. I ſay, that the Angle E F H, made by the Plane E I F G, is greater than E K H. † 9. 7. El.

Dem. Now, E H, the common Interſection of the two Planes I F G and I K L, together with the Interſections E K and K H, of the Plane I K L with the two Planes A C and C D, form a Triangle; and ſo does the ſame line E H with the two Interſections E F and F H, made by the Plane I F G, with the ſame Planes A C and C D.

But H F K is a Triangle, and, the Angle H F K is preſumed to be a Right one; wherefore, H K is longer than H F;‡ and, for the ſame reaſon, E K is longer than E F; and conſequently, the Triangle E F H, having one Side (EH) common with the other Triangle E K H, and, having the other two Sides, EF and F H, leſs than the two Sides E K and K H, reſpectively, they, therefore, contain a larger Angle, *viz.* E F H than E K H. — — Cor. to 14. 1. El. ‡ 12. 1. El.

But, the Plane I F G is perpendicular to both the Planes, A C and C D; and the Plane I K L is perpendicular to one of them (A C) only.

Therefore, the Angle EFG, made by the ſection of a Plane which is perpendicular to both the other, is larger than any Angle made by any other Section, of a Plane perpendicular to one of them only; and conſequently, the Section E F G meaſures the true Angle of Inclination of the Planes A B C and C B D. Fig. 10.

N.B. The Angle made by the ſection of a Plane inclined to both Planes, A C and C D, may be either greater or leſs than E F G.

5. Although it is not poſſible to conceive an Idea of a Plane abſtracted from one or other of the three Poſitions I have explained, yet, in the application of Planes, in Perſpective, as in Geometry, no particular regard is had to them; for one Plane is ſaid to be perpendicular to another, if it makes right Angles with the other Plane, as H to V: each of which is ſaid to be perpendicular to the other, notwithſtanding one of them (H) is horizontal. Fig. 11.

The Planes X and Y are alſo ſaid to be perpendicular to each other, although both are inclined to the Horizon; and, whatever their Inclination to the Horizon may be, it matters not, if they make right Angles with each other, as at C. For, if the Plane Y was turned up, on AB, its interſection with the Horizon, into the vertical Poſition W, and, along with it, the Plane X, into the horizontal Poſition Z; their Poſition, in reſpect of each other, is not altered, if the Angle, at D, be ſtill a Right one, as before, at C.

* E I is the true Interſection, perpendicular to the Plane A C; and conſequently, E H would be I E produced. But, as it would run into the Figure below, I thought it beſt to diſpenſe with it, as the Demonſtration is the ſame.

6. So

PlateIII. 6. So likewiſe, one Plane is ſaid to incline to another, if they do not interſect at right Angles, as H and X, or would not if produced, as X and V.

The Plane X being inclined to both H and V, (Art. 3. of Planes) they are, for the ſame reaſon, both inclined to X; yet, one is horizontal and the other vertical; for, the inclination of two Planes is mutual. So that, when it is ſaid that one Plane is perpendicular or inclined to another, it means nothing more, than, that they are at right Angles, or otherwiſe with each other; no regard being had to the horizontal or vertical Poſition of either; except the Poſition of one is previouſly known or determined, to which, the other is ſaid to be perpendicular or inclined.

7. In Perſpective, it is alſo frequently ſaid, that, Lines are perpendicular to certain Planes; whereas, if the Plane be vertical, it is eaſy to conceive, from what has been ſaid, that all Lines, which are perpendicular to a vertical Plane, are horizontal,
Fig. 12. and parallel amongſt themſelves; as A B, E F, and C D, to the Plane G I K. Yet, the Planes, in which theſe Lines are, may be either horizontal, as ABFE; vertical, as EFCD; or inclined to the Horizon, as A B C D.

If the Plane be horizontal, the Lines perpendicular to it are really perpendicular i. e. to the Horizon; as A J, E D and F C. But, if the Plane be inclined to the Horizon, then, the Lines, which are perpendicular to it, are alſo inclined to the Horizon, yet parallel amongſt themſelves.

Suppoſe the Plane ILMN vertical, and perpendicular to the inclined Plane GH; the Lines LI and MN, which are at right Angles with its Interſection, IN, are perpendicular to the Plane GH. But the line IP, which is perpendicular to the Horizon, is inclined to the Plane G H, in the Angle LIO. And OI, at right Angles with IP, is horizontal; but, it is alſo inclined to the Plane GH, in the Angle OIN equal LIP.

Dem. Let NI be produced, towards H. LI is perpendicular to NH.
Then, becauſe LIN is a Right Angle, LIH is, alſo, a Right one - C. 2. 1. El.
Conſequently, LIO, equal LIN—OIN, is equal to PIH, or LIH—LIP.

Every Right Line, therefore, which is neither parallel nor perpendicular to a Plane, is conſequently, inclined to that Plane; and its Inclination may be known, by drawing a Perpendicular from the extreme, or any other Point, as M, in the Line IM, to the Plane GH; the Complement of the Angle IMN, i. e. IML
* 4. 1. El. equal MIN*, is the Inclination of IM to the Plane GH.

Or, If a Plane (ILMN) be drawn, through the Line IM, perpendicular to the Plane GH; the Angle MIN, which the line IM makes with IN, the interſection of the perpendicular Plane with GH, is the Angle of its inclination to the Plane GH.

N. B. IN is the Seat of the Line IM or IO, and alſo of ML, or, of the Plane NL, on GH; produced by the Interſection of a perpendicular Plane paſſing through the Line, as above. Therefore, the Angle which any Line makes with its Seat, on a Plane, is the Angle of its inclination to that Plane.

8. PQRS is a horizontal Plane, cuting the inclined Plane KPQ in the Line PQ. But, PQ is not the Seat of the Plane PR, on KQ, it being inclined to KPQ, in the Angle SPT; for, if ST and RU be drawn, perpendicular to the Plane KPQ, the Lines PT and QU (joining the Points, T and U, where the perpendiculars cut the Plane, with P and Q) are the Seats of the Lines PS and QR; TU is the Seat of the Line RS, or of the Plane TURS; and conſequently, PQUT is the Seat of the Plane PQRS, on KPQ. Alſo, turs is the Seat of that part which is over the Ground Plane.

I would adviſe the young Student, who is not well verſed in theſe things, to make them familiar to him; for which, the reading over a ſecond time, with due attention to the Figures, will be ſufficient.

I have been more particular on this Subject, becaufe I have frequently known Pupils to be miflead, by calling Lines and Planes perpendicular, imagining them to be really fo, i. e. to the Horizon; from the common acceptation of the Term, Perpendicular, (to hang down, as a plumb Line) not confidering the Pofition of that Plane or Line, to which, the other Planes or Lines are faid to be perpendicular; and to which, due attention muft firft be given.

9. Suppofe the Object A I K C to be, in the lower part, a right angled Parallelopiped (the moft general form for Buildings, or the feveral Parts of a Building) the Planes B E D C, and A I B, of the Front and End, and their oppofites, are vertical; for they are perpendicular to the Horizon, or Ground (confidered as a Plane) on which it ftands. Fig. 13.

Now, the Lines A B and F E, in the Plane A I B, are perpendicular to the Plane B E D C; and the Lines B C, E D, and I K, are all perpendicular to the Plane A I B (i. e. the originals of thofe Lines are fo, in the real Object) yet, they are all parallel to the Horizon, and the three laft, parallel between themfelves, though in different Planes; § for, a Plane may pafs through any two Lines which are parallel. (Ax. 5.) § 4. 7. El.

10. In the Practice of Perfpective, it is often neceffary to fuppofe the Object, we are delineating, tranfparent, as if the whole Object was Glafs; and, the Planes or parts of the Building, which are adjacent, are fuppofed to be feen through the hither Planes; as ABCH, the ground Plane, and FEDG parallel to it; AFGH parallel (or otherwife) to BEDC, and HGDC oppofite to AFEB.

Thefe fix Planes, forming a right angled Parallelopiped, compofe the Body of the Building; every Angle of which, A, B, E, D, &c. is a folid Right Angle; each being compofed of three plane Right Angles, as A B C, A B E, and E B C, of the Angle B; confequently, AB, BC, and B E are each perpendicular to the other.

By means of this fuppofed tranfparency, the connection of the feveral parts of an Object are diftinctly feen, and delineated with greater accuracy; which, in fome Cafes, could not, without that expedient, be fo certainly afcertained.

11. The Roof, FIKDGE, is a triangular Prifm, in its conftruction. The Planes, EIKD on this fide, and FIKG on the other fide, are inclined to the Horizon; yet, thefe Planes may be perpendicular to each other, if the Angles, FIE, GKD, are Right. (See Art. 5.)

They are alfo inclined to the Front and its oppofite, which are vertical Planes; but obferve, that, their Inclination is not the internal Angle IEB, or IFA, for thofe Angles are obtufe; but, if either of the Planes be produced, as BEDC to LM, there is made an acute Angle (IEL) with that Plane, equal to the Complement of IEB to two Right Angles; which, is the Angle of Inclination of thofe Planes.

The inclined Planes, of the Roof, as well as the horizontal and vertical Planes AC, EC, &c. are perpendicular to the Plane AIB, and to HCK, its oppofite.

This, I hope, is fufficient to make all I have faid, relative to Planes and Lines (in refpect of their pofitions, and affinity to each other) clearly underftood; if fo, it is a very material point gained towards underftanding Perfpective clearly; which will be found a moft neceffary Introduction to the Theory, and will, alfo, greatly facilitate the Practice.

Plate IV.

SECTION II.

Of the feveral kinds of PROJECTION, and other introductory Matters.

PROJECTION is the defcription or delineation of Objects in Plano, or on a Plane, according to a certain Law; by means of Right Lines, called Rays, fuppofed to be drawn from every Angle of the Object to fome Plane.

Fig. 14. As A a, B b, C c, &c. which, taken altogether, are called the SYSTEM OF RAYS.

When thofe Rays are all united in a Point (as A O, B O, &c.) it is called a CONE or PYRAMID OF RAYS; and that Point (O) being fuppofed an Eye, and the Right Lines O A, O B, &c. Vifual Rays, the Syftem of Rays is then called, the OPTIC CONE, or PYRAMID (See the 4th Definition, in Optics; Book I. Page 9.)

The Figure defcribed, as a e f b, on the Plane V, a b c e, on the Plane Z, or, a b d e, on the Plane X, is the Projection of the Object A B C.

N. B. When the Object to be delineated is circular or globular, the Syftem of Rays is then properly called a Cone, for it is really fo; but when the Object is right lined, it is more properly called a Pyramid, as in Fig. 6. Plate 3; the Bafe, A, only, being confidered as the Object, and a B, b B, &c. as Vifual Rays to the Eye, at B, in the Vertex of the Pyramid, a B d, formed by the Rays.

ICHNOGRAPHY, or Ichnographic Projection, is that which is defcribed by Right Rines, parallel amongft themfelves and perpendicular to the Horizon, from every Angle of the Object, on a Plane parallel to the Horizon.

Fig. 14. No. 1. As A a, B b, C c, &c. from the Angles A, B, C, &c. of the Object, to the Plane Z, which is horizontal. The points, a, b, c, d, &c. where the perpendicular Lines, or Rays, cut that Plane, being joined by Right Lines, a b, b c, c d, &c. and diagonal wife, as a c, b d, &c. is the Ichnographic Projection of the Object, A C B.

The Figure, a e c b, projected on the horizontal Plane Z, is likewife called the PLAN, or SEAT, of that Object on the Ground Plane. The Points a, b, c, &c. are the Seats of the Angles, A, B, C, of the Object. The Line a b is the Seat of the Side A B, and b c of B C, &c. alfo the Diagonal a c is the Seat of A C; by which means, the Plan of the whole Figure is formed.

ORTHOGRAPHY. If the projecting Rays are parallel to the Horizon, and fall perpendicularly on a Plane (confequently vertical) the Figure defcribed, on that Plane, by the interfection of the Rays, is the Orthographic Projection.

Fig. 14. No. 2. The Lines A a, B b, &c. are parallel to the Horizon and to each other; which, falling perpendicular on the vertical Plane V, defcribe the Orthography of the Object A B C, on that Plane.

Of this kind of Projection there may be infinite variety; for, if either the Plane or the Object be turned, though ever fo little, the Figure will be varied on it.

The Figure, a e f b, thus projected, may likewife be called the SEAT of the Object A B C on that Plane; as a e c b on the Ground Plane.

a is the Seat of the Angle A, and b of the Angle B, &c; a b is the Seat of the Side A B, a e of A E; and, the Diagonal a f of the Side A F.

a e may alfo be confidered as the Projection, or Seat, of the Plane A E C, and a b of A B C; for, if they were produced, they would cut the Plane V in thofe Lines.

N. B. The Seat of a Point, Line, or Plane, may be had on any Plane in whatever Pofition; by drawing Right Lines from each extreme of the Line, &c. perpendicular to the Plane.

b f on the Plane Z, and b f on the Plane Y, are the Seats of the fame Line B F, in the Object A B C; and fo of the reft.

Ortho-

Orthographic Projection is usually called the ELEVATION. But, when it exhibits the End of a Building, or part of the End only, shewing the Projectures of the several parts, from the main Body of the Building, the Contour of the curves of Moulding, &c. it is called a PROFILE. And, when the Building, or other Object, is supposed to be cut by a vertical Plane, in any direction through the Building, the hither part being supposed to be removed, and the inside exposed to view, shewing the thickness, &c. of the Walls and Floors, the structure of the Roof and proportion of the Timbers, &c. it is called, a SECTION.

All these different kinds of Projection are entirely geometrical, and supposes the Eye of the Spectator at an infinite Distance; the projecting Rays being parallel.

SCENOGRAPHY is the Projection made by a Cone or Pyramid of Rays, Right Lines, from every Angle of the Object, converging to a Point. As O A, O B, &c. Fig. 14. No. 3.

Those Rays being cut by a Plane (X) passing, in any direction, between the Object and the Vertex (O) the Figure projected, by the Rays, on that Plane, is called the Scenographic Projection of the Object A B C.

If the Vertex, (O) of this Pyramid of Rays, be considered as the Eye of a Spectator, and the Right Lines O A, O B, &c. as Visual Rays, the optic Pyramid being cut by the Plane X, considered as a Picture, the Figure, a b d e, projected thereon, is the PERSPECTIVE REPRESENTATION of the Object A B C.

STEREOGRAPHY comprehends the whole Art of representing Solids * on a Plane; which differs from geometrical Projection, in what was explained in the preceding Definition; that, the projecting Rays, supposed to proceed from every part of the Object, terminate in a point, at any distance from the Object; which Point, is always considered as the Eye of a Spectator, and the intersecting Plane as the Picture. The section of the Rays is the Representation.

By this kind of Projection, the several dimensions of Bodies, *viz.* length, breadth and thickness, are all represented at one View, in the greatest degree of perfection that Art is capable of, or Nature exhibits.

As the Stereographic Projection (of Objects) is various, I shall explain it, more fully, under the three several Heads, Perspective, Projected Perspective or Projection, and Transprojection. And first, of PERSPECTIVE, which is, more immediately, the Subject of the following Treatise; that part of Stereography being more particularly adapted to the purpose of delineating all kinds of regular Objects, than either of the other; which are, therefore, but seldom practised; and indeed, because they are but little understood, or the difference between them known; yet, they have their several uses in delineating, particularly in projecting shadows.

PERSPECTIVE is a Science founded on Geometry; which teaches how to delineate, or draw, on a plane Superficies (or, simply, on a Plane) the true representations of Objects, according to their Distance and Situation, and Bearings of the Objects to each other; from any Station, at pleasure, real or imaginary.

To give a clear and perfect Idea of the Principles on which the Science of Perspective is built, it is necessary that the Objects, to be represented, are considered as being beyond, or on the other side of the Picture; that is, having fixed on the Station, from which you intend to delineate an Object, imagine a transparent Plane interposed between the Eye and the Object, or Objects; through which you have distinct Vision, of the Objects on the other Side.

It is evident, to all who have considered it, that if, with a steady hand, you trace every Line, of Objects, accurately, as they appear on the transparent Plane (which may be supposed a Window, or a perfectly regular Plate of Glass) keeping the Eye fixed in a Point, there will be true, linear perspective, Representation of all the Objects, on that Plane; which, is considered as the Picture. Now, the performance of this, by geometrical Rules, is what is properly called PERSPECTIVE.

* Every Object, having length, breadth, and thickness or depth, comes under the Denomination, geometrically, of a Solid; the concavity, (as of a Box, or House, &c.) not being considered; but only the external Form, consisting of Planes or other Surfaces, variously disposed.

Plate V. N.B. Every Reference, to Figure 15, refers likewise to the Apparatus.

Fig. 15. Let B F I K L be an Object having the three Dimensions, length, breadth, and thickness; which may be supposed a Building, or what you please, to be represented on the Plane, or Picture, M N O P, standing perpendicular on the Ground Plane, S R Z; whose Intersection with it, (M P) is at right Angles with the line of Station, (S L.) The Picture, M N O P, is, therefore, direct, between the Object and the Spectator, E S. E A, E B, E F, &c. may be considered as Visual Rays, in which the vision of the Object, B F I L, is conveyed to the Eye, at E.

2. It is obvious, that if this Plane, or Picture, be transparent, the Representation, a i f c, on that Plane, of the Object, B F I L, on the other side, would, by means of the Right Lines E A, E I, E F, &c. from each Angle of the Object to the Eye, exactly coincide, and agree in every part, with the original Object.

For, as it is not possible for Vision to be conveyed but in Right Lines to the Eye (except through dense, refracting Mediums) the Angle A, of the Object, must necessarily appear at a, on the Picture, B will appear at b, and F at f, &c. where the Right Lines, from the Angles of the Object to the Eye, pass through the Picture; and is the supposed reason for the genesis of a Point on the Picture, in Theory. The Right Line a b, or b g, on the Picture, joining the representations of the Angles, or Points, A and B, or B and G, in the Object, will also coincide with, and hide the Original Line, A B, or B G, from the Eye, at E; and is, therefore, its Representation; and so of all the other Lines on the Picture.

This, I think, needs no other Demonstration, for it is ocular, and evident, that the Angle, made by the Pyramid of Rays forming a solid Angle (A E F, F E C) at E, is, not only equal, but, the same, under which, both the Object and its Representation are seen.

So likewise, the representation, f i or f d, on the Picture, of any Line, F I or F D, in the Object, is seen under the same Plane Angle, I E F or F E D. Consequently, f i coincides with F I, and f d with F D; the representations, a b g h of the Plane A B G H, f g h i of F G H I, and b f c of the Plane B F C, coincide with each other, respectively; and consequently, the whole Representation, a i f c, or Projection of the Object, A I F C, perfectly coincides with the Original, in the Point of View E, in that Position and Situation of the Picture and Object.

Wherefore, since the Eye is affected, in the same manner, by the Lines and Angles on the Picture, as by the corresponding Lines and Angles of the Object, it is evident, that, if the Picture had the same degree of Light and Shade, and also the true teint of Colour, as the Object, it would be impossible for the Eye, at E, to distinguish whether it was a Picture, delineated on the Plane M N O P, or the real Object, on the others side, that was perceived.

3. Hence it is manifest, that there is, and may be, great deception in Vision; and also, that there may be various Representations, of the same Object, from the same Station or Point of View; which, notwithstanding their difference in figure and dimensions, will have the same Appearance in the true Point of View.

For, if any other Plane, as *M N O P*, be placed between the Eye and the Object, not parallel to M N O P, the Representation, of the same Object, projected on that Plane, by its interfection with the Visual Rays, will not only be smaller, but, also, very different in Figure and Proportion. For, having drawn the Right Lines S A, S B, S C, from the Foot of the Spectator, at S, to each angle of the Object, towards the Picture, on the Ground Plane, they will determine the extreme width of the representation of that Object on each Picture; and also the proportion, of the representation of the Plane A G to that of G C, which is, as k l to l m, on the Picture M N O P, where the lines S A, S B, and S C cut the bottom edge of the Picture; and, on M N *O P*, the Proportion is as n o to o p; but, M *P* is not parallel to M P; consequently, the proportion, of n o to o p, is not as k l to l m.

If the Picture was placed on M Q, it is obvious that the Reprefentation would be ftill lefs, and alfo different in its Figure and Proportion; as the parts Q r, r q, intercepted between the lines S A, S B, and S C, fufficiently evince.

Thus, may there be as many different Reprefentations, of the fame Object, as you pleafe, and from the fame Point of View; from the different Pofition and Diftance of the Picture; all which, will affect the Eye alike, at the Point E, in the Vertex of the Optic Cone or Pyramid of Rays.

PROJECTED PERSPECTIVE, or PROJECTION.

If the Vifual Rays E B, E C, E D, &c. are fuppofed to be produced, or projected beyond the Object, A B C D (a quadrangular Pyramid) and there fall on, or are cut by a Plane (V) the Reprefentation (a b d c) of the Object, projected on that Plane, is the Projection of the Object A B C. Fig. 14. No. 4.

Projected Perfpective is the very fame, except in the operation, as common Perfpective, i. e. when the Rays are cut by a Plane paffing between the Object and the Eye, with only this difference, that, in common Perfpective, the Reprefentation is always lefs than the Object; becaufe the fection of the Rays, by the Plane X, is on this fide, towards the Eye; in projected Perfpective, the Reprefentation muft neceffarily be larger than the Object, becaufe the Plane of the Section (V) is beyond the Object; but, if the Planes are parallel between themfelves, whether the Rays are cut on this or on the other fide of the Object, or both, the Reprefentations will be perfectly fimilar. No. 5. No. 4.

a, b, c, and d are the projective Reprefentations of the feveral Angles of the Pyramid, A, B, C, D; which, joined by Right Lines, is the Projection of that Object on the Plane V.

F G H I is the Plan or Seat of the Object A B C D, on the Ground Plane, to which the Bafe, A C d D, is parallel.

By means of the Seat and the Station Point, S, the Reprefentation, a b c d, on the Plane V, may be projected.

The difference between Perfpective and Projection is very obvious.

In Perfpective, the Object, to be reprefented, is always fuppofed to be beyond the Picture; in Projection, the Picture is beyond the Object; which is projected, or fuppofed to be thrown forward to the Picture, and which is full as rational to fuppofe. Nor is there occafion, in this Cafe, to fuppofe the Picture tranfparent, as in the former, when the Picture is fuppofed to be on this fide of the Object, and feen through; which indicates the meaning of the term Perfpective.

The difference in the operation is very little, and is illuftrated by frequent ufe in the practical part of this Work.

TRANSPROJECTION. If the Vifual Rays, from the Object (A B C D) are fuppofed to pafs through the Eye (at E) forming an oppofite Pyramid of Rays (a E b) and there fall on, or are cut by a Plane (Y) the Figure (a c b d) projected on that plane, by its interfection with the Rays, is the Tranfprojection of the Object (A B C D). Fig. 14. No. 6.

It is evident, that, in this kind of Projection, the Reprefentation may be either larger or fmaller, or equal to the Object, as the Plane, Y, is removed farther from or nearer to the point E; or, as the point E is removed nearer to, or further from the Object. For, if the point E be in the middle, between the Object and the Plane of the Section, the Reprefentation will be equal to the Object; and, whether it be nearer to, or further from the Object, the Reprefentation will have that Proportion, to the Original, as their Diftances from the point E.

It is alfo evident, that Projections of this kind muft neceffarily be inverted; as the Rays all pafs through one common Point (a is the tranfprojected place of the Angle A, on the Plane Y, c of C, and b of the Vertex, B, of the Pyramid) in the fame manner as optical Philofophers endeavour to account for Vifion; by fuppofing an Image, of every Object perceived, formed in the back part of the Eye,

Plate IV. on the Retina, in the ſame inverted Poſition. The Eye being in this reſpect like a *Camera Obſcura*, which is a kind of artificial Eye; in which, the Picture is always inverted. (See Page 10, 1ſt and 2nd Par. Alſo, ſee Fig. 13, Plate II.)

Notwithſtanding, if the Plane Y, of the tranſprojective Picture, be parallel to either the perſpective, (X) or the projective, (V) the Repreſentation thereon will be ſimilar to the other, or to both, if they are all parallel amongſt themſelves.

An ORIGINAL OBJECT is any Object whatever, which is the Subject of the Picture we are delineating.

Fig 15. B F I L is an original Object, of which, the Projection, a i f c, on the Plane M N O P, or *a i f c*, on the Plane M N O *P*, are Repreſentations.

Alſo, parts of an entire Object, as a Column, a Chimney, &c. are the Originals of their ſeparate Repreſentations.

By ORIGINAL PLANE is meant, not only the Ground, or other Planes upon which Objects are ſeated, but alſo, the plane Surfaces of original Objects to be delineated.

A B G H and B F C. &c. in the Original Object B F I L, are Original Planes, as well as the Ground Plane (Z) on which it ſtands.

It may be neceſſary to make a diſtinction between Figures and Objects, although every Plane Figure may be called an Object; but, I think, that Term is more properly, applicable to Solids than to Plane Figures.

By ORIGINAL FIGURE, I ſhall therefore mean, only, Figures in Original Planes, as Doors, Windows, &c. in Original Objects; or the Figure of the Original Plane itſelf. As F G H I, or B G F D C, &c.

Any geometrical Plane Figure whatever, in Original Objects, is, therefore, an Original Figure.

ORIGINAL LINE is any Line in an Original Object, whether Right Line or curved; as the bounds or limits of all Original Planes, or Figures, are Lines.

A B, B G, F G, F D, &c. are Original Lines, in the Original Planes A B G H and B F C, each, of which, is likewiſe in another Plane; for, each is the common Interſection of two Planes; conſequently, each Line is in two Planes.

By Th. 1.7. El. the common Interſection of two Planes is a Right Line, 3. 11. Euc.

A B is the common Interſection of the Ground Plane and the Plane A B G H, and is, therefore, in both Planes; and, B G is, for the ſame reaſon, in both the Planes A B G H and B F C; and ſo of all the reſt.

The Interſection of two Lines is a Point; and the extremes of Lines are alſo Points; wherefore, the Angles F, G, H, &c. of the Object B F I L, are Points; for they are the Interſections, and alſo the extremes of Lines; and are called Original Points.

All Objects, whatever, that are formed by Art, as Buildings of all kinds, or other regular Objects, (which are the fiteſt ſubject for Perſpective) are compoſed either of Planes or of curved Surfaces, or both. Every Building and parts of Buildings come under the denomination of ſome one or other geometrical Solid; as Parallelopiped, Priſm, Pyramid, Cylinder, Cone, or Sphere; or they are compounded of ſeveral, together. For, the Body of the Building is either one Parallelopiped, or it is compoſed of ſeveral, variouſly diſpoſed, at the diſcretion of the Architect. The Roofs are, generally, either triangular Priſms, or Pyramids. The Planes, of which Roofs are chiefly compoſed, are either Triangles, Parallelograms, or Trapezia. The Planes of the Fronts and Ends of a Building are, for the moſt part, Rectangles; ſometimes Pentagons (as B G F D C) or other Poligons; which contain other geometrical Figures, variouſly diſpoſed, as Doors, Windows, &c.

which are generally Rectangles, fometimes Circles, Ellipfes, or mixed Figures.

Temples, in Gardens, Cupolas, &c. are either cylindrical, or poligonal Prifms; the Roofs, of which, are either pyramidal, or fpherical, or mix'd curved Surfaces.

Columns approach nearly to Cylinders; in the lower part they are perfectly fo.

Thus, may every part of a Building, or other regular Object, be reduced to fome geometrical Solid or other. Solids are compofed of Planes or other Surfaces, all which, may again be reduced to their firft Principle, Lines; for, as the boundaries of Solids are Planes, or other Surfaces, fo the bounds of Planes, &c. are Lines.† Alfo, the Figures in Planes, whether Doors, Windows, or other Figures, are formed of Lines; all which, are ORIGINAL LINES. † Def. 55, Geo.

So likewife, Mouldings, Steps, &c. are reprefented by Lines; which, in the Originals, are generated either by Planes, only, as ftreight Steps, or by Planes and curved Surfaces, as in Mouldings and circular Steps. Each Moulding, if it be right lined, is compofed of Planes and cylindrical Surfaces; which, by their parallel interfections, generate Right Lines. Circular Mouldings, in Cornices, &c. are compofed of Planes, with cylindrical and other curved Surfaces; which, by their regular interfections, generate circular Lines.

The Edges of Columns and other cylindrical Objects, which are reprefented by Lines, on the Picture, have no real exiftence in the Originals, but are only apparent; for there is no real Line; as it is but one continued Surface, which returns again into itfelf, without cuting or interfecting, by which Lines are generated. The Lines which form the reprefentations of the Bafes, &c. of Columns are, in the Originals, fome of them real and fome only apparent; as the Contour of the curve of the Torus, &c. and have various forms, on the Picture, according to the fituation of the Eye, or Picture.

Having thus reduced compound Original Objects into their firft Principles or Elements, *viz.* Planes and Lines; the next thing, to be confidered, is the Pofition and Situation of thofe Planes and Lines, in refpect of the Picture and of each other; which being premifed and well confidered, the whole myftery of linear Perfpective will be found comprifed in a fmall Compafs, both in Theory and Practice. The Principles on which the Theory is built are few, but they are general, and applicable in all pofitions of the Picture and fituation of the Object, or of the Eye. Wherefore, the Diftance and Situation of the Object being determined; that is, the Station being fixed, from which an Object is to be delineated, and the Pofition of the Picture determined, the Reprefentation is alfo determinable; which, Reprefentation, is in proportion to the Diftance of the Picture.

Thus far I have proceeded, by way of Introduction. I have called it an Introduction to Perfpective, becaufe, it cannot be called a part or branch of that Science; feeing, all which it contains may be, and is, known to feveral, who are not acquainted with one Theorem, or any Rules for the practice of Perfpective. Neverthelefs, as this Work is intended to be a perfect Tutor for young Students, I am well convinced, that, the knowledge they will acquire by this Introduction is by no means to be difpenfed with; being as effential to be previoufly known, as the Definitions of the Terms ufed in Perfpective; which are elementary. It is certainly poffible, by Rules laid down, for a Perfon to practice Perfpective without knowing what Perfpective means; but that is not compatible with my Defign, having entitled this Work a Compleat Treatife, which could not poffibly be, without accounting for the Rules given; which I fhall do, as briefly as is confiftent with the Subject, not dwelling on any unneceffary part of the Science; but certainly, every neceffary knowledge, previous thereto (which being, perhaps, no part of any other diftinct Science) fhould firft be inculcated.

SECTION

Plate IV.

SECTION III.

Containing the ELEMENTS OF PERSPECTIVE.

IN order to inveſtigate the Theory of Perſpective, with clearneſs and preciſion, it is neceſſary to have recourſe to certain imaginary Planes, which may be conceived to paſs through the Eye of a Spectator, or Point of View, from which an Object is ſuppoſed to be delineated, in all Poſitions as occaſions require.

The Picture, as it has already been obſerved (under the Article Perſpective) is ſuppoſed to be between the Eye and the Object.

Three of thoſe imaginary Planes, together with the Picture and any Original Plane whatever, are ſuppoſed to be conſtructed as they are repreſented in Plate IV, Fig. 16, 17, 18, 19, 20 and 21. The Planes, thus conſtructed, I ſhall firſt define; afterwards, the Lines generated by their Interſections; and laſtly, the Points produced by the interſections of the Lines; all, which, are ſo eſſentially neceſſary, that, in ſhort, without the aſſiſtance of theſe five fundamental Planes, and the Lines and Points generated by their Interſections with each other, Perſpective would be a very intricate and perplexed Study; and, in Theory, a moſt imperfect Science.

I would particularly advertiſe young Students, not to pay the leaſt regard to the general poſitions of the Planes I am about to define, but only their poſitions in reſpect of each other; for which reaſon I have given them in various Poſitions, and adviſe the Reader not to give particular attention to the firſt. For, in the Theory of Perſpective, the poſition they have to the Horizon is not conſidered, at all; as the Theorems are general, and applicable in all poſitions and ſituations of the Picture, whatever; ſince (as Dr. Brook Taylor, in the Preface to his ſecond Treatiſe, juſtly obſerves) all Planes, ſimply as Planes, are alike in Geometry, and have the ſame properties however ſituated.

DEFINITIONS.

Fig. 16. 17, 18, 19, 20 and 21. I. Let ABGH be conſidered as a part, or a continuation of an ORIGINAL PLANE; being ſuppoſed to be produced (if neceſſary) from any Original Object to the Picture; and, till it cuts a Plane, paſſing through the Eye of a Spectator, parallel to the Picture.

DEFINITION II.

The PLANE of the PICTURE is conſidered as the Board, Paper, or Canvas, on which is to be delineated the repreſentation of ſome Original Object, or Plane Figure. As ABLM, or the Plane X, is the Picture.

The PICTURE is, generally, and for the moſt part, vertical, and direct between the Eye and the Object to be delineated; as in Fig. 16, 18 and 19; but may be in any Poſition whatever; as in Fig. 17, 20 and 21.

DEFINITION III.

VANISHING PLANE. If a Plane be imagined to paſs through the Eye, parallel to any Original Plane, it is called the VANISHING PLANE of that Original Plane; or, ſimply, the PARALLEL of the Original Plane.

As V, or IKLM, parallel to Z or ABGH, Fig. 16, 17, &c. E is the Eye.

And, a Perpendicular (EC) from the Eye to its Interſection with the Picture, is the RADIAL of the Vaniſhing Plane.

DEFINITION IV.

The DIRECTING PLANE is an imaginary Plane, ſuppoſed to paſs through the Eye of a Spectator, parallel to the Picture.

G H I K, or the Plane Y, parallel to the Picture (X) is the Directing Plane; the Eye of a Spectator being ſuppoſed at E, in the Directing Plane.

N. B. The Diſtance of the Directing Plane, from the Picture, is always equal to the Diſtance of the Eye (as E C) and being parallel to the Picture, it makes equal Angles with the Original Plane, as the Picture. (I H A equal M A N).

DEFINITION V.

The VERTICAL PLANE (E C D F) is ſuppoſed to paſs through the Eye, perpendicular to the Original Plane and to the Picture. Wherefore, it cuts all the four preceding Planes at right Angles, in all poſitions of the Picture whatever.

DEFINITION VI.

RADIAL PLANE is an imaginary Plane, paſſing through the Eye and any original Right Line, whatever.

As E V *I D*, or E *C P D*; which, being produced, would paſs through the original Line O N, or Q P. Fig. 20.

In Fig. 16. the Original Plane (A G) is ſuppoſed horizontal, and the Picture (A L) vertical; conſequently, the Varniſhing Plane (M K) and Directing Plane (K H) have the ſame Poſitions, and cut each other at right Angles.

In Fig. 17. ſuppoſe the Picture (A B L M) and the Directing Plane (H I K G) in the ſame vertical Poſition; and ſuppoſe the four Planes, *viz.* the Original Plane, the Picture, the Vaniſhing and the Directing Planes, ſo fixed together, as to be moveable on their Interſections, A B, G H, I K, and L M, as on hinges; and then, let us ſuppoſe, the Picture and Directing Plane (and along with them the Vaniſhing Plane) puſhed into an inclined poſition, on either ſide of the original, vertical Poſition; the Interſections, A B and G H, remaining as they were, unmoved.

It is evident, that the paralleliſm of the Planes is not deſtroyed by this motion; for, the Vaniſhing Plane (I K K M) is ſtill horizontal as before, though removed lower, to i k l m; and, the Directing Plane, H i k G or H *I K* G, is ſtill parallel to the Picture, A m l B or A *M* B, both being inclined to the Original Plane and its Vaniſhing Plane, in equal Angles.

In Fig. 18. the Picture (X) and the Directing Plane (Y) are ſtill vertical, but the Original Plane (Z) and its Vaniſhing Plane (V) are inclined to them, and to the Horizon; making equal Angles with each other, as in the former Caſe.

Nor is there underſtood to be any difference, in their Poſition, between this and the preceding Figure; for, either Plane (A L or H K) may be ſuppoſed the Picture, and the other the Directing Plane; by which means, it has both the Poſitions of the former; only, ſuppoſing the Original Plane horizontal, the Picture and Directing Plane are conſequently inclined.

In Fig. 19. both the Original Plane and Picture (N B H, and A B L M) are ſuppoſed vertical; therefore, the Vaniſhing and Directing Planes are alſo vertical; and the vertical Plane (C D F E) in this Caſe, is conſequently horizontal.

In Fig. 20. the Planes are all inclined to the Horizon, and alſo to each other; excepting the Vertical Plane; which always cuts the other four at right Angles, and is, therefore, perpendicular to them all.

This Figure I recommend, more particularly than any of the other four, to be contemplated by the young Student (although it is the ſame in all) in order to diveſt him, entirely, of partiality to any particular Poſition, reſpecting the Horizon.

In Fig. 21. The four primary Planes are all moveable, and may be put into all the Poſitions of the former; either at right Angles, as Fig. 16; or inclined, on either ſide, as in Fig. 17, in any Angle at pleaſure.

Plate IV. As thefe Planes are all marked with the fame Characters, as in the five preceding Figures, it would be fuperfluous to particularize them here; and, if a Plane be fuppofed to pafs through the four Points, E, C, D, and F, it will be vertical or perpendicular to them all, in every Pofition.

N. B. Any one of the four Planes may be the Original Plane, the oppofite one is, confequently, its Vanifhing Plane; either of the other may be the Picture, and the oppofite to it is the Directing Plane; E C D F is ftill the Vertical Plane.

Of LINES, generated by the Interfections of the five elementary Planes.

DEFINITION VII.

INTERSECTION of the PICTURE, with an Original Plane, is the Line in which any Original Plane cuts the Picture; or, in which an Original Plane, being produced, would cut the Picture.

Fig. 16, 17, 18, 19, 20 and 21. AB; is the Interfection of the Picture ABLM. with the Original Plane ABGH.

DEFINITION VIII.

VANISHING LINE is a Line produced by the Interfection of an imaginary Plane, paffing through the Eye parallel to any original Plane, with the Picture.

L M, the Interfection of the Vanifhing Plane, I K L M or V, with the Picture A B L M or X, is the Vanifhing Line of the Original Plane, A B G H or Z.

DEFINITION IX.

PARALLEL of the EYE is the Line I K, in which the Vanifhing Plane (V) and the Directing Plane (Y) interfect each other.

As both thefe Planes are imagined to pafs through the Eye (at E) confequently, their Interfection (I K) paffes through the Eye; and, becaufe the Directing Plane is parallel to the Picture, the Parallel of the Eye is parallel to the Picture.

DEFINITION X.

DIRECTING LINE is the Line G H, in which, an Original Plane (Z) cuts, or would, if produced, cut the Directing Plane (Y).

DEFINITION XI.

Fig. 16, &c. The VERTICAL LINE is the Line C D, in which, the Vertical Plane (E C D F) cuts the Picture; at right Angles with the Vanifhing Line and Interfection of the Original Plane.

DEFINITION XII.

The DIRECTOR of an Original Line. If an Original Line be produced till it cuts the Directing Plane, a Right Line paffing through the Eye and that Point is the Director of the Original Line.

E *D* (Fig. 20 and 21) is the Director of the Line N O, being produced to *D*.

DEFINITION XIII.

VISUAL RAY. With optical writers, this Term fignifies an imaginary Ray of Light; by which, Vifion is fuppofed to be conveyed from the Object to the Eye; therefore, in Perfpective, it is a Right Line imagined to be drawn from any Point, in an Object, to the Eye.

E A, E I, E F, &c. (Fig. 15.) and E N, E O, &c. (20 and 21.) are Vifual Rays.

DEFINITION XIV.

RADIAL LINE is the parallel of any Original Line, producing its Vanifhing Point. (See Def. XXII.) As E V (Fig. 20, and 21) parallel to N O.

DEFI-

DEFINITION XV.

DIRECT RADIAL is a Right Line, from the Eye or Point of View, perpendicular to the Picture. As E C.

N. B. If the Original Plane be at right Angles with the Picture (as in Fig. 16.) the Direct Radial (E C) is the common Intersection of the Vanishing Plane, (I K L M) and the Vertical Plane (E C D F); and is, always, in the Vertical Plane.

Of POINTS, and their Distance from the Eye.

DEFINITION XVI.

The POINT of SIGHT is that Point where the Eye, of a Spectator, ought to be placed to look at a Picture; for, in that Point, only, can a perspective Picture be seen perfectly.

E is the place of the Eye, or Point of View, to look on the Picture A B L M.

It is the Point where the three imaginary Planes, *viz.* the Vanishing, the Directing, and the Vertical Planes, intersect; consequently, the Eye is in all the three. Or, it is the Point of Intersection between the Parallel of the Eye (I K) and the Prime Director (E F)

N. B. It is the Vertex of the optic Pyramid of Rays (E A, E B, E F, &c.) the only Point in which the Images, or Representations (a i f c,) on the Planes or Pictures, M N O P or *O P*, can exhibit a true Appearance of the Original Object (B F I L) on the other side. Fig. 15.

DEFINITION XVII.

The CENTER of the PICTURE is the Point C, in which a perpendicular Line from the Eye, or Point of Sight, is cut by the Picture.

The Direct Radial (E C) being perpendicular to the Picture, is, therefore, in the Vertical Plane (E C D F) consequently, the Center of the Picture, produced by the Perpendicular E C, is in the Vertical Line (C D) and, when the Original Plane is perpendicular to the Picture, it is the Intersection of the Vertical Line (C D) and its Vanishing Line (L M). Fig. 16.

DEFINITION XVIII.

DISTANCE of the PICTURE, or principal Distance, is the Direct Radial, or perpendicular Line (E C) from the Eye to the Picture, or to its Center.

N. B. The Center and Distance of the Picture are most essential, and ought to be well understood; for, except the Intersecting and Directing Points, all the rest are dependant on them.

DEFINITION XIX.

CENTER of a VANISHING LINE is the Point where it is cut by a perpendicular Line from the Eye or Point of Sight.

E C (Fig. 15 and 16) or E *C* (Fig. 18, 19 and 20) being a Perpendicular from the Eye (E) to the Vanishing Line (L M) C or *C* is, therefore, its Center.

DEFINITION XX.

DISTANCE of a VANISHING LINE, is the Perpendicular, E C or E *C*, from the Eye to its Center; the shortest Line that can be drawn to the Vanishing Line.

DEFINITION XXI.

POINT of INTERSECTION is that Point in which any Original Line (being produced) cuts the Picture; or the Plane of the Picture produced, if necessary. Fig. 20, and 21.

I is the Intersecting Point of the Line N O, produced to the Picture; *B* and *F* are the Intersecting Points, of the Lines A B and F I, on the Pictures M N O P. Fig. 15.

DEFINITION XXII.

VANISHING POINT. If a Line be drawn from the Eye, parallel to any original Right Line, the Point, where it cuts the Picture, is the Vanishing Point of that Original Line. (See Theo. 2. Sect. 3. Book 1.)

Plate IV. Fig. 20, and 21. N O is an Original Line, in the Original Plane H N B G; E V is a Line from the Eye, parallel to N O, cuting the Picture; V is, therefore, the Vanifhing Point of N O, and E V is its RADIAL, or PARALLEL of the Original Line; confequently it is in the Vanifhing Plane, or parallel of that Plane the Original Line is in.

Fig. 15. EC or E V being parallel to the Original Lines A B, G H, F I, &c. C or V, where it cuts the Picture, is, therefore, the Vanifhing Point of thofe Lines.

N. B. The RADIAL EC, or E V, is the Diftance of the Vanifhing Point, C or V.

DEFINITION XXIII.

DIRECTING POINT is that Point in which any Original Line, being produced, would cut the Directing Plane.

Fig. 20. and 21. *D* is the Directing Point of the Original Line N O, or P Q, being produced to the Directing Plane (G H I K). It is the Interfection of the Original Line and the Directing Line, of the Plane the Original Line is in.

In Fig. 23. O, P, and Q are the Directing Points of the Lines CB, AB, and AD, where thofe Lines cut the Directing Plane, *G I* K *H*, produced.

DEFINITION XXIV.

STATION POINT is the Point in which a perpendicular Line, from the Eye or Point of Sight, cuts the Ground, confidered as a horizontal Plane; or any other horizontal Plane, on which an Object, to be delineated, is feated.

Fig. 15. S, is the real Point of Station, at the foot of a Spectator (E S.)

In Theory, it is that Point in which a Right Line, from the Eye, cutst he Directing Line, perpendicularly.

Fig. 16, 17, &c. As F, in the Directing Line G H; E F being, fuppofed, perpendicular to G H.

AXIOMS.

I. One part of a Right Line cannot be in any Plane, and another part of the Line out of that Plane, being produced.

II. Two Planes which are parallel cannot interfect.

III. The common Interfection of two Planes is a Right Line.

IV. The Interfection of two Lines is a Point.

V. Two Right Lines, that are parallel, are in the fame Plane: i. e, a Plane may pafs through both Lines.

VI. Two Right Lines, meeting in a Point, or which, if produced, would interfect, may be in the fame Plane.

VII. Two Right Lines, being parallel, and both cut by another Right Line, are all in the fame Plane.

VIII. Three Right Lines, meeting or interfecting each other, are all in the fame Plane; confequently, every right lined Triangle is in a Plane.

Several of thefe Axioms are demonftrable Propofitions, in the eleventh Book of Euclid's Elements; but, having a true Idea of a Plane, they are fo very evident, that it is but trifling, to little purpofe, to attempt to prove them. Neverthelefs, as the Student is fuppofed to underftand Geometry, they may, without fcruple, pafs for Axioms here.

SECTION IV.

Containing the THEORY of Rectilinear PERSPECTIVE.

THEOREM I.

AN Original Plane, which is parallel to the Picture, has no Interfection with the Picture, nor Vanifhing Line.

DEM. The Original Plane being parallel to the Picture cannot interfect it. Ax. 2.
And, a Plane paffing through the Eye parallel to the Original Plane, which fhould produce the Vanifhing Line, by its Interfection with the Picture, is, in this Cafe, alfo parallel to the Picture, and the fame with the Directing Plane; confequently, it can never cut the Picture, and therefore, cannot produce a Vanifhing Line. Q. E. D.

EX. Let MN*OP* be the Plane of the Picture, parallel to the Plane BFC, of the Original Object (BFIL.) Fig. 15.

Then, becaufe the Plane BFC is parallel to the Picture, it cannot cut the Picture (though produced infinitely) therefore can have no Interfection with it. Ax. 2.
And a Plane (RSTU) paffing through the Eye (at E) parallel to the Original Plane (BFC) is alfo parallel to the Picture; therefore, it cannot cut the Picture and produce a Vanifhing Line; (agreeable to Def. 8.) for, it is the Directing Plane of that Picture; which is parallel to the Picture. — Def. 4.

COR. *Original Lines, which are parallel to the Picture, have neither Vanifhing Point, Interfecting, nor Directing Points.*

For, an Original Line, parallel to the Picture, is, or may be, in a Plane that is parallel to the Picture; and, a Line paffing through the Eye parallel to the Original Line, which fhould produce its Vanifhing Point ‡, is, in this Cafe, alfo parallel to the Picture; and is, therefore, in the Directing Plane, which is parallel to the Picture. Wherefore, it can never cut the Picture and produce a Vanifhing Point; or, it may be fuppofed at an infinite Diftance. ‡ Def. 22.
Neither can the Original Line, for the fame reafon, cut the Picture or Directing Plane; therefore, it has neither Interfecting nor Directing Point.

EX. Becaufe the Plane BFC is parallel to the Picture, every Line in that Plane is alfo parallel to the Picture; therefore, BG, GD, &c.
Confequently, fince one part of a Right Line cannot be in any Plane, and another Part of it out of the Plane †, the Line BG, or GD, &c. can never cut the Picture and Directing Plane, and produce an Interfecting and Directing Point; agreeable to Definitions 21 and 23. † Ax. 1.
For the fame reafon, a Line paffing through the Eye parallel to the Original Line (BG, GF, or GD, &c.) muft neceffarily be in the Directing Plane (RSTU) confequently, it can never cut the Picture and produce a Vanifhing Point; agreeable to Definition 22.

Plate IV.

THEOREM II.

The Vaniſhing Line, the Parallel of the Eye, and the Directing Line of any Original Plane, not parallel to the Picture, are parallel to the Interſection of that Plane with the Picture.

Fig. 16. The Conſtruction of the five elementary Planes, with their uſes and relation to each other (as explained or defined) 'tis preſumed, is well underſtood.

Suppoſe, then, the firſt four, *viz.* the Original Plane (Z) its Parallel, or Vaniſhing Plane (V) the Picture (X) and Directing Plane (Y) forming a right angled Parallelopiped; or that part of them, only, which lies between their Interſections (A B, M L, K I, and G H) with each other.

DEM. Now, the Picture and the Directing Plane are parallel, and they are cut by an Original Plane and its Vaniſhing Plane, which are alſo parallel. Def. 3 & 4.

But, if two Planes, being parallel, are cut by another Plane, their common Sections are parallel. - - - - P. 8. 7. El.

Conſequently, if two parallel Planes are cut by two parallel Planes, their Interſections are all parallel amongſt themſelves.

But, A B, the ſection of the Original Plane (Z) and the Picture (X) is the Interſection of the Picture. - - - - Def. 7.

M L, the ſection of the Picture (X) and the Vaniſhing Plane (V) is the Vaniſhing Line of the Original Plane (Z) - - - Def. 8.

K I, the ſection of the Vaniſhing Plane (V) and the Directing Plane (Y) is the Parallel of the Eye. - - - - - Def. 9.

And, G H, the ſection of the Directing Plane (Y) and the Original Plane, (Z) is the Directing Line. - - - - - Def. 10.

Therefore, the Vaniſhing Line (M L) the Directing Line (G H) and the Parallel of the Eye (K I) are all parallel amongſt themſelves, and to the Interſection (AB) of the Original Plane with the Picture. Q. E. D. 4. 7. El.

N. B. Whether the Original Plane be at right Angles, or otherwiſe, with the Picture, the Demonſtration holds good, in all poſitions whatever; ſince, the Directing Plane is always parallel to the Picture, and the Vaniſhing Plane to the Original Plane; conſequently, their Interſections with each other are ſtill parallel amongſt themſelves.

Fig. 21.

To aſſiſt the Imagination, I have given a Figure with the four principal Planes, moveable; which may be raiſed up, to a Right Angle or any other, at pleaſure.

In every Poſition, it is evident, that the paralleliſm of the Planes, and conſequently of their Interſections, remains; and the diſtance of the Interſections between each two, which are in the ſame Plane, is invariable.

The Vertical Plane may be ſuppoſed to paſs through the middle, and perpendicular to them all; cuting the Vaniſhing and Directing Plane, in E C and E F; the Picture and Original Plane, in C D and D F.

COR. 1. *If any one of the four Lines be given or determined, and one Point, in any of the other, be alſo given or found, the whole Line is determined.*

For, it is a Right Line, drawn through that Point, parallel to the given Line; ſeeing, they are all parallel amongſt themſelves, by the Theorem.

COR. 2. *If the Original Plane paſſes through the Eye, its Interſection and Vaniſhing Line is the ſame; and, the Parallel of the Eye is alſo the Directing Line.*

For,

For, ſeeing that the Original Plane (U, or a b c d, being produced) paſſes through the Eye (E) there can be no other Plane paſs through the Eye parallel to it; but, in this Caſe, the Original Plane and its Parallel coincide; conſequently, its Interſection (L M) with the Picture (A *L M* B) is alſo its Vaniſhing Line. And, for the ſame reaſon, the Parallel of the Eye (I K) coincides with the Directing Line; being produced by the ſame Original Plane. In which caſe, no Figure in the Original Plane can be repreſented. Plate VI. Fig. 22.

For, as the Original Plane paſſes through the Eye, the whole Repreſentation of that Plane, and every Figure in it, is its Interſection with the Picture; and is the reaſon why, Figures, in horizontal Planes, which are on a level with the Eye, cannot be repreſented; for they are all in the Vaniſhing Line of horizontal Planes, and, therefore, cannot appear in the Picture.

THEOREM III.

All Original Planes, which are parallel amongſt themſelves, but not to the Picture, have the ſame Vaniſhing Line.

DEM. Vaniſhing Lines, are produced, in Theory, by imaginary Planes, paſſing through the Eye or Point of View, parallel to the Original Planes. Def. 8.

Wherefore, a Plane paſſing through the Eye parallel to any Original Plane, producing its Vaniſhing Line, is alſo parallel to every Plane which is parallel to the Original Plane; and ſince this parallel Plane can produce but one Line on the Picture, by its Interſection with it, that Line is, conſequently, the Vaniſhing Line of all the Planes to which it is parallel. Q. E. D.

EX. V, U, and W, are three Original Planes, parallel amongſt themſelves; A *L M* B is the Picture, and E the Eye, in the Directing Plane (I K G H.) Fig. 22.

I K L M paſſing through the Eye at E, is a continuation of the Plane U; conſequently, it is parallel to the Planes V and W; and, ſeeing that no other Plane can be drawn through the Eye parallel to them, therefore L M is the Vaniſhing Line of the three parallel Planes V, U and W. - Def. 8.

This may be further illuſtrated by Fig. 15, and by the Apparatus.

Suppoſe a Plane (R S T U) to paſs through the Eye (E) parallel to the Planes B F C and A I L in the Original Object B F I L, cuting the Picture, M N O P, produced, in R U; which is, therefore, the Vaniſhing Line of thoſe parallel Planes; and, conſequently, of all other Planes parallel to them. Plate V.

For the ſame reaſon, a Plane (T O P S) paſſing through the Eye, parallel to the Planes A B G H and C D K L, cuting the Picture M N *O P* in *O P*; and the Picture R U O P in O P; *O P*, or O P, is the Vaniſhing Line of thoſe Planes, on each Picture, reſpectively.

SCHOL. Horizontal Planes have horizontal Vaniſhing Lines; except, the Picture be alſo horizontal; in which Caſe they have no Vaniſhing Line, the Picture being parallel to the Original Plane.

It is alſo evident, that all vertical Planes have, when the Picture is alſo vertical, perpendicular or vertical Vaniſhing Lines; i. e. the Vaniſhing Lines are, in ſuch caſe, perpendicular Lines on the Picture, and at right angles with ſuch as are horizontal.

For, 1ſt. The Interſection of a horizontal Plane, with any other Plane, is neceſſarily a horizontal Line, ſeeing it is produced by a horizontal Plane, in which the interſecting Line muſt always be.

2nd. The common Interſection of two vertical Planes, is a Right Line perpendicular to the Horizon; ſeeing that, the Line of ſection, is, in both Planes.

Plate V. COR. *All Original Lines which are parallel amongſt themſelves, and not to the Picture, have the ſame Vaniſhing Point.*

For, by the ſame reaſoning as in the Theorem, it is evident, ſince the Vaniſhing Point of an Original Line is produced, in Theory, by an imaginary Line paſſing through the Eye parallel to the Original Line †, it neceſſarily follows, that, a Line paſſing through the Eye, producing the Vaniſhing Point of any Original Line, by its Interſection with the Picture, being parallel to the Original Line, is alſo parallel to all Lines which are parallel to the Original Line; and, ſince it can produce but one Point on the Picture, it is, conſequently, the Vaniſhing Point of them all. Theor. 2. Book 1ſt.

† Def. 22.

Fig. 15. In the ſame Figure, the Original Lines A B, G H, and F I, are parallel amongſt themſelves; E C V is a Right Line from the Eye, parallel to them all, cuting the Picture R U O P in V, and M N *O P*, in C, its Center; the Points C and V, on each Picture reſpectively, are, therefore, the Vaniſhing Points of the Original Lines A B, G H, and F I.

The ſame may be ſaid in reſpect of the parallel Lines G F and H I, whoſe Vaniſhing Point, on the Picture R U O P, produced, is W.

Alſo, of G D and B C, whoſe Vaniſhing Point is Y, on the Picture R U O P; but, being parallel to M N *O P*, they have, conſequently, no Vaniſhing Point on that Picture, but are parallel to their Repreſentations.

N. B. It may be neceſſary to obſerve here, that, in this laſt, and in moſt of the Schemes or Diagrams made uſe of in this Theory, (being drawn perſpectively) whenever any two, or more, Lines are ſaid to be parallel, if they are not really ſo, they tend to the ſame Point, agreeable to Coroll. 1. For, I think it unpardonable, in a Treatiſe on Perſpective, to give an Appearance of Objects, or Schemes, not truly and perſpectively drawn; merely, for the ſake of preſerving the paralleliſm of certain Lines, as ſeveral Authors have done. Unluckily, for them, they cannot alſo preſerve the true Angles, which are worſe repreſented by that means.

Therefore, whenever Angles are ſaid to be Right, or otherwiſe; and ſuch or ſuch Lines, to be parallel or perpendicular, if they are not really ſo, they muſt be underſtood to be ſo; and, I am perſuaded, that they will always appear to be ſo, when truly delineated, perſpectively.

THEOREM IV.

The Vaniſhing Lines of all Planes, which are perpendicular to the Picture, paſs through the Center of the Picture.

DEM. For, ſince the Vaniſhing Line of a Plane is produced by an imaginary Plane paſſing through the Eye parallel to the Original Plane †, and the Center of the Picture is determined by a Perpendicular, from the Eye to the Picture §, the Original Plane being, in this Caſe, perpendicular to the Picture, it follows, that its Parallel or Vaniſhing Plane (producing the Vaniſhing Line) ſeeing it paſſes through the Eye, muſt, neceſſarily, paſs through the Direct Radial, which is perpendicular to the Picture ‡, and conſequently, through the Center of the Picture.

† Def. 8.

§ Def. 17.

‡ Def. 15.

Therefore, the Vaniſhing Line, produced by the Interſection of every ſuch Plane with the Picture, muſt paſs through the Center of the Picture. Q. E. D.

Fig. 15. EX. 1. All horizontal Planes are perpendicular to a vertical Picture; wherefore, the Plane G H K D, and alſo the Ground Plane (Z) are perpendicular to both Pictures, M N O P or *O P*; their Vaniſhing Plane, therefore, paſſes through the Direct Radial (E C) of each Picture, and conſequently, through C, their Centers.

C X and Y V (on each Picture, reſpectively) parallel to thoſe Planes, drawn through the Centers (C) of both Pictures, are the Vaniſhing Lines of thoſe Planes. For, by Theo. 3d. the Vaniſhing Line of every Plane is parallel to its Interſection; and conſequently to the Original Plane.

EX. 2. The Planes A H G B and C D K L are perpendicular to M N O *P*, only; their Vanifhing Plane (S T O *P*) therefore, paffes through its Radial (E C) and their Vanifhing Line (*O P*) through the Center (C) of that Picture.

As thefe two Examples are particular Cafes; *viz.* when the Original Plane is either horizontal or vertical, I fhall give another general Cafe, which will illuftrate the Theorem univerfally. Raife up the Planes of No. 2. A O N B is the Picture, G E H the Directing Plane, and E the Eye.

An Original Plane, *IKLM*, is moveable on the Line T U, its common Section with the Ground, or other horizontal Plane, at right Angles with A B, the Interfection of the Picture with the Plane Z. Fig. 15. No. 2.

It is evident that, as the Original Plane, *IKLM*, is turned (on T U) it is always perpendicular to the Plane or Picture (A O N B) † for, in every pofition of the Plane, in that revolution, the Line T U is ftill perpendicular to the Picture. † 9. 7. El.

Now, if the Plane I K L M, revolving, pafs through the Eye (at E) and, being all the while parallel to the Original Plane, it muft revolve on E C, the Direct Radial, which is parallel to T U; and, in every Pofition, its Interfection with the Picture muft neceffarily pafs through C, the Center of the Picture; produced by the Perpendicular E C.

Let the Original Plane (*IKLM*) be raifed, making the Angle P Q R.

If the Plane I K L M, paffing through the Eye, makes the fame Angle with the Horizon, i. e. if it be parallel to the Plane *IKLM*, it will cut the Picture in *O* N, the Vanifhing Line of that Plane, and pafs through C, the Center of the Picture; making the Angle, N C M, with L M, the Vanifhing Line of horizontal Planes, equal to the Angle P Q R, of the Original Plane with the Ground Plane.

After the fame manner, the Vanifhing Lines of all Planes, which are perpendicular to the Picture, may be afcertained and drawn on the Picture; knowing the Angle which the Original Plane makes with any horizontal Plane; or with a vertical Plane, which is perpendicular to the Picture.

N. B. The Center of the Picture is the Center of all Vanifhing Lines, of Planes perpendicular to the Picture; and they have all the fame Radial and Diftance (E C) by Def. 19. and 20.

COR. *Hence it is evident, that the Center of the Picture is the Vanifhing Point of all Lines which are perpendicular to the Picture.*

For, E C, a perpendicular Line from the Eye, producing the Center of the Picture, is the common Radial of all Vanifhing Planes, that are perpendicular to the Picture; and confequently, parallel to all Lines that are perpendicular to the Picture (for, they are all parallel amongft themfelves, and to the Direct Radial, E C) as A B, G H, F I, &c. Fig. 15.

Therefore, fince E C is parallel to all fuch Lines, the Point (C) which it produces on the Picture, is their Vanifhing Point; by Def. 22.

But, C is the Center of the Picture; confequently, the Center of the Picture is the Vanifhing Point of all Lines that are perpendicular to the Picture.

In this fingle and particular Vanifhing Point, many Artifts feem to reft all their knowledge of Perfpective (commonly called the Point of Sight) and, whenever a number of Lines tend to the fame Point, it is, by them, called a Point of Sight; not confidering, or perhaps knowing, that the Center of the Picture is a Vanifhing Point, in common with all other, only by virtue of the general Definition (Def. 22.) And, becaufe the knowledge of it is more general, and, as they imagine, the practice much eafier, we find it more ufed than any other; as moft regular Objects, particularly Buildings, are right-angled; fo that, having one Side or Front parallel to the Picture, (as is ufually the cafe) all the horizontal Lines in the adjoining Sides are, confequently, perpendicular to the Picture, and therefore vanifh in its Center; which, from the certainty of the Pofition of fuch Lines, is determined and fixed at pleafure, though often very injudicioufly.

Plate VI.

THEOREM V.

Original Planes, whofe common Interfection is parallel to the Picture, have parallel Interfections with the Picture, and parallel Vanifhing Lines.

DEM. The common Interfection of the Original Planes being parallel to the Picture, a Plane may be fuppofed to pafs through that Interfection which is alfo parallel to the Picture.

Now, the Original Planes being produced, through their common Section, to the Picture, each Plane will cut the Picture in a Line parallel to their common Section (8. 7. El.) Wherefore, fince the Interfection of each Original Plane, with the Picture, is parallel to the common Section of both Planes, they are, confequently, parallel between themfelves. Q. E. D. 4. 7. El.

2. But the Vanifhing Line of each Original Plane is parallel to its Interfection with the Picture. - - - - - - Theorem 3.

And the Interfections are parallel between themfelves. Proved.

Therefore the Vanifhing Lines are parallel between themfelves.

Fig. 22. EX. a b c d and c d e f are two Original Planes, whofe common Section (c d) is parallel to the Picture (A B L M); g h i is a Plane, fuppofed to pafs through their common Section, parallel to the Picture.

Now, the Plane a b c d being produced to the Picture, cuts it in L M; and the Plane c d e f, being produced, cuts it in *AB*; wherefore, fince the Picture and the Plane g h i are parallel, and are both cut by the Planes *A* d c *B* and L a b M, the Interfections (*AB* and L M) are, each, parallel to the common Section, c d; and therefore they are parallel between themfelves.

EX. 2. *LM* is the Vanifhing Line of the Plane cdef, and LM of the Planes V, U, & W; which are, refpectively, parallel to the Interfections A B, J F, &c. (by Theo. 4.) and therefore they are parallel, alfo, between themfelves.

3. After what has been faid, it is manifeft that the Directing Lines of Planes, in fuch Cafe, are alfo parallel.

For, fince their Interfections with the Picture are parallel, confequently, if they were produced till they cut the Directing Plane, their Interfections with it would likewife be parallel; the Directing Plane being parallel to the Picture.

COR. 1. *The common Interfection of a Plane inclined to the Horizon, with any horizontal Plane whatever, being parallel to the Picture; the Vanifhing Line, of that inclined Plane, is a Line parallel to the Horizon.*

For, becaufe the common Interfection is parallel to the Picture, the Vanifhing Lines of both Planes are parallel to each other; by Theorem.

But, one of thofe Planes is horizontal, therefore its Vanifhing Line is parallel to the Horizon; by Def. 8.

Confequently, the other Vanifhing Line is, alfo, parallel to the Horizon.

COR. 2. *All Original Planes, that have parallel Vanifhing Lines, have the fame Vertical Plane; and, confequently, the fame Vertical Line; and, alfo, the fame Parallel of the Eye.*

I K L M is the parallel of the Plane W, cuting the Picture in L M, the Vanifhing Line of W; and, I K *L M*, is the Parallel, or Vanifhing Plane of the

the Original Plane X, paſſing through the Eye (at E) and cuting the Picture in *L M*, the Vaniſhing Line of that Plane. Alſo, I K l m being parallel to the Horizon, its Section with the Picture (l m) is the Vaniſhing Line of horizontal Planes; to which, L M and *L M* are parallel, by the Theorem.

But, R S T is the Vertical Plane of all thoſe Original Planes, and of all other parallel to them; i. e. it cuts them all at right Angles, being perpendicular to their common Sections (c d, f g, &c.) and it cuts the Picture in C D the Vertical Line of them all; for the ſame Plane cannot cut the Picture in two Lines.

I K is alſo the Parallel of the Eye of all thoſe Planes; for, it is the common Section of all their Vaniſhing Planes with the Directing Plane.

This Theorem, and the laſt Corollary, may alſo be illuſtrated by Figure 15.

1. B G, the common Interſection of the two Planes A B G H and B F C, is parallel to both Pictures (M N O P or *O P*) their Interſections (*BE* and *BK*) with M N O P, are therefore parallel; alſo their Vaniſhing Lines, R U and O P, on that Picture. The other (M N *O P*) being parallel to the Plane B F C, has but one Vaniſhing Line (*O P*) which is parallel to the Interſections *B G* and MN. Fig. 15.

2. The two Planes, A B G H and B F C, being vertical, a Plane vertical to them, is, conſequently, horizontal; for, no other Plane can cut both, or either of them and the Picture, perpendicularly. (See Fig. 19.)

Wherefore, they have the ſame Vertical Plane; which, in reality, is horizontal; and, conſequently, they have alſo the ſame Vertical Line; which, in this Caſe, is the Vaniſhing Line of horizontal Planes; *viz.* C X or V Y, on either Picture, reſpectively.

N. B. It is the ſame in all Poſitions, whatever, of the Original Planes.

THEOREM VI.

The Planes, which produce the Vaniſhing Lines of two Original Planes, are inclined to each other in the ſame Angle as the Originals; and have their common Interſection, paſſing through the Eye, parallel to the common Interſection of the Original Planes.

DEM. For, firſt, ſince the Planes, producing the Vaniſhing Lines of any two Original Planes, are, reſpectively, parallel to the Originals, they have conſequently the ſame Inclination to each other, as the Originals.

For, if all the Planes are produced (*viz.* the Original Planes and their Parallels) they will cut each other in parallel Lines, and form a Parallelopiped, between their Interſections. - - - - 8. 7. El.

Therefore, they have the ſame Inclination to each other, reſpectively. 13. 7.

EX. U, or W, and X, are Original Planes, and K M, K *M* their Vaniſhing Planes; being parallel to them, reſpectively.

The Angle, which the Original Plane (X) makes with W, is d P N (P d and P N being both perpendicular to their common Section P Q) and it is equal to the Angle L K *L* which their Vaniſhing Planes (K M and K *M*) make with each other Fig. 22.

Becauſe KL is parallel to NP, and K*L* to d P, the Angle LK*L* is equal to dPN ‡. ‡ 6. 1. El.
Alſo, the Angle *L* K L is equal to L d P, which is equal to d P N †. † 4. 1. El.
Therefore, the Planes, extended through thoſe Lines, being reſpectively parallel, are inclined to each other in the ſame Angle; i. e. K *M* has the ſame inclination to K M, as the Plane X has to U or W.

Plate V. Secondly. Becaufe, in this Cafe, the common Sections (c d, N O, &c.) of the Original Planes are parallel to the Picture, the Interfection (I K) of the Vanifhing Planes, paffing through the Eye (E) is alfo parallel to the Picture; and, confequently, to the common Sections of the Original Planes.
For it is in the Directing Plane, which is parallel to the Picture; by Def. 4.

To illuftrate this Theorem more familiarly, by moveable Planes.

Fig. 15. No. 2. 1. Raife up the Plane of the Picture A O N B, perpendicular, or otherwife, to the Horizon. The Directing Plane (G E H) will be parallel to it; and I K L M to the Plane of the Horizon.

No. 3. Then, if the Plane *A D F B* (any Original Plane) whofe Interfection (*A B*) with the Horizon, or any horizontal Plane, is parallel to the Picture, be raifed up, on *A B*, making any Angle (P Q R) with the Horizon, and, a Plane, (*I K* O N) pafs through the Eye (at E) parallel to that Original Plane; it is manifeft, that this Vanifhing Plane (*I K* O N) makes the fame Angle with I K L M, as *A D F B* with the Horizon; and *I K* (the parallel of the Eye) the Interfection of the Vanifhing Planes, is parallel to *A B*, the common Interfection of the Original Planes.

2. The Picture, &c. remaining in the fame Pofition, perpendicular to the Ground Plane, and to the common Interfection (T U) of the Original Plane *I K L M*; making any Angle with the Horizon, at pleafure.

No. 2. Then, the Vanifhing Plane (I K L M) being placed parallel to the Original Plane *I K L M*, it is evident, that it will make the fame Angle with the Horizon, or with a Plane paffing through the Eye, parallel to the Horizon, and cuting the Picture in L M (the Vanifhing Line of horizontal Planes, as the Original Plane (*I K L M*) makes with the Horizon.
Alfo, E C (the Direct Radial) the Interfection of thofe Vanifhing Planes, is parallel to T U, the common Interfection of the Original Planes.
For, they are both perpendicular to the Picture.

In Fig. 23. this Theorem is quite general, i. e. the Pofition, the Original Planes have to the Horizon, or to the Picture, is not regarded.

Fig. 23. X and Y are two Original Planes, both inclined to the Horizon, and to the Picture (I T U *N*) which is alfo inclined to the Horizon, towards the Eye (at E.) A B is the common Interfection of thofe two Planes.

Then, if a Plane (I K L M) be fuppofed to pafs through the Eye (E) parallel to the Plane, X; and another Plane (*I K L M*) parallel to A B D, or Y, cuting the Picture in the Vanifhing Lines, L M and *M L*, of thofe Planes; they will be inclined to each other in equal Angles, refpectively; i. e. the Vanifhing Planes, as the Original Planes; and the common Section (E V) of the Vanifhing Planes, be parallel to the common Section (A B) of the Original Planes.

COR. 1. *The Radials, producing the Vanifhing Points of any two Original Lines, make the fame Angle, at the Eye, as the Original Lines make with each other.*

For, they are refpectively parallel to their Originals (Def. 22.) and, confequently, they make equal Angles, refpectively. - - 5. 7. El.

Fig. 15. EX. E V is the Radial or Parallel of A B, and E Y of C B.
V and Y are, therefore, the Vanifhing Points of A B and C B; the Radials (E V E Y) being parallel, refpectively, to the Originals (A B and C B) Def. 22.
Confequently they make the fame Angle at the Eye (E) as the Original Lines make with each other; *viz.* V E Y equal A B C. - - 5. 7. El.
But, A B C is a Right Angle; therefore, V E Y is a Right Angle.

 This

Plate III.

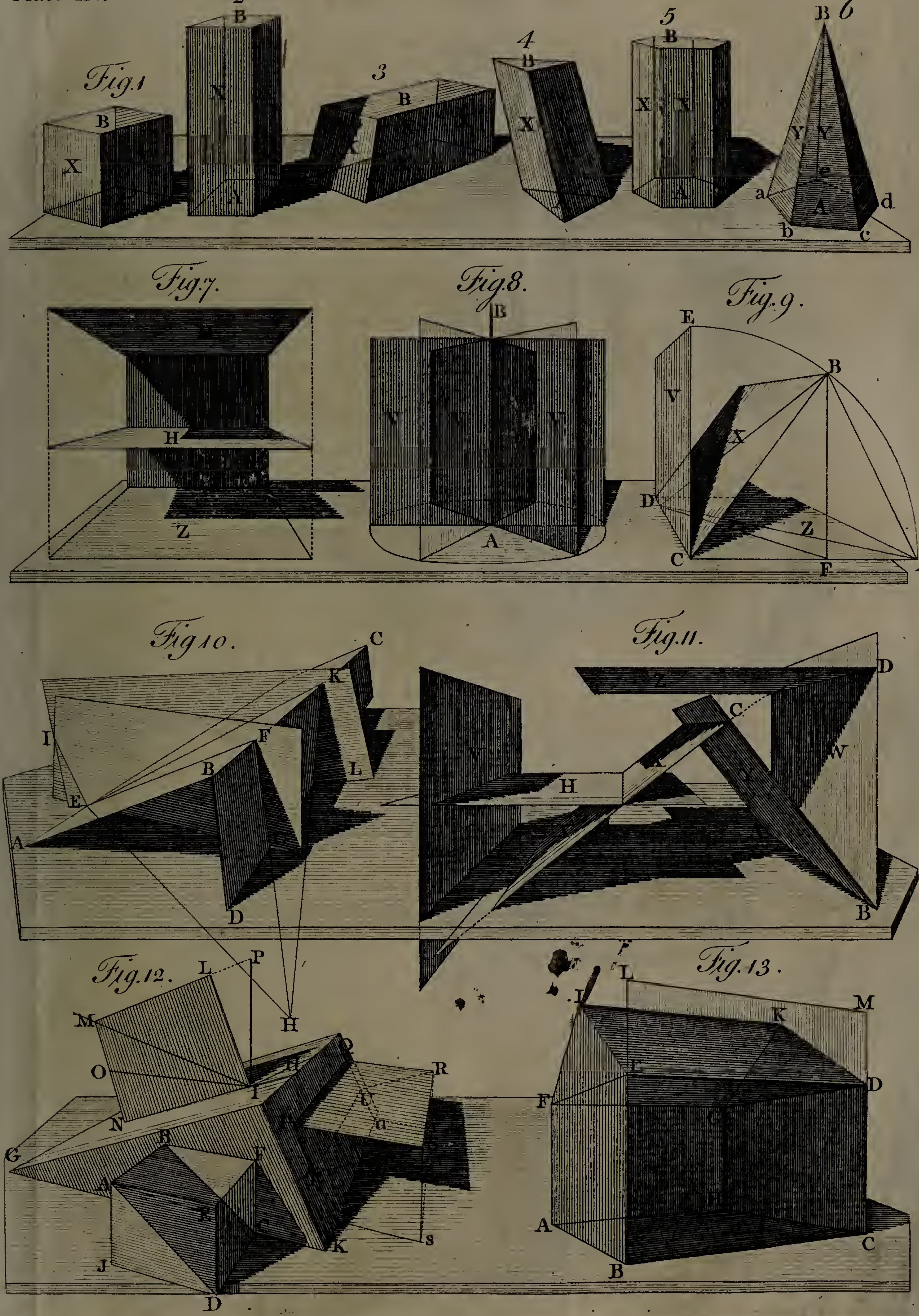

Plate IV.

Fig. 14.

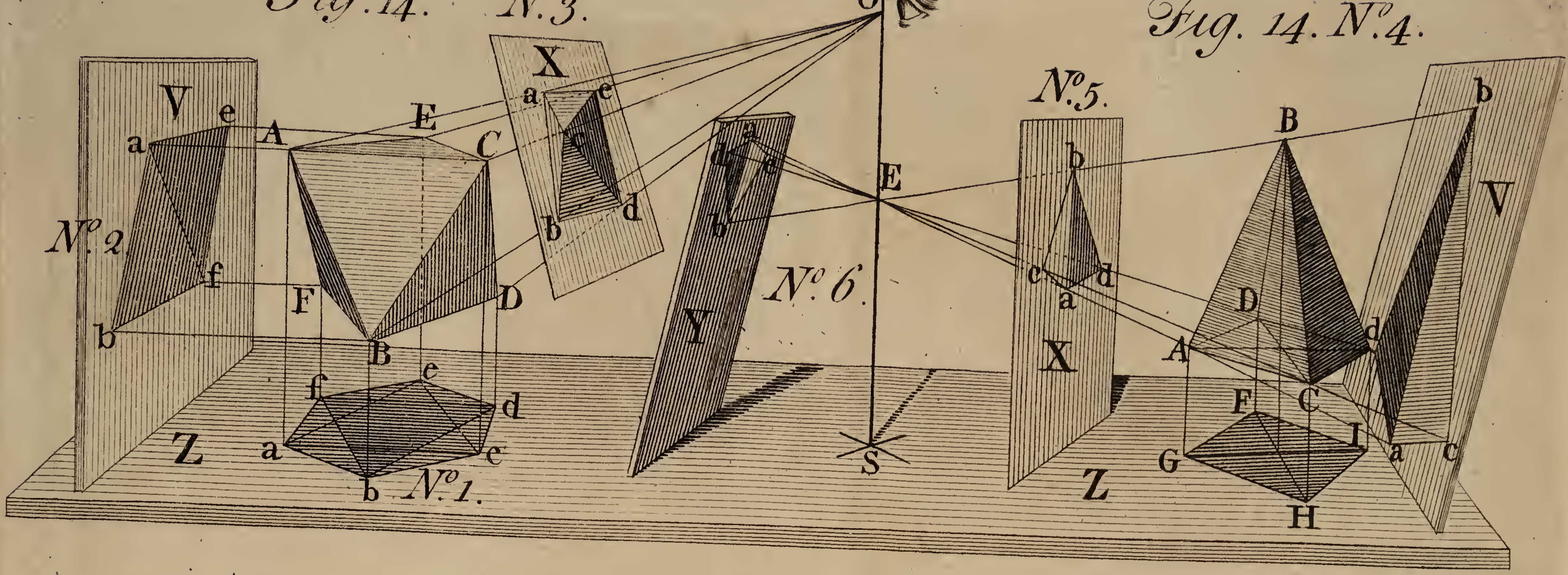

Fig. 16.

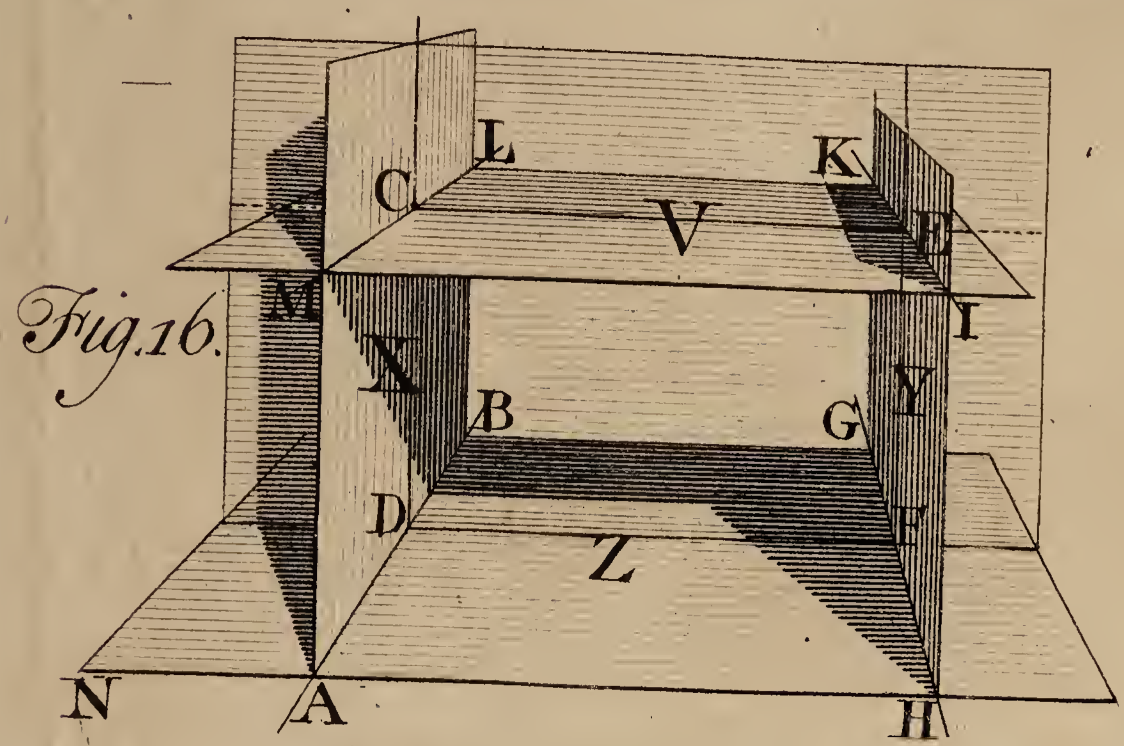

Fig. 17.

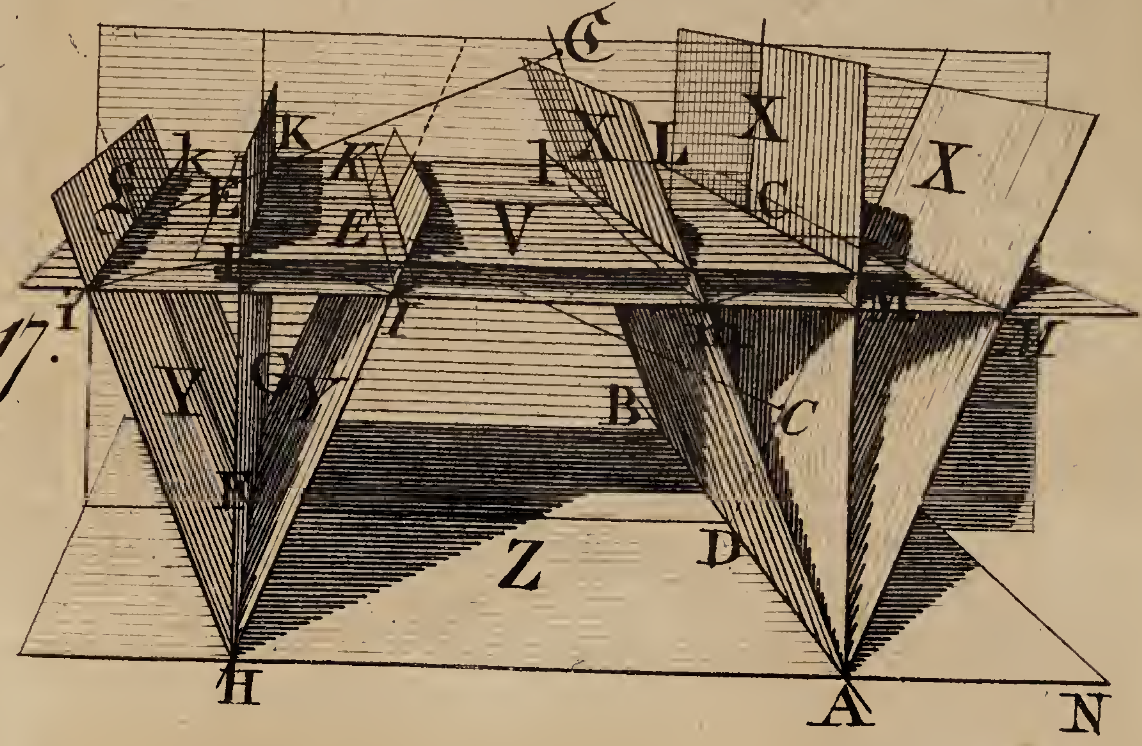

Fig. 19.

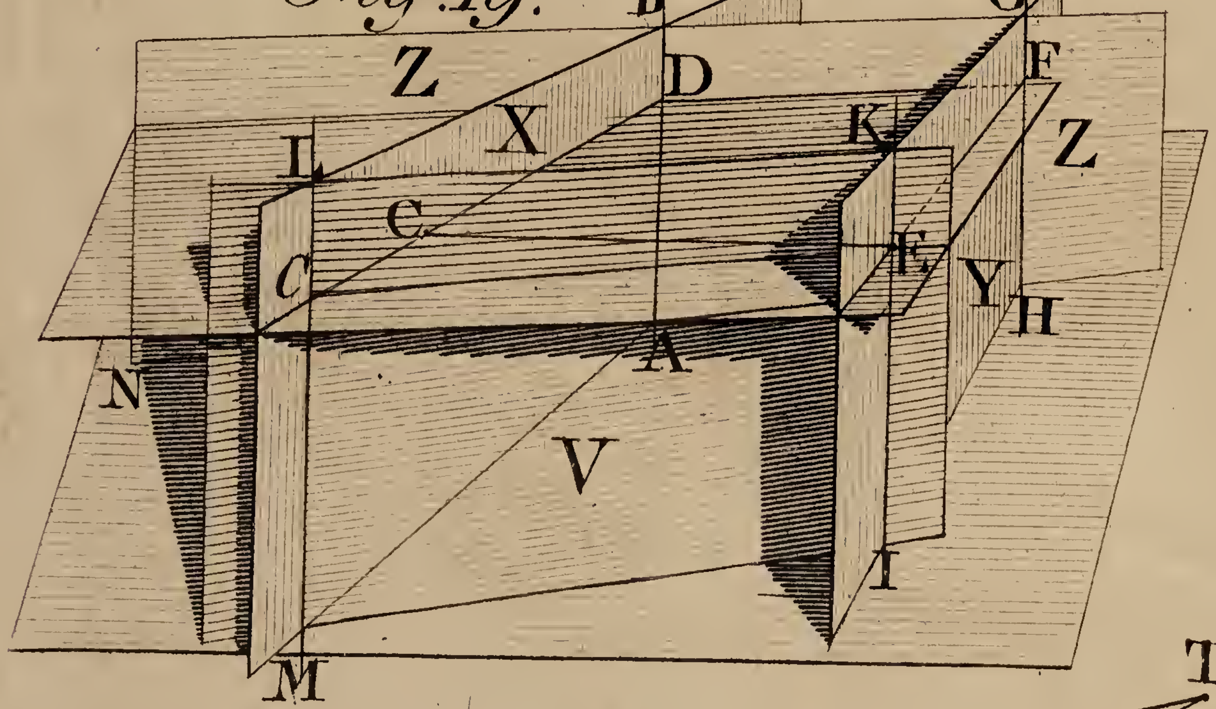

Fig. 18.

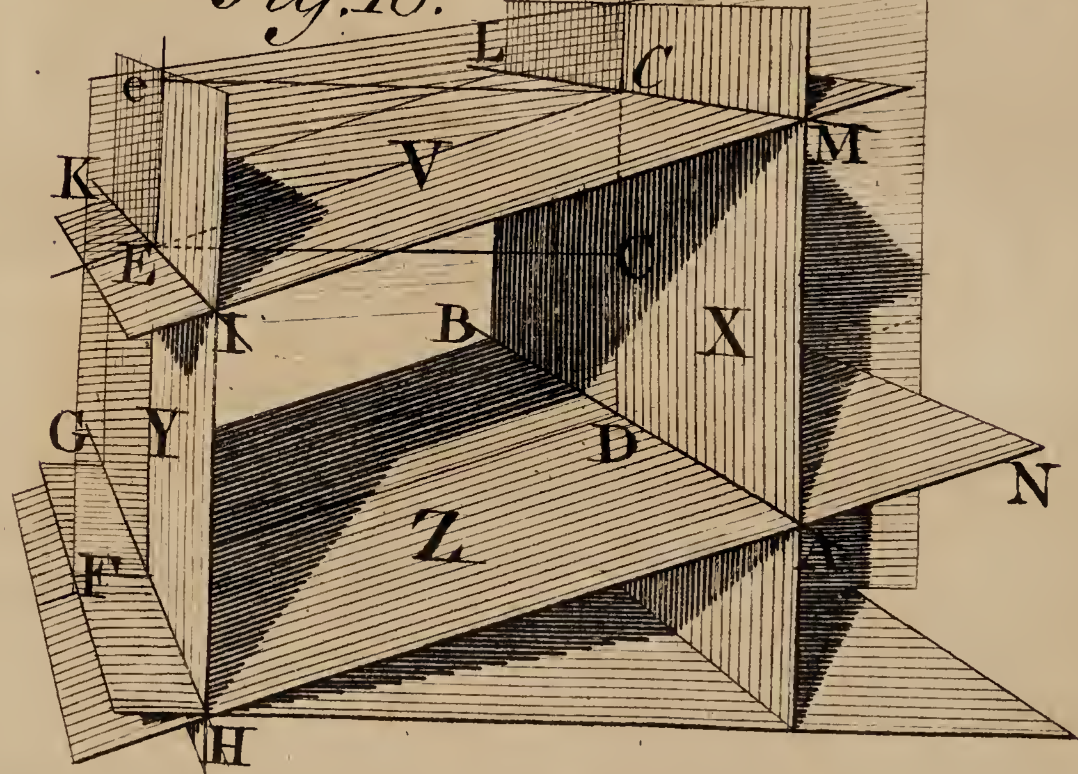

Fig. 20.

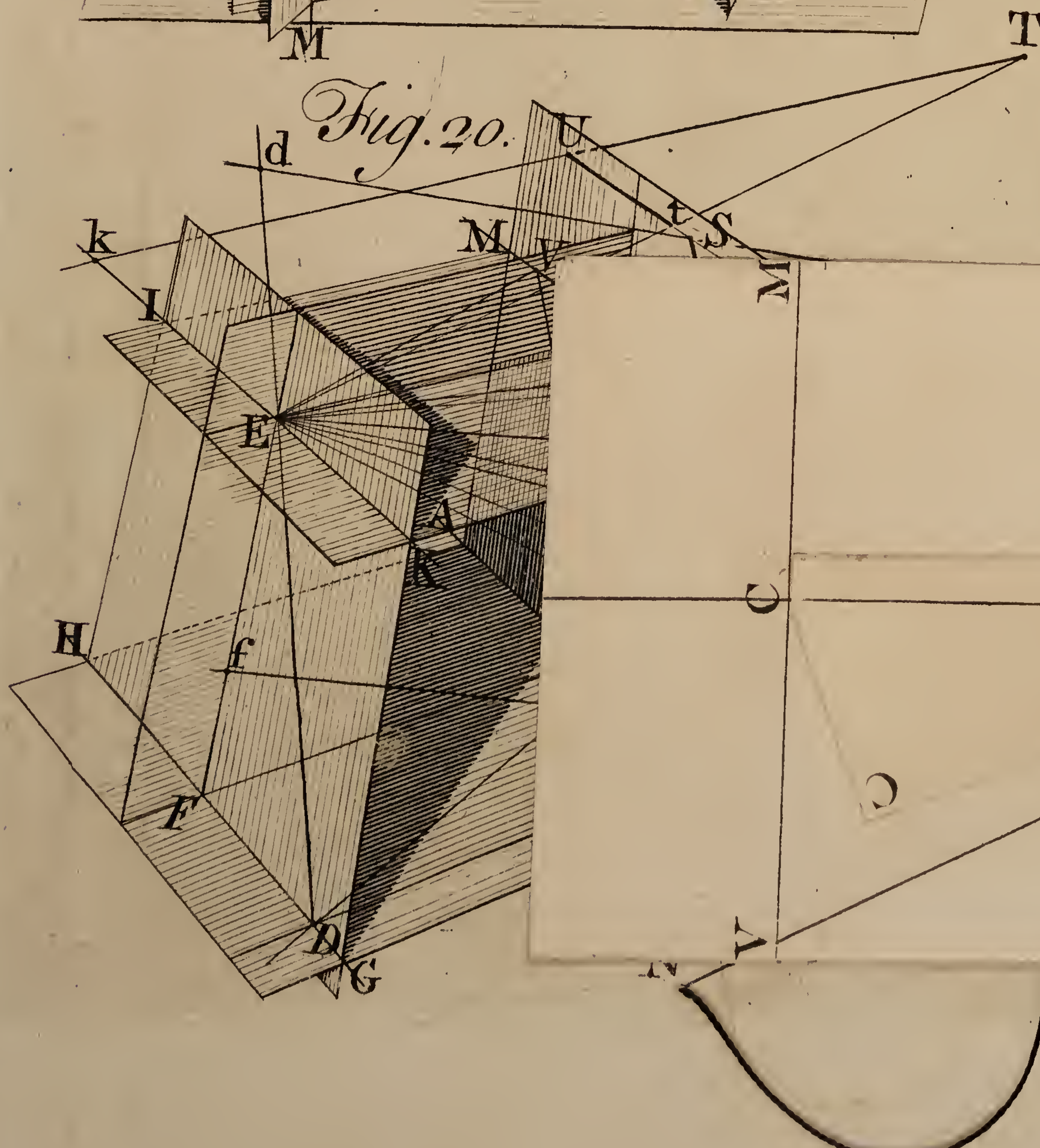

Fig. 21.

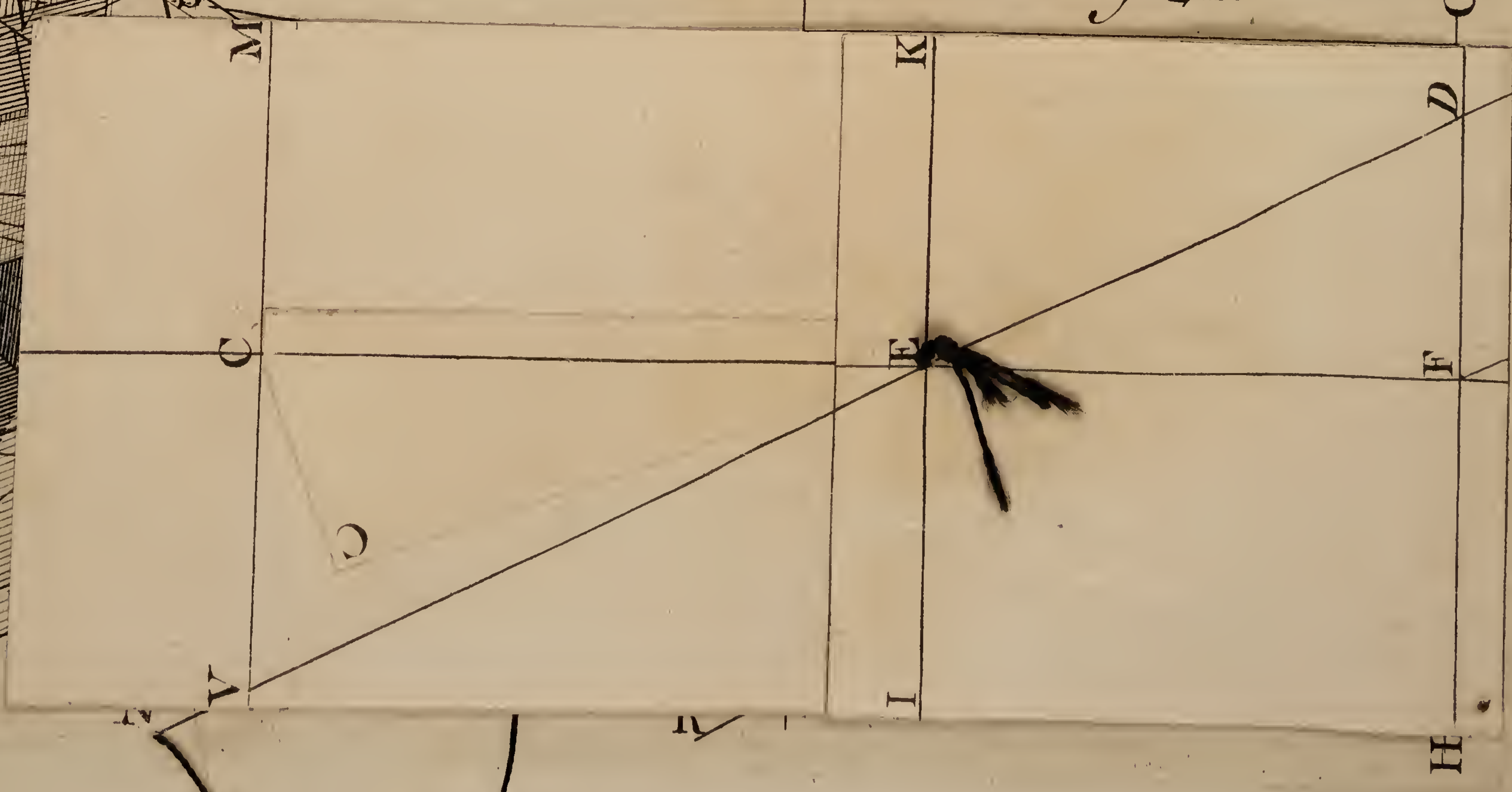

Plate V.

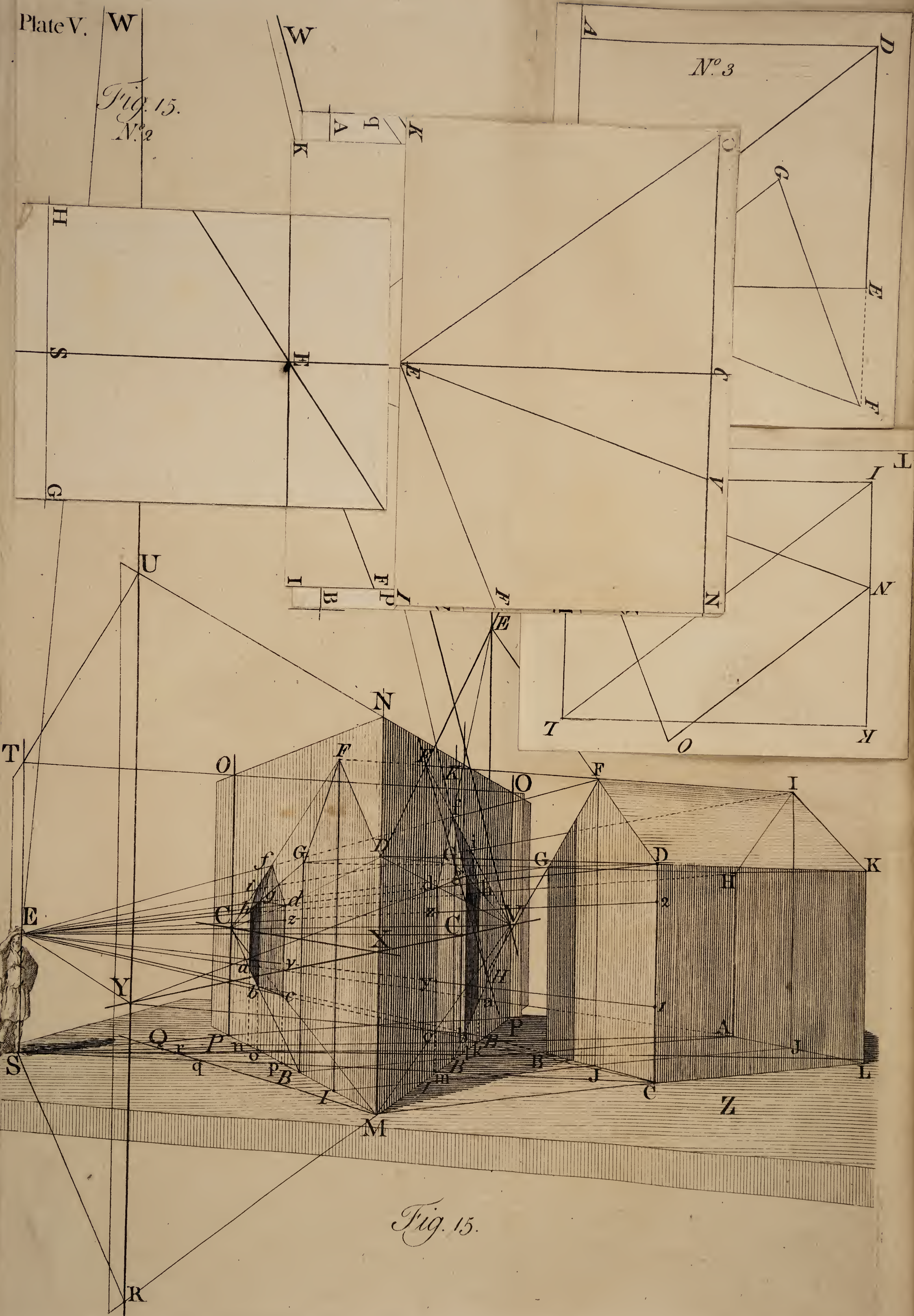

Fig. 15.

This is also illustrated in Fig. 23, and, by changing Y for M, it is described in the foregoing; save only, that A B C is not a Right Angle, consequently V E M is not a Right one, being equal to A B C.

SCHOL. If the Original Lines (not being in the same Plane) are so situated, that they cross, or would cross, each other, without cuting; then, the Angle, which the Radials of such Lines make with each other, is equal to the Angle that would be made between the Originals, if either of them be moved, parallel to itself, in any direction, till it cuts the other.

COR. 2. *The Vanishing Point of the common Interfection, of two Original Planes, is the Point in which their Vanishing Lines interfect each other.*

For, the Interfection of the parallel Planes, producing the Vanishing Lines, passes through the Eye, parallel to the common Interfection of the Original Planes (by the Theorem) and consequently, will cut the Picture in that Point where the Vanishing Lines interfect.

But, the Vanishing Point of a Line is that Point, in which, a parallel Line, from the Eye, cuts the Picture. - - - - Def. 22.

Therefore, the Point of Interfection of the Vanishing Lines is the Vanishing Point of the common Interfection of the Original Planes.

EX. A B is the common Interfection of the Planes X and Y. M L is the Vanishing Line of the Plane X, and *ML* of the Plane Y; their interfecting Point (V) is, therefore, the Vanishing Point of A B; for E V is parallel to A B. Fig. 23.

The Visual Rays E A, E B, together with the Line A B, form a plane Triangle, which cuts the Picture in a b (the Representation of A B) whose Vanishing Point is V; for, it is the Section of a Radial Plane, passing through the Original Line and the Eye, with the Picture; which must necessarily pass through E V, the Radial of A B †, and cut the Picture in J V. † Ax. 5.

M m is the Vanishing Line of the vertical Plane W, and M q of horizontal Planes; both which, cut M L, the Vanishing Line of the inclined Plane (X) in M; therefore, M is the Vanishing Point of C B, the common Interfection of the Planes W and X, and also, of a horizontal Plane (C B D.)

EX. 2. Let the triangular piece (in the Apparatus) be fixed to the direct Picture (of which, it is supposed to be a continuation) and draw the thread (at W) through the Eye of the Spectator; also, bring forward the folding part, into the same Plane.

Then, V Y is the Vanishing Line of horizontal Planes; R W, of the vertical Planes (B F C and A I L) and V W, of the inclined Plane F G H I. V, Y, and W, are, therefore, the Vanishing Points of the common Interfections of those Planes, respectively; *viz.* V, of G H, the common Interfection of D G H K and F G H I; and also of A B G H, whose Vanishing Line is O P. Y is the Vanishing Point of the common Interfection of horizontal Planes, and of the vertical Planes, B F C and A I L; and, W is the Vanishing Point of G F and HI, the common Interfections of those vertical Planes and the inclined Plane F G H I.

SCHOL. This Corollary, being so very essential in Practice, cannot be too much enforced; for, having found the Vanishing Point of a Line (in any Plane whatever) which is either the common Interfection of that Plane with some other adjoining Plane, or a Line parallel to it (as V) the Vanishing Line of that other Plane must, necessarily, pass through that Vanishing Point; and, if the Position of the other Vanishing Line be known, its place on the Picture is determined (as O P) otherwise, it is necessary to have the Vanishing Point of some other Line in the same Plane (as W, of the Line F G) in order to determine the Vanishing Line (V W) of the Plane (F G H I).

Plate IV.

THEOREM VII.

A Right Line drawn from the Center of the Picture, to the Center of a Vaniſhing Line, is perpendicular to that Vaniſhing Line.

DEM. For, becauſe the Direct Radial, producing the Center of the Picture, is perpendicular to the Picture §; and, the Center of every Vaniſhing Line is produced by a Right Line from the Eye, perpendicular to the Vaniſhing Line ‡; conſequently, the Radial of the Vaniſhing Line, producing its Center, and the Direct Radial, together with the Line which joins the two Centers, form a Triangle, and are all in the ſame Plane † which Plane is perpendicular to the Picture. - - - - - P. 9. 7. El.

§ Def. 15. ‡ Def. 19. † Ax. 8.

For, it paſſes through the Direct Radial, which is perpendicular to the Picture. And, it is alſo perpendicular to the Vaniſhing Line; becauſe, it is perpendicular to the Picture, and alſo to the Plane which produced the Vaniſhing Line. - - - - - Cor. 1. 9. 7. El.

Wherefore, ſince the Radial of the Vaniſhing Line, and the Right Line joining the Center of the Picture and the Center of the Vaniſhing Line, are both drawn to the ſame Point in a Plane, at which Point the Vaniſhing Line is perpendicular to that Plane; conſequently, the Line, joining the Center of the Picture and the Center of the Vaniſhing Line, is perpendicular to the Vaniſhing Line. Q. E. D. - - - 2. 7. El.

Fig. 18. and 19.

EX. L M is the Vaniſhing Line of the Original Plane Z; *C* is its Center, and C the Center of the Picture. Draw C*C*.

I ſay, that the Line C*C* is perpendicular to the Vaniſhing Line L M.

By Def. 17. E C producing the Center of the Picture is perpendicular to the Picture; wherefore, it is in the Vertical Plane (E *C* D F.)

For, it is perpendicular to the Picture (X) and alſo to the Vaniſhing Plane (V) †.

† Def. 5.

Conſequently, L M. the common Section of the two Planes, V and X, is perpendicular to the Vertical Plane (E *C* D F.) - - 9. 7. El.

Wherefore, E*C*, the Interſection of that Plane and the Vaniſhing Plane (V) is perpendicular to L M. - - - - - 2. 7. El.

§ Def. 19.

But, E*C* produced the Center (*C*) of the Vaniſhing Line L M §; and conſequently, C*C*, the Interſection of the Vertical Plane with the Picture, is, alſo, perpendicular to L M; for, E*C* and C*C* are two Lines, cuting each other, in the ſame Plane (E *C* D F) to which the Vaniſhing Line (L M) is perpendicular, at the Point (*C*) of their common Section.

COR. 1. *When the Original Plane is perpendicular to the Picture, the Center of the Picture is the Center of the Vaniſhing Line.*

Fig. 16.

For, a perpendicular (E C) from the Eye to the Vaniſhing Line (ML) producing its Center is the Direct Radial, which is perpendicular to the Picture; and, conſequently, produces the Center of the Picture; which is the Center of every Vaniſhing Line, of Planes perpendicular to the Picture. See Th. 4.

COR. 2. *The Diſtance of every Vaniſhing Line, which does not paſs through the Center of the Picture, is the Hypothenuſe of a Right angled Triangle, whoſe Catheti, or Legs, are the Diſtance of the Picture, and the Diſtance between the Center of the Picture and the Center of the Vaniſhing Line.*

Fig. 18. and 19.

For, E C is the Diſtance of the Picture, C*C* is the Diſtance between the two Centers; and, E*C*, the Hypothenuſe, of the Right angled Triangle E C *C*, is the Diſtance of the Vaniſhing Line L M; being the ſhorteſt Line that can be drawn from the Eye to the Vaniſhing Line. - Cor. 2. 12. 1. El.

To set this matter in the clearest light possible I have illustrated it by moveable Planes, in Fig. 21.; which, let be raised up, in any Angle at pleasure.

Underneath the Vanishing Plane (I K L M) is a triangular Plane (E C *C*) which, being turned down, perpendicular to the Vanishing Plane, will represent a Part of the Vertical Plane, and determines the Center of the Picture, in a certain angle of inclination, inclined from the Eye, in the Angle E*C*C.

The Triangle E C *C* is right angled, at C; consequently, E C is perpendicular to the Picture, and C is therefore its Center.† † Def. 17.

It is evident that *C*E and *C*C are both perpendicular to L M, or V M, the Vanishing Line of the Original Plane, N G H.

But, *C* is the Center of the Vanishing Line, and C is the Center of the Picture; therefore, a Right Line joining the Center of the Picture and the Center of a Vanishing Line is perpendicular to that Vanishing Line.

Above the Vanishing Plane is another Right angled Triangle; which being erected, perpendicular to the Vanishing Plane and the Picture, determines its Center, when the Picture is inclined towards the Eye, in the Angle E *C* C.

In which case, the Center of the Picture necessarily falls above the Vanishing Line; as in the former case it falls below.

Both these Positions are represented in the 17th Figure, and also in the 18th, considering either Plane, X or Y, as the Picture, and E or *C* the place of the Eye.

But, when the Picture and its Directing Plane are perpendicular to the Original Plane, as in the 16th, and the 17th Figure also shews the same, the Center of the Vanishing Line is the Center of the Picture.

N. B. In this Scheme, and some of the following Diagrams, the original Plane (N G H) must not be considered as necessarily being horizontal; for, the whole Theory, here given, is quite general, and applicable to all positions whatever, either of the Original Plane or of the Picture.

It may perhaps, to some, seem unnecessary to dwell so long on this Proposition, which, at first, may not appear of much consequence; but, I know it to be of the greatest consequence, in the most difficult part of Perspective, *viz.* in the practice on inclined Planes; the Theory, of which, being well digested and understood, the Practice will be found almost as easy as in the most familiar cases, being founded on the same invariable Principles, as will be exemplified in Practice; and therefore, the pains I have been at to enforce it, is by no means to be dispensed with.

The Reader may be well assured, that every Part of the Theory, given in this Work, is of utility, and necessary to be known; and that, the better it is understood, the progress in practical Perspective will be thereby facilitated.

THEOREM VIII.

The Perspective Projection of every original Right Line is a Right Line, in every Position or Situation whatever.

DEM. Suppose a radial Plane to pass through the Eye and any original Right Line.

If the Line be situated on the other side of the Picture, the imaginary Plane must, necessarily, cut the Picture; and, if it lie on this side, the Plane will cut the Picture if produced.

But, the Intersection of two Planes is a Right Line - - Ax. 3.

And, since the Eye is in this imaginary Plane, the whole of that Plane appears but a Line. (See Definition of a Plane, Page 41.)

But, the original Line is supposed to be in that Plane; consequently, the Original Line and every Line in the radial Plane, have their Representations in the Intersection of that Plane with the Picture; seeing, there can be no other Line common to both Planes. Therefore it is a Right Line.† Q. E. D. † Ax. 3.

Plate V. Fig. 15. EX. Let, B F I L be a rectilinear Object, bounded by Planes in various positions. Let M N O P be the Picture, placed direct between the Spectator (at E S) and the Object (B F I L).

F I, F G, or F D, is an Original Line (or, take any other Line in the Original Object) E is the Point of Sight, or place of the Eye; and E I, E F, E G, &c. are Visual Rays; in which, the extreme Points, I, F, G, &c. are seen, forming the Optic Angles F E I, F E G, &c.

Now, since the Original Line (F I, F G, &c.) is on the other Side of the Picture, the Visual Rays (E F, E I, &c.) must necessarily cut the Picture.

But, F E I is a Triangle; for, it is three Right Lines touching or cuting each other, and is, therefore, in a Plane. - - - Axiom 8.

The Ray E F cuts and passes through the Picture in the Point f, and E I in the Point i, &c. the Points f, i, g, &c. are, therefore, the Representations of the Original Points F, I, and G; for, they coincide with their respective Originals.

Wherefore, the Line f i, or f g, joining the Points f and i, or g, is the Intersection of the Plane Triangle F E I, or F E G, with the Picture.

But, the common Intersection of two Planes is a Right Line. - - Ax. 3.

And, f i is the Representation, or perspective Projection of the original Line F I, or f g of F G. (See Projection, Scenography, Page 49.)

For, to the Eye, at E, the Point f coincides with the Original Point, F, i with I, and g with G; consequently, the Line f i coincides with its Original F I, and f g with F G.

THEOREM IX.

The Representation, on the Picture, of a Line parallel to the Picture, is parallel to the Original; and, it has that proportion to the Original, as the Distance of the Picture to the Distance of a Plane, passing through the Original Line, parallel to the Picture.

DEM. The Perspective Projection of every Right Line on the Picture, is a Right Line; and it is produced, in Theory, by the Intersection of a Plane, with the Picture, passing through the Eye and the Original Line. - Theo. 8.

Wherefore, since the Original Line is, in this case, supposed parallel to the Picture, a Plane may pass through that Line, also parallel to the Picture.

But, if two Planes being parallel, are both cut by another Plane, their Intersections are parallel. - - - - - 8. 7. El.

But the Original Line is one of the Sections, and its Projection, on the Picture, is the other (the cuting Plane being supposed to pass through the Original Line and the Eye).

Therefore, the Representation of a Line, which is parallel to the Picture, is parallel to the Original. Q. E. D.

Fig. 22. EX. N O is an Original Line, parallel to the Picture (A L M B) E is the Eye.

Imagine a Plane, passing through the Original Line N O, parallel to the Picture; and, E N, E O, Visual Rays, from the Eye, to each extreme of the Original Line; which, are in the same Plane with N O. - Ax. 8.

Wherefore, N E O is a Radial Plane, passing through the Original Line and the Eye, and cuting the Picture in n o, the Representation of N O. - Th. 8.

But, the Plane Y is supposed parallel to the Picture, and N O is in that Plane; consequently, n o, its Representation, which is the intersection of the Radial Plane (N E O) with the Picture, is parallel to the Original Line (N O).

For, they are the common Sections of two parallel Planes, by another Plane.

In the same manner, pq, the Representation of PQ, may be proved parallel to PQ.

EX. 2. A H, B G, C D, &c. are Original Lines, parallel to both Pictures (M N O P or O *P*) E B and E G are Visual Rays, to the Line B G; which are all in the same Radial Plane, cuting the Pictures in b g, and *b g*; which are, therefore, the Representations of B G, on each Picture, respectively (by Theo. 8th.) Fig. 15.

But, B G, the Original Line, is perpendicular to the Horizon or Ground Plane, (S M Z) and the Pictures are vertical Planes.

And, the Radial Plane GEB is also vertical, seeing it passes through G B. 9. 7. El.

Consequently, the common sections of those Planes are Right Lines, perpendicular to the Ground Plane. - - - Cor. to 9. 7. El.

and, consequently, they are parallel between themselves. - 3. 7.

Therefore, b g or *b g*, the Representation of B G, is parallel to B G.

Also, c d, or *c d*, is parallel to C D; and a h, or *a h*, to A H.

After the same manner, the Representation of any other Line (B C, G D, or G F, &c.) in the Plane B F C, which is parallel to the Picture M N *O P*, only, may be proved parallel to their respective Originals.

DEM. 2. The Representation, n o (of N O) is proved parallel to N O, by the 1st Part. Fig. 22. No. 2.

Wherefore, the Triangles N E O, n E o, are similar;

and n o : N O : : E n : E N, or as E o : E O. - - Cor. 3. 2. 6. El.

But, E C f is perpendicular to the Picture A B, and consequently, to the Plane O N S, which is parallel to the Picture, cuting the Picture in C, its Center, and the Plane O N S in f. E f is therefore the shortest Line that can be drawn to those Planes, and consequently measures their Distances. 12. 1. El.

For, E C is the distance of the Picture, and E f of the Plane O N S.

But, if two or more Right Lines are cut by parallel Planes, they will be cut proportionally (whether they proceed from one Point or not) - 10. 7. El.

wherefore, E C : E f : : E n : E N, or, as E o is to E O.

But, n o : N O : : E n : E N. Th. n o : N O : : E C : E f, by equality of Ratios.

That is, the Representation, n o, has that Proportion to N O, its Original, as the Distance of the Picture, E C, has to E f, the Distance of a Plane passing through the Original Line parallel to the Picture.

N. B. The same Demonstration holds good if *N O* be considered as the Original Line, on this Side of the Picture, and projected to the Picture. (See Projected Perspective, page 49)

COR. 1. *From the former part, it is evident, that the Projections of any number of Lines, which are parallel amongst themselves and to the Picture, are also parallel amongst themselves and to the Originals.*

A H, B G, and C D, are parallel amongst themselves, and to both Pictures; their Representations, on both, are therefore parallel. Fig. 15.

Also, B C and G D are parallel between themselves and to the Picture M N O *P* only; their Representations, on that Picture, are therefore parallel between themselves and to the Originals.

COR. 2. *From the second part of this Theorem, may be clearly deduced; that if an Original Line parallel to the Picture, be any how divided, the Representations of the several Parts will have the same Proportion to each other, and to the whole Representation, as the Parts of the Original Line have to each other, and to the whole Line.*

If from the divisions, 1 and 2, of the Line C D, the Visual Rays E 1, E 2 be drawn, they will cut the Representations, c d, on both Pictures, to which the Original Line is parallel, in the same Ratio, in the Points y and z.

For, because c d is parallel to C D, the Triangles C E D, c E d are similar. And, for the same reason, C E 1, C E 2, are similar to cEy, cEz, &c. C. 3. 2. 6. El.

Wherefore, cy : C 1 : : c z : C 2 : i. e. as c d : C D : - 4. 6. El.

And, consequently, c y : y z : z d : : C 1 : 1 2 : 2 D; or, as c d to C D.

Plate VI. COR. 3. *Hence it is manifeſt, that if the Eye be moved, ſtill keeping the ſame Diſtance from the Picture*; i. e. *let the Eye be any where in the Directing Plane; the whole Repreſentation, and each Segment, will ſtill have the ſame proportion to the Original Line.*

For, by the ſecond part of the Theorem, the repreſentation of a Line parallel to the Picture has the ſame Proportion to the Original, as the Diſtance of the Picture to the Diſtance of a Plane, paſſing through that Line, parallel to the Picture. And the Diſtance of either is not varied, whilſt the Eye is in the Directing Plane; therefore, &c.

COR. 4. *The Angle, which the Repreſentations of any two Original Lines, that are parallel to the Picture, make with each other, is equal to the Angle made by the Original Lines. For the Repreſentations are reſpectively parallel to their Originals.*

Fig. 22. No. 2. In Fig. 22. No. 2. Let the Original Lines, O N and S R, be produced till they interſect (in P) making the Angle O P S.

Becauſe n o is parallel to N O and r s to R S, being produced, they will make the Angle o p s equal to O P S, made by the Original Line. 5. 7. El.

Fig. 15. In Fig. 15. the Angle *g f d*, on the Picture M N *O P*, to which the Plane B F C is parallel, is equal to the Angle G F D; and *f g d* to F G D, &c.

For, *f g* is parallel to F G, *f d* to F D, and *g d* to G D; by the Theorem.

COR. 5. *Original Figures, in Planes which are parallel to the Picture, have their Repreſentations ſimilar to the Originals.*

Fig. 22. No. 2. The Plane NOS is parallel to the Picture, A B; wherefore, the Repreſentation o p is parallel to the Original Line, O P; p s is alſo parallel to P S; and if o s, O S be drawn, o s will be parallel to O S, - by Theo. Part 1ſt.

By Cor. 4. the Angle o p s is equal to O P S; and, by the ſecond Part of the Theorem, o p : O P : : p s : P S; for, each is as E C to E f.

Conſequently, the Triangles p o s, P O S are equi-angular; the Angle o is equal O, and s equal S; and conſequently, o s : O S : : o p : O P, or as p s : P S.

1. 3. El. Therefore, the Repreſentation, p o s, is ſimilar to the Original Triangle, POS.

For, S E O P is a Pyramid, cut by a Plane (A B) parallel to its Baſe (POS).

The Plane B F C, in the Object B F I L, is parallel to the Picture M N O *P*.

Fig. 15. By Cor. 1. *b g* and *c d*, on that Picture, are parallel between themſelves, and to their Originals (B E and C D) and ſo are *g f* and *f d* to their Originals.

And by Cor. 4. the Angles *b g f* and *g f d*, &c. are equal, reſpectively, to the Angles B G F and G F D, &c.

But, it is demonſtrated, that, *b g* : B G : : *g f* : G F; or, *g d* to G D, &c. for, each is to its Original, as the Diſtance of the Picture to the Diſtance of the Plane they are in; by the Theorem, part 2.

Therefore, ſince the Sides are directly proportional, and the Angles contained by correſponding Sides, or Diagonals, equal; the Triangle *g f d* is ſimilar to G F D; and the whole Repreſentation (*b g f d c*) to the Original.

N. B. All Lines, that can be drawn in a Plane, which is parallel to the Picture, are parallel to the Picture; and their Repreſentations have all the ſame Proportion to their reſpective Originals.

THEOREM X.

All Right Lines, in any Original Plane, not parallel to the Picture, have their Interfecting Points in the Interfecting Line, and their Directing Points in the Directing Line of that Plane.

Alfo; the Vanifhing Points, of Original Lines, are all in the Vanifhing Line of the Planes the Original Lines are in.

DEM. If a Right Line be not parallel to the Picture, it will, if produced, cut the Picture, and the Directing Plane, in its Interfecting and Directing Points. § § Def. 21. and 23.

Now, fince one part of a Right Line cannot be in a Plane and another part of it out of that Plane; † confequently, the Original Line muft cut the Picture, fomewhere, in the Interfection of the Plane, it is in, with the Picture. † Ax. 14.

And, for the fame reafon, it will cut the Directing Plane, in the Interfection of the Plane it is in with the Directing Plane, i. e. in the Directing Line of that Plane. Therefore, all Lines, &c. Q. E. D.

Secondly. All Original Lines, in the fame Plane, have their Radials in the Vanifhing Plane of that Plane; their Interfections with the Picture are, confequently, in the Interfection of the Vanifhing Plane with the Picture.

But, the Interfection of the Vanifhing Plane, of any Original Plane with the Picture, is the Vanifhing Line of that Original Plane. - Def. 8.

Therefore, the Vanifhing Point, of every Original Line, is in the Vanifhing Line of the Plane that Original Line is in. Q. E. D.

Examples, of this felf evident Theorem, are in the Corollaries.

COR. 1. *Hence, a Right Line, drawn through any two Interfecting Points, is the Interfecting Line; and, a Right Line drawn through two Directing Points is the Directing Line of the Plane, in which the Original Lines are fituated.*

It is alfo manifeft, that a Right Line, drawn through any two Vanifhing Points of Lines in any Original Plane, is the Vanifhing Line of that Plane.

EX. A B, being produced, cuts the Picture in J, and N A cuts it in *I*; wherefore, a Right Line, drawn through J and *I*, is the Interfection of the Plane X. Fig. 23.

For, J and *I* are the Interfecting Points of Right Lines in that Plane; and if two Points in a Right Line are given, the whole Line is determined. (Cor. 1. Th. 2).

C B, produced, being in the fame Plane (X) cuts the Picture in d, in the Line J *I*; and fo, it is manifeft, would every other Line in that Plane.

A D, produced, cuts the Picture in *N*; J *N* is, therefore, the Interfection of the Plane Y; for AB is alfo in that Plane.

P is the Directing Point of A B, and O of C B; O P is, therefore, the Directing Line of the Plane X; and P Q of the Plane Y.*

The Plane I K L M, paffing through the Eye (at E) parallel to the Plane X, produces its Vanifhing Line, M L. - - - Def. 8.

E V is the Radial of A B, and E M of C B; which being parallel, refpectively to thofe Lines, are in the Vanifhing Plane I K L M.

And, the Vanifhing Points, V and M, are, confequently, in the Line of its fection with the Picture, which is the Vanifhing Line of that Plane; wherefore, if two Vanifhing Points (V and M) of any two Lines are determined, the Vanifhing Line (M L) of the Plane they in, is alfo determined.

For, the Vanifhing Point of every Line, in any Plane, is in its Vanifhing Line.

* The Point Q fhould be where A D would cut the Directing Plane, in a Line parallel to J *N*; i. e. where the Interfection, J *N*, and the Vanifhing Line, *M L* (which reprefent parallel Lines) interfect, which Point is out of the Plate. Therefore, P Q is not the true Directing Line, of the Plane A B D.

Plate VI. COR. 2. *All Right Lines, which are parallel to any Original Plane, have their Vaniſhing Points in the Vaniſhing Line of that Plane.*

Becauſe, a Plane may be drawn through any Right Line, parallel to any Plane to which the Line is parallel; and two or more Planes, being parallel, have the ſame Vaniſhing Line; by Theorem 3d.

COR. 3. *The Interſecting Point of the common Interſection, of two Original Planes, is the Point in which the Interſecting Lines of thoſe Planes cut each other.*

For, the common Section of two Planes is a Right Line, in both Planes. Wherefore, the Interſecting Point, of that Line, is in the Interſections of both Planes; conſequently, it is that Point in which the Interſections cut each other. For, there is no other Point common to both Lines.

Fig. 23. EX. J *I* is the Interſection of the Plane X, and J *N* of the Plane Y. J, the Point of their common Interſection, is, therefore, the Interſecting Point of A B, the common Interſection of the Planes, X and Y.

J *I* is the Interſection of the Plane X; and i d of the Plane W; d is, therefore, the Interſecting Point of C B, the common Interſection of the Planes X and W.

Fig. 15. 2. F D and F G are the Interſections of the vertical Plane B F C, and the inclined Planes, F K and F H; wherefore, *E* and *H* are the interſecting Points of thoſe Lines, on the Picture M N O P; where the vertical Interſection (*B E*) of the Plane B F C, is cut, by the Interſections, *D F* and *F G*, of the inclined Planes.

COR. 4. *The Directing Point of the common Interſection of two Planes, is the Interſection of their Directing Lines.*

Fig. 23. EX. P O is the Directing Line of the Plane X, and P Q of the Plane Y; wherefore, P, the Point of their Interſection, is the Directing Point of A B.

Hence, it is evident, may alſo be deduced the 2nd Corollary to the 6th Theorem, *viz.* The Vaniſhing Point of the common Interſection of two Original Planes, is the Interſection of their Vaniſhing Lines. Which, I adviſe the Student to be quite clear in; for, by that means, he will be able to determine the Vaniſhing Lines of contiguous Planes; and ſo proceed, in the delineation of Objects, from one Plane to another, with the greateſt facility.

This Diagram, though apparently intricate, is the completeſt in the Work, for rendering the knowledge of Vaniſhing Lines and Points, Interſections, &c. general; and I think, it cannot fail of anſwering the end I aimed at, *viz.* to diveſt the Student of that partiality which moſt have to the horizontal Vaniſhing Line; as it is manifeſt, that the ſame Principles extend to all, nor is there any difference in the proceſs; all the difficulty, if any, is in finding and determining them, on the Picture; which, when a clear Idea is inculcated, as by this Diagram, will readily be acquired. It is not poſſible to give, at once, ſo perfect an Idea, in Repreſentation, as in a Model, as is obvious in Figure 15, of the Apparatus; yet, when regularly analized, as in this Theorem, I am perſuaded, that all the intricacy will vaniſh.

THEOREM XI.

The Radial, or Parallel, of an Original Line producing its Vaniſhing Point, makes the ſame Angle with the Parallel of the Eye and Vaniſhing Line, as the Original Line makes with the Interſection and Directing Line, of the Plane that Original Line is in.

DEM. Since the Interfection and Vaniſhing Line, the Parallel of the Eye and Directing Line, of every Original Plane, are parallel amongſt themſelves (Th. 2.) and the Radial, producing the Vaniſhing Point of an Original Line, is parallel to the Original (Def. 22.); it neceſſarily follows, that, whatever inclination the Original Line has to the Interſection and Directing Line (which are all in the ſame Plane) the Radial of that Line has the ſame Inclination to, or makes equal Angles with, the Parallel of the Eye and Vaniſhing Line, which are all in a parallel Plane. - - - 4. of 1. and 7. El.

EX. N O is an Original Line, which, being produced, cuts the Interſection and Directing Line (A B and G H) in equal Angles. - - 4. 1. El. Fig. 20. and 21.

E V is the Radial of N O, producing its Vaniſhing Point (V) in the Vaniſhing Line (M L) of the Plane N G H, making the Angles E V L, with the Vaniſhing Line, and V E I, with the Parallel of the Eye, equal.

And, they are alſo equal to the Angles N *I* A, N *D* H, which the Original Line, (N O) makes with A B and G H, the Interſecting and Directing Lines.

Becauſe, the Original Line (N O) and its Radial (E V) are parallel (Def. 22.) and, becauſe they cut parallel Lines in parallel Planes.

2. It is alſo manifeſt, that, the Angle (V E *C*) made by the Radial (E V) of the Original Line (N O) and E *C*, the Radial of the Vaniſhing Line (V L) producing its Center (*C*) is equal to the Angle which the Original Line, (N *I D*) makes with F D, the Interſection of the Vertical Plane (E *C* D F) with the Plane N G H, in which the Original Line (N O) is ſituated.

For, theſe Angles are the Complements of the Angles made by the Original Line and its Radial, with the Interſection of the Picture, and the Parallel of the Eye or Vaniſhing Line.

3. From the ſame reaſoning, it is evident, that the Angles K E *D*, E *D* H, made by the Director (E *D*) of the Line N O, with the Parallel of the Eye and Directing Line, are equal to the Angles L V *I*, V *I* A, which the indefinite Repreſentation (*I* V) of the Original Line (N O) makes with the Interſection (A B) and the Vaniſhing Line (V L).

For, they are parallel Lines in parallel Planes, and are cut by parallel Lines.

COR. *The Center of every Vaniſhing Line is the Vaniſhing Point of Lines, which are perpendicular to the Interſection of thoſe Planes of which it is the Vaniſhing Line.*

For, by the Theorem, the Radial of a Line makes the ſame Angle with the Vaniſhing Line, as the Original Line makes with the Interſection of the Plane it is in. And in this caſe, the Original Line makes right Angles with the Interſection.

But, the Center of a Vaniſhing Line is produced by a perpendicular from the Eye to the Vaniſhing Line. - - - - Def. 19.

Therefore, it is the Radial of all Lines, which are perpendicular to the Interſection of the Plane they are in, &c.

I would recommend it to young Students, to be very clear in this Theorem, as it is moſt eſſential in Practice. For, from the knowledge it inculcates, the Vaniſhing Points of Lines, in any Plane whatever, may be readily found; knowing the Angle which the Original Line makes with the Interſection of the Plane it is in, with the Picture; or with a Line paſſing through the Station Point, and cuting the Interſection at Right Angles; making an equal Angle, at the Eye, with the Parallel of the Eye; or, with the Radial of the Vaniſhing Line, producing its Center.

Plate VI.

THEOREM XII. Defin. 25th.

The Indefinite Reprefentation, or Projection on the Picture, of an Original Right Line, not parallel to the Picture, is a Line drawn through its Interfecting and Vanifhing Points.

DEM. For, every Right Line, not parallel to the Picture, will, if produced, cut the Picture, fomewhere in the Interfecting Line of the Plane it is in.

And the Radial of every Line producing its Vanifhing Point muft cut the Picture, in the Vanifhing Line of the Plane the Original Line is in. Theo. 10.

Wherefore, a Radial Plane, paffing through the Original Line and the Eye, muft neceffarily pafs through the Radial of that Original Line, and confequently through its Vanifhing Point. - - - Ax. 5.

For, the Radial of every Line is parallel to the Line. - - Def. 14.

But, the Perfpective Projection of every Right Line, is a Right Line, produced by the Interfection of a Radial Plane with the Picture. Theo. 8.

And, the Radial Plane, paffes through its interfecting and vanifhing Points.

Confequently, the Indefinite Reprefentation of the Original Line, on the Picture, paffes through its Interfecting and Vanifhing Point. Q. E. D.

Fig. 24. EX. Let H L be a Right Line, cuting the Picture (A C B) in I, and the Directing Plane in D; let E be the Eye; E V is the Radial of H L, cuting the Picture in its Vanifhing Point (V) by Def. 22.

Let a Radial Plane (N O P Q) be fuppofed to pafs through the Original Line and the Eye (at E) confequently through the Radial E V (for it is parallel to HL) and alfo through I, the Interfecting Point of H L.

Now, the Points I and V are in this imaginary Plane, and they are alfo in the Picture; therefore, a Right Line, drawn through the Points I and V, is the Interfection of the Plane N O P Q with the Picture (A C B).

But, the Plane (N O P Q) paffes through the Eye (E) therefore, the Eye is in that Plane; and confequently, the whole of that Plane, and every Line in it, appears but a Right Line.

Wherefore, the Interfection (g V r) of that Plane with the Picture, is the indefinite Reprefentation of the Original Line (H L) on the Picture; feeing that, every Line drawn through E, to any Point (N, M, F, K, &c.) in that Line (except the Point D) is in the Plane N O P Q, and will, if produced, cut the Picture, fomewhere in I V, produced both ways, to g and r;

But, the Points n, m, f, &c. where the Vifual Rays, or Right Lines, E N, E M, E F, &c. cut the Picture, are the Reprefentations of the Points N, M, F, &c. in the Original Line†; confequently, the Reprefentation of every Point in the Line H L, except the Point D, where it cuts the Directing Plane, is in the Interfection g V r, produced indefinitely.

† Perfp. Art. 2.

But, the Interfecting Point, I, and V, the Vanifhing Point of the Original Line, are in that Interfection.

Therefore, the Indefinite Reprefentation of every Right Line is a Right Line, drawn through its Interfecting and Vanifhing Points.

COR. 1. *The Reprefentation of an indefinite Right Line, fituate on the other Side the Picture, lies between its Interfecting and Vanifhing Points.*

If that part of the Original Line (I N) which lies beyond its Interfecting Point (I) was produced infinitely, towards H, its whole Reprefentation, on the Picture, is I V.

For, the Point I is its own Reprefentation; I m is the Reprefentation of I M, and m n of the part M N; and, if the Point H be fuppofed at any finite Diftance, its Reprefentation h, muft lie between n and V.

5 For,

For, fince the Radial, E V, producing the Vanifhing Point (V) is parallel to the Original Line (I N) the Angle V E H will always be equal to E H D.

And, if the Point H be fuppofed at an infinite Diftance, its Reprefentation, (h) will coincide with V; the Angle V E H will not be fenfible, and confequently the Point H will vanifh, or be loft to fight; and, therefore, the whole indefinite Reprefentation of I H, infinitely produced, lies between its Interfecting Point (I) and its Vanifhing Point (V).

COR. 2. *The Perfpective Reprefentation, of a finite Right Line, is part of a Line drawn from its Interfecting to its Vanifhing Point.*

For the Vifual Rays (E M, E N, E H, &c.) from the feveral Parts of the Original Line (M N, N H, &c.) to the Eye, determine the reprefentations of thofe Parts; and, fince the whole infinite Line, from I, its Interfecting Point, is reprefented between its Interfecting and its Vanifhing Points (I and V) confequently, the Reprefentation of every finite Part is a part of I V. Fig. 24.

I m, reprefents the Part I M, m n reprefents MN, and, nh reprefents NH; the remaining Part, h V, reprefents all the Line beyond H, infinitely produced.

COR. 3. *The Projective Reprefentation of that Part of an Original Line* (I D) *which lies between the Picture and the Directing Plane, falls on the other fide of its Interfecting Point*; *and, its whole Reprefentation is infinite.*

EX. The Point F will be projected to f, where the Vifual Ray E F, produced, cuts the Picture; and the Point G to g. - - (See Proj. Perf. P. 49).
I f is, therefore, the projective Reprefentation of I F, and f g of F G.

If any Point (K) be taken, near D, its Reprefentation will be projected at a great diftance from the Interfection I; and, the nearer the Original Point is to D, the farther will it be projected, from I; for, E k will always make an Angle with I g equal to D E K; but, when the Point K coincides with D, it will be projected to an infinite Diftance.

Therefore, the Part, I D is reprefented by I g, infinitely produced; and confequently, the Directing Point, D, can have no Reprefentation.

SCHOL. It may be obferved, that, as the perfpective Part (IV) of the Indefinite Reprefentation, reprefents all that Part beyond its Interfecting Point, to an infinite Diftance; fo, the finite part of the Original Line, I D, between the Interfecting and Directing Point, is infinite in its Reprefentation, from I through g.

COR. 4. *All that part of the Original Line, which lies on the other fide of the Directing Plane, infinitely produced from its Directing Point, is tranfprojected to the Picture, beyond its Vanifhing Point*; *and the whole Reprefentation is infinite.*

EX. The reprefentation of the Point Q is at q, and of R at r; and, the nearer any Point (S) is taken to D, the farther will its Reprefentation be from V; but, when S coincides with D, its Reprefentation will be at an infinite Diftance. (See Tranfprojection, Page 49).
Alfo, the farther any Point (L) is taken beyond Q, the nearer its Reprefentation (C) will approach to V; and, at an infinite Diftance, it will coincide with V.

SCHOL. Here, it may be obferved, that an indefinite Original Line is reprefented by an indefinite Line, the Terms being inverted; and that, a Point, on the other Side of D, will be tranfprojected to an infinite Diftance, on one fide of the Vanifhing Point, V. On this fide, it will be projected to an infinite Diftance on the other fide of the Vanifhing Point; whereas, but one Point (D) lies between; and alfo, that the Reprefentations of two Points, H and L, being confidered as the extremes of a Right Line, at infinite Diftances, both ways, from the Interfecting Point (I) coincide at V.

Plate VI. 2. The tranfprojected part of the indefinite Projection, C V, may reprefent a Line in the fame Plane, above the Eye; from the Point C produced indefinitely, through O; which muft alfo be infinite, if it be parallel to E V, before the Points o and V coincide.

3. If the Original Line T V paffes through the Eye, its whole Reprefentation is in its Vanifhing Point; for, its Interfecting and Vanifhing Point is the fame; and, the Point of Sight (E) is its Directing Point.

COR. 5. *The whole indefinite Reprefentation* (I V) *of an Original Line, from its Interfecting Point* (I) *is not varied, the Eye being in any Part of its Radial* (E V).

Suppofe the Eye, at E, removed to *E*, in the Direction V E.

E V, being parallel to I N, the Original Line, the Vanifhing Point, V, remains the fame; and, I, being its Interfecting Point, is invariable; confequently, the Indefinite Reprefentation (I V) will be the fame at any Diftance of the Eye.

But, it muft be obferved, that, it is only fo in refpect of the whole of I V; for the finite Parts of the Reprefentation are continually varied, while the Eye moves from V to *E*; m being the perfpective Reprefentation of the Original Point M, from the Eye at E; but, removed to *E*, it will appear at m^2; confequently, I m is the Reprefentation of the finite Part I M, of the Original Line, from the firft Point of View, and I m^2 from the Point *E*. And fo, of any other part of the whole Line.

Again, if the Eye move from E, or *E*, in the direction of E M, or *E* M, the Reprefentation m, or *m*, remains the fame, and the indefinite Reprefentation is varied. For, at *E*, the indefinite Reprefentation is I V; but, at E^2, it is I n; yet I m^2, the Reprefentation of the finite part I M, is the fame, from both Points of View.

COR. 6. *Hence, it is alfo evident, that, if the Eye be removed to any other Point in* E D (*the Director of the Original Line,* I N) *its whole Indefinite Reprefentation, as well as the finite Parts are varied from every Station.*

For, if the Eye be raifed to E^3, the Radial is E^3 C; confequently, I C is the whole indefinite Reprefentation; but if it be depreffed to E^2, then is E^2 n the Radial, and I n the whole indefinite Reprefentation; and the finite Parts I *m*, *m n*, &c. are raifed higher, or the Points, m, n, fall nearer to the Interfection, I, as the Eye is raifed or depreffed.

COR. 7. *The Reprefentation of the common Interfection of two Original Lines, is the Point, in which the Indefinite Reprefentations interfect each other.*

Fig. 23. For, the Point B, of the Interfection of the two Original Lines, AB and B C, is common to both Lines; wherefore, its Reprefentation is in the Indefinite Reprefentations (J V and d M) of both Lines; and, confequently, in the Point (b) of their mutual Interfection; for, there can be no other Point common to both. Alfo, a, the common Interfection of J V and *NV*, the indefinite Reprefentations of A B and A D is the Reprefentation of A, the Interfection of thofe Lines.

As this Theorem, and what is deducible from it, with the next, contain the whole effence of practical, rectilinear Perfpective, refpecting all Lines which are not parallel to the Picture, I would advife the Student to make himfelf particularly well verfed in them before he proceeds to Practice; for if thofe Theorems be clearly underftood and retained, he will find the Practice eafy to be acquired, and, at the fame time, rationally accounted for.

THEOREM XIII.

The Diftance between the Interfecting Point of an Original Line, and the Reprefentation of any Point in that Line, is to the whole Indefinite Reprefentation; as the Diftance between the Original Point and the Interfecting Point, is to the Diftance between the Original Point and the Directing Point, of that Line.

This Theorem will be beft demonftrated in the Example.

M N is an Original Line produced to D; I is its Interfecting, and D its Directing Point; and V is the Vanifhing Point of that Line, the Eye being at E; E V is, therefore, its Radial. Fig. 24.

Now, if any Point, M or N, be taken in the Original Line, apart from its Interfecting Point; its Reprefentation, on the Picture, will be, fomewhere, between I, the Interfecting, and V, the Vanifhing Point of the Original Line.

I fay, that, the Diftance of m or n, from I, the Interfecting Point, is, in proportion to the Indefinite Reprefentation, I V; as M I or N I, the Diftance of the Original Point from the Interfecting Point, is to M D or N D, the Diftance of the Original Point from the Directing Point.

Having drawn the Vifual Ray, E M or E N; the Reprefentation, m or n, of the Point M or N, is where the Vifual Ray cuts the Picture. Perfp. Art. 2.

DEM. Now, E V is parallel to N M D †; and, I V is parallel to E D. 8. 7. El.
Confequently, I V E D is a Parallelogram‡; and the Triangles N E D, N n I are fimilar; wherefore, I n : E D : : I N : N D. - - - 4. 6. El.
But, I V is equal to E D||; therefore, I n : I V : : I N : N D. Q. E. D.

† Def. 22.
‡ Def. 33. Geom.
|| 15. 1. El.

Or, it may be demonftrated thus:

Becaufe E V is parallel to I N, the Triangles I N n, n E V are fimilar.
Wherefore, I n : n V : : I N : V E; and confequently, - - 4. 6. El.
In : I n + n V (= I V) : : I N : I N + V E (= N D) i.e. I n : I V : : I N : N D.

N. B. This Proportion is invariable, whether the Eye be farther from (as at *E*) or nearer to the Picture; or whether the Eye be raifed or depreffed, as at *E*[3] or *E*[2]; for, E[3] D being ftill parallel and equal to the Indefinite Reprefentation (I C) confequently, I *n* : I C : I N : N D; and I *m* : I C : : I M : M D.

COR. 1. *From this Theorem, the Indefinite Reprefentation of a Line being given or drawn, and the Diftance of any determinate Point, in the Original Line, from the Picture, known, the Reprefentation of that Point, on the Picture, is alfo determinable.*

For, whatever Plane the Original Line is in, is not material, the Diftance of the Point, in queftion, from the Picture, anfwers the fame purpofe, as its Diftance from the Interfecting Point of the Line it is in; or, from the Interfection of any Plane with the Picture in which that Line is fituated. e. g.

EX. N M is an Original Line cuting the Picture, in the Point I, in the Interfection, (A B) of the Plane (N B C) that Line is in; and the Directing Plane (D E C) in its Directing Point (D). Fig. 25.

I fay, that N F (the Diftance of the Point N from the Picture, A V B) is to F G, the Diftance of the Picture, as N I to I D (the Diftance of the Point N

U from

Plate VI. Fig. 25.

from the Interfecting Point, to the Diftance between the Interfecting and Directing Points, of the Line M N) or, as N A to A C, its Diftance from the Interfection, to the Diftance between the Interfection and Directing Line, of the Plane the Original Line is in.

DEM. For (having joined I F and D G) becaufe the Directing Plane is parallel to the Picture, I F is parallel to D G - - - 8. 7. El.
(for the Triangle G N D is a Plane cuting them both ; Ax. 8.)
Wherefore, the Triangles G N D, F N I, are fimilar ; and, for the fame reafon, G N C, F N A, and alfo, D N C, I N A, are fimilar ;
confequently N F : N G : : N I : N D ; or, as N A : N C. - - - 4. 6. El.
Wherefore, the Diftance of the Original Point from the Picture, and the Diftance of the Picture being known ; the Diftance of its Reprefentation, from the Interfecting Point, is a fourth Proportional, *viz.* as the Diftance of the Original Point from the Picture, added to the Diftance of the Picture, i. e. as the Diftance N G (of the Original Point, N, from the Directing Plane) is to N F (the Diftance of that Point from the Picture) fo is the Indefinite Reprefentation, I V (of the Original Line M N) to I n ; the Diftance of n (the Reprefentation of the Point N) from I, the Interfecting Point of the Original Line. Q. E. D. It is, therefore, as N G : N F : : I V : I n.
For, (by the Theorem) it is, inverfely, as N D : N I : : I V : I n.
Or, from the Vanifhing Point, it will be, as N G : F G : : I V : n V.

In Numbers, it is thus calculated. The Diftance (N F) of the Original Point from the Picture, and the Diftance of the Picture (FG) being added (equal NG). The Indefinite Reprefentation, I V, being multiplied by N F, and that Product divided by N G (the Diftance of the Original Point from the Directing Plane) gives I n ; the Diftance of n, the Reprefentation of the Original Point, from the Interfecting Point, of the Line it is in. For, N G : N F : : I V : I n ; as above.
Wherefore, the Rectangle, under N G and I n, is equal to the Rectangle under N F and I V ; and confequently, $NG \times In = NF \times IV$; - 9. 6. El.

It more frequently occurs, in Practice, that the Reprefentation of a certain portion of a Line is required, from fome Point already found ; the Interfecting Point of the Original Line not being within reach, nor attainable, but only the Vanifhing Point. For, the Interfection of every Plane is neither wanted nor can be had, when the Vanifhing Line of the Plane is abfolutely neceffary ; it is the fame in refpect of the Vanifhing and Interfecting Points, of Original Lines ; as it will be exemplified in the practical part of this Treatife.

COR. 2. *If the whole Indefinite Reprefentation be not given, but only a part, from fome determinate Point, in the Original Line* ; *any other portion, of that Line, (from the Point determined) may be perfpectively proportioned, geometrically.*

The Reprefentation, m, of the Point M, in the Original Line (M N) being given or found, and the Indefinite Reprefentation (m V) from that Point drawn (V being the Vanifhing Point) the Reprefentation, n, of any other Point, N, in the Original Line, will be ; as N M : M D : : m n : n V ;
or, as N M : N D : : m n : m V.

DEM. For, as N I : N D : : I n : I V ; and, as M I : M D : : I m : I V ; by Theo.
But, - N D : I D : : I V : n V ; and, - M D : I D : : I V : m V ; 4. 6. El.
for, the Triangles N E D, V E n are fimilar ; DE is equal to IV, and ID to VE (Becaufe N D E V is a Radial Plane, paffing through the Original Line (N M) and the Eye (E) cuting the Picture and Directing Plane ; in I V and E D ; and, E V, producing the Vanifhing Point (V) is parallel to N D ; by Def. 22.)

Now,

Now, N D : I D :: I V : n V ; also M D : I D :: I V : m V ; Fig. 25.
inversely, - - - - I D : MD :: m V : I V.
Wherefore, by inordinate equality, N D : MD :: m V : n V.
Consequently, - N D—M D (=N M): MD :: m V × n V. (= m n) : n V.
that is, - - - - N M : MD :: m n : n V, as it was affirmed.
Wherefore, by addition, - - N M : N D :: m n : m V ;
and consequently, by inversion, N D : N M :: m V : m n.

Wherefore, the Distance of M (whose Representation, m, is given) and the Distance of any other Point, N, in the Original Line, from the Picture, being known ; the difference being N*m*, it will, consequently, be, as N G : N*m* :: m V : m n ; Q. E. D. For, N G : N*m* :: N D : N M.

Thus may the perspective Proportion, of the known proportions of any Original Line, from any Point in it, be ascertained, geometrically ; having the Representation of that Point, in the Indefinite Representation given, and the Distance of that Point, with any other Point in the Original Line, from the Picture, known ; seeing, that the Distances of the several Points, from the Picture, are in the same Ratio, as their Distances from the Intersecting Point ; and the Distance of those Points, from the Picture, may be had, when, often, their Distances from the Intersecting Point of that Line cannot ; for several reasons.

The Distances of the Points M and N, from the Picture, being known, and the Distance of any other Point, in that Line (as O) required ; it will be, as MN : M O or N O :: *m*N : *mo* or N *o* ; (O *o* being supposed parallel to M *m*).

So that, if *m* F, the Distance of the Point M from the Picture, be added, we have the Distance of the Point O, from the Picture, equal *o* F.

And the Distance of the Picture (F G) being known, the Representation, o, of the Point O, is determinable ; for, it will be, as *om* : *m*G :: mo : oV ;
and, by addition, - - - - as *om* : *o*G :: mo : mV.

By which means, the perspective Representations of several Divisions in a Right Line may be had, geometrically, their Distance from each other being known.

Let A B C be an Original Line, and C D E G a Radial Plane, passing through the Eye (E) and also through the Original Line ; cuting the Picture in I V, their common Section. Fig. 26.

E V, parallel to A C, is its Radial ; I V is, therefore, the whole Indefinite Representation of A B C ; I is its Intersecting, D its Directing, and V its Vanishing Point.

Now, if the Representation (a) of any Point (A) in the Original Line, be determined, all other Divisions (B, C, &c.) in that Line, are also determinable, without the Intersection I ; the Distance of A and B, from the Picture, and the Distance of the Picture being known.

For, a b : a V :: A B : B D ; a c : a V :: A C : C D ; and also, b c : b V :: B C : C D.

DEM. For, (having drawn A F and B G, parallel to I V) because a V is parallel to A F, a b : a V :: A*b* : A F ; also, a c : a V :: A *c* : A F ;
and, for the same reason, b c : b V :: *bc* : *b*F ; and, as B*C* : B G. } Cor. to 6. 6. El.

But, the Triangles G E *C*, *C* C B are similar, for E G is parallel to C B ;
wherefore, - - - B*C* : *C*G :: B C : E G (equal B D) 4. 6. El.
Consequently, by compounding, B*C* : B G :: B C : C D, equal C B + G E.
But, as B *C* : B G :: b c : b V ; therefore, b c : b V :: B C : C D.

c is, therefore, the Representation of the Original Point C, and b of B ; which are determined from the given Representation of A ; the Distance of A and any other Original Point from the Picture, being known ; no regard being had to the Distance of the Point A, from the Intersecting Point (I) or from D, the

Plate VI. the Directing Point of the Original Line; seeing that, the Distance of the Original Points, from the Picture and Directing Plane, are in the same Ratio, as their Distances from the Intersecting and Directing Points of the Original Line.

In this Theorem is the Perfection of Practical perspective. It is, at the same time, the most mathematical, and the Demonstrations the most perfect, elegant, and convictive, of the whole Theory. By it, we not only know that there is Analogy of Ratios, between the several Distances of Points, in an Indefinite Representation, from the Intersecting Point, and the whole Indefinite Representation; to the several Distances of the Original Points from the Intersecting and Directing Points, or from the Picture and Directing Plane; but, by its means, the Representations of the several portions of an Original Line, in the Indefinite Representation, are determined, with the greatest facility, accuracy, and expedition, geometrically; or they may be determined numerically, by a Scale of equal Parts; but that is seldom practiced, the method of doing it geometrically, being much readier and more accurate than it is possible to calculate, by Numbers.

THEOREM XIV.

The Perspective Projection, or Representation, of every Right Line, is parallel to its Director.

Dem. The Radial of an Original Line, producing its Vanishing Point, is parallel to the Original Line; — — — — Def. 22.
wherefore, a Radial Plane may pass through the Original Line and its Radial; consequently the Eye is in that Radial Plane. — — Ax. 5.

Now, since one part of a Right Line cannot be in a Plane, and another part of the Line out of that Plane, the Radial Plane will cut the intersecting Line (of the Plane the Original Line is in) in the intersecting Point, and the Directing Line, in the Directing Point, of the Original Line. — Ax. 1.

But, the Radial Plane passes through the Eye; — — Def. 6.
and, consequently, through the Vanishing Point; seeing it passes through the Radial Line, producing the Vanishing Point.

Wherefore, the Section of this Radial Plane, with the Picture, is the Indefinite Representation of the Original Line; seeing, it passes through its Intersecting and Vanishing Points. — — — Theorem 12.

But, it also passes through the Eye, and Directing Point, of the Original Line; consequently, it cuts the Directing Plane in the Director of the Original Line. — — — — — Def. 12.

But, the Directing Plane is parallel to the Picture. — Def. 4.

Wherefore, the Sections of the Radial Plane, with the Picture and Directing Plane, are parallel to one another. — — — 8. 7. El.

But, its Section with the Picture is the Indefinite Representation; and, its Section with the Directing Plane is the Director of the Original Line; as above.

Therefore, the Perspective Projection of an Original Line, being a part of its indefinite Representation, is parallel to its Director. Q. E. D.

Fig. 20. EX. Imagine a Plate (N*D*EV) passing through the Eye and the Original Line, NO. This Radial Plane must necessarily cut the Picture and Directing Plane, in *I*V and E*D*; for EV is parallel to *DI*N, i. e. to NO; wherefore, EV*ID* is a part of the Radial Plane; which, being produced, would pass through NO, the Original Line.

But, *I* is the Intersecting, and, V is the Vanishing Point of NO; Def. 21. and 22.
wherefore, *I*V is the Indefinite Representation of NO; — Def. 25. Th. 12.
and, since the Radial Plane passes also through the Eye and Directing Point,

(E and D) its Section with the Directing Plane (ED) is the Director of N O ‡. ‡ Def. 12.
Consequently, IV, or n o, a part of IV, is parallel to E D. - 8. 7. E l

In Fig. 21. this is more perfectly illustrated, by means of Visual Rays; from the Extremes (N and O) of the Original Line, N O.

For, NO being produced to the Directing Plane, D is its Directing Point; and E D, being drawn, is the Director of N O, as before.

But E N D is a Triangle, consequently it is a Plane; Ax. 8. which cuts the Picture in n o, or IV, the Indefinite Representation of N O; and, consequently, it is parallel to E D, its Director.

COR. 1. *If two, or more, Right Lines cut the Directing Plane in the same Point, they will have parallel Representations.*

For they have the same Director, to which they are all parallel, by Theorem.

EX. P Q cuts the Picture in P, and the Directing Plane in D, the Directing Point of N O; and, because it is perpendicular to A B (the Intersection of the Picture) C is, therefore, its Vanishing Point. (E C being perpendicular to L M, is therefore parallel to P Q, being in parallel Planes.)

Consequently, P C is the indefinite Representation of P Q; which is parallel to IV, the indefinite Representation of N O; by Theorem, and 4. 7. El.

And also, by Theo. 5; for, E D, the Director of both (N O and P Q) is the common Intersection of the Radial Planes, $DEVI$ and $DECP$, passing through both Lines. Therefore, their Intersections, IV and PC, are parallel.

N. B. This Corollary is true if the Lines, NO and PQ, are not in the same Plane.

COR. 2. *All Lines, not having the same Directing Point, but, which cut the same Director, indefinitely produced (through the Eye) both ways, have parallel Representations.*

For, if two Original Lines, N O, or P Q, and R S, cut the Directing Plane in the two Points D and d, in such wise, that a Right Line drawn through the Eye and the Directing Point of one, will also pass through the Directing Point of the other, their Representations, n o and p q, are parallel.
Because they have the same Director, D E d.

E V is the Radial or Parallel of N O, V is therefore its Vanishing Point; and E v is parallel to R S; therefore v is its Vanishing Point; and, S v its Indefinite Representation.

COR. 3. *All Right Lines cuting the Directing Plane in* E F, *the Intersection of the Vertical and Directing Planes, will have their Representations parallel to the Vertical Line;* (C D.)

Because, they all have the same Director (EF) to which the Vertical Line (CD) is parallel; being produced by the section of the Vertical Plane (E C D F) with the Picture and Directing Plane. - - - 8, 7. El.
As P Y, parallel to R S, cuting E F, the Prime Director, in f.
E v is its Radial and P v its indefinite Representation.

COR. 4. *All Lines which cut the Parallel of the Eye of any Original Plane, have their Representations parallel to the Vanishing Line of that Plane.*

Because, the Parallel of the Eye is the Director of all such Lines, and it is parallel to the Vanishing Line; by Theorem 2nd.

T U is an Original Line, cuting IK, the Parallel of the Eye, of the Original Plane N B H, in k; E x is its Radial, x is therefore its Vanishing Point, and U x its indefinite Representation; which is parallel to L M, the Vanishing Line of the Plane N B H.

 For,

† Theo. 2.

For, IK is parallel to L M †, and it is the common Section of the Plane, IKLM and E k U x; therefore, U x is parallel to L M, by Theorem 5th.

COR. 5. *The Reprefentation of any Original Line makes equal Angles with the Interfection and Vanifhing Line, of the Plane it is in, as the Director, of that Line, makes with the Parallel of the Eye and the Directing Line of that Plane.*

Becaufe the Reprefentation is parallel to its Director, by the Theorem; and becaufe they cut parallel Lines in parallel Planes.

COR. 6. *If the Reprefentations of any two Lines are parallel, the Originals are either parallel between themfelves and to the Picture, or they have the fame Director.*

For, if the Original Lines are parallel between themfelves and to the Picture, their Reprefentations will be parallel (Theo. 9.); but, if they are not parallel to the Picture, they muft have the fame Director; feeing, there can be but one Line drawn in the Directing Plane, parallel to both Reprefentations.

N. B. Lines, which are parallel to the Picture, have no Directing Point, but the Director of every fuch Line, is a Line drawn through the Eye parallel to the Original Line.
For, the Reprefentations are parallel to the Original; by Theorem 9th.

This laft Theorem (and the Corollaries deducible from it) contains the whole Theory of the Directing Plane; as, in it and the thirteen preceding Theorems is contained the whole knowledge of rectilinear Perfpective; or, at leaft, all that I conceive to be really ufeful, in delineating. It has been my aim, not, merely, to amufe or to fhew my knowledge in it, but to give ufeful Inftruction; and, I dare venture to affirm, that if the whole of this Theory be clearly underftood, the Student will feldom be at a lofs in Practice.

It is a miftaken notion which many entertain of Perfpective, that, the Theory is unneceffary to a Practitioner. It is certainly poffible to practice Perfpective, in all common Cafes, without being able to account, or give a reafon for any rule that is followed; for, the Rules, being deduced from the Theory, will, undoubtedly, if ftrictly followed, produce certain effects though we know not how to account for it: as there are many Perfons very acute in Menfuration, Gauging, Surveying, &c. who know nothing of Geometry, the foundation of the whole. The cafe is very different in Perfpective; for I am well convinced that it is of great ufe to underftand the Theory well, in the firft place; and that, the Practice will, by that means, be fooner acquired, and more fecurely retained. For want of Theory, the Pupil is frequently bewildered, and knows not what he is about; every different Example appearing difficult and ftrange, though, perhaps, founded on the fame invariable Principles. In fhort, the neareft and moft certain road, to Perfpective, is to go through the Theory to Practice: and, I will venture to ftake all my knowledge in it, that when acquired, the lofs of Time (if it be any) will never be regreted.

I fhall give three more Theorems, on Circles and fpherical Bodies, and then proceed to Practice. If any Perfon require further knowledge, or a more extenfive Theory, I refer him to the elaborate Work of Mr. Hamilton, which is deferving of the higheft encomiums, if it was as ufeful as it is ingenious and learned in the Science; for, he has certainly faid all that can be faid of it, in Theory: and, I am perfuaded, more than any other Perfon would ever have thought on, and much more than is of real ufe; for, I think I have omited nothing that can be ufeful or neceffary to be known, by any Practitioner, whatever.

SECTION V.

Of the THEORY of CURVILINEAR PERSPECTIVE.

IN this Section, I ſhall chiefly conſider the Theory of Perſpective relative to circular Objects; which are the moſt common, and moſt uſeful of all curve lined Figures. Other Curves cannot be compriſed in any certain Theory, by which their perſpective Repreſentations can, with certainty, be aſcertained; or if they could, it would anſwer no purpoſe to an Artiſt, ſeeing that, irregular curved Figures or Objects but ſeldom occur, in Practice. I would not be underſtood to mean the curves of the apparent Contours of human or other Figures (endowed with Life or not) which occur in almoſt every Picture, but which, can never be reduced to Rules, for Practice, from an eſtabliſhed Theory, but, irregular curved Figures, in Planes or other Surfaces; winding or ſerpentine Rivers, Rocks, Mountains, Trees, &c. which are bounded by irregular curved Lines and Surfaces, cannot be reduced to Practice, in delineating them, by any Theory in the Science of Perſpective. Notwithſtanding they may be delineated with great accuracy, by any Perſon, who is a little accuſtomed to ſketch by ſight only, by means of an Apparatus, which I ſhall deſcribe in an Appendix to this Work.

To treat, at large, of the various Curves which the Repreſentation of a Circle may take, ſuch as the Parabola or Hyperbola, is foreign to my Deſign; as it ſo rarely aſſumes thoſe forms. Nor is the knowledge thereof of any real uſe in delineating; ſeeing that, the ſmall part of the Repreſentations of ſuch Circles as are or can be repreſented, when they do aſſume either, could not readily be diſtinguiſhed from a portion of an Ellipſis; which Curve, as it is the moſt general and uſeful, ſo it is the eaſieſt to deſcribe, and the only one of real uſe, in Perſpective.

In order to a clear underſtanding of the nature of an Ellipſis and its Properties, it is neceſſary to be acquainted with the Conic Sections; ſince every Repreſentation of a Circle or Sphere, in Perſpective, is either one or other of the Sections of a Cone. But, as a thorough inveſtigation of it is not neceſſary, here, I ſhall refer the Reader, who deſires to be perfectly acquainted with the Conic Sections, to Mr. Steel's, or to a later Work, by Mr. Emerſon.

Nevertheleſs, I find it impoſſible to treat the Theory of the Circle, in Perſpective, without having recourſe to them, in ſome degree; therefore I ſhall, in the firſt place, define what is a Cone, and, the difference between a Right Cone and a ſcalene or oblique Cone; for, without that knowledge, all that can be ſaid of it would be to little or no purpoſe.

The methods of deſcribing an Ellipſis, and all which appertains to it, are treated fully, yet briefly, in ſix Problems in an Appendix to the practical Part (Book ſt) of my Treatiſe on Geometry; together with a conciſe Theory, of its moſt eſſential Properties; to which I refer the Reader; I ſhall conſider it, here, only as being the perſpective Repreſentation of a Circle or Sphere.

DEFINITIONS.

A CONE is a geometrical Solid, whoſe Baſe is a Circle, which terminates in a Point, called its VERTEX; and, a Right Line, paſſing through the Vertex and the center of its Baſe, is called its AXE or AXIS*.

* The Axis of any thing is either a real or imaginary Right Line paſſing through its middle, in a certain and determined Poſition.

If the Object be a Plane Figure, its Axe may be either perpendicular to, or in the Plane of the Figure. If the Axe be in the Plane of the Figure, it is divided, by the Axe, into two equal and ſimilar Figures. Any Diameter of a Circle may be its Axe, an Ellipſis has but two Diameters which are Axes, viz. the Tranſverſe and its Conjugate, at Right Angles with each other.

It

Plate VII. It may be confidered as a Pyramid whofe Bafe is a Polygon of an infinite number of Sides; every Section of which, by a Plane parallel to its Bafe, will, confequently, be a fimilar Figure; wherefore, the fection of a Cone, parallel to its Bafe, is a Circle.

2. A Right CONE is that which is formed, or fuppofed to be generated, by the revolution of a right angled Triangle, on one of its Legs.

Fig. 27. No. 1. A B C is a right angled Triangle; C is the Right Angle; B C is, therefore, perpendicular to A C, its Bafe.

If you fuppofe the Triangle A B C to be revolved quite around, on B C, as an Axis, its Bafe (A C) will defcribe the Circle A E D F, which is the Bafe of the Cone A B D; and the Hypothenufe (A B) will have defcribed the Surface of the Cone; which is every where various, from the Vertex (B) to the Periphery of its Bafe; its Sides are equal every where, as B A, B E, B D, &c. The Perpendicular (B C) which is the Axe of the Cone, remains at reft; one extreme in B, the Vertex, the other in C, the Center of its Bafe.

No. 2. 3. An Oblique CONE has, alfo, a Circle for its Bafe, but its Axe is inclined to its Bafe, as *B* C to A D.

As if the Vertex (B) of a Right Cone, was drag'd on one fide, out of its perpendicular pofition; confequently, the real Axe, of fuch a Cone, does not pafs through the center of its Bafe, yet *B* C is called its Axe.

A Right Line bifecting any Angle of a Triangle is called its Axe; wherefore, *B* E, bifecting the Angle A *B* D, is the Axe of the Triangle A *B* D, and confequently of the Cone; for it paffes through the middle of fuch a Solid, whofe rotundity is elliptical, feeing, its Dimenfions, perpendicular to its Axe, are unequal; and, the more the Axe is inclined to the Bafe, the more its Dimenfions differ (its Bafe remaining the fame) and the real Axe is removed farther from the Center (C) towards A.

If a Section of an oblique Cone be made, by a Plane, perpendicular to its real Axe (*B* E) it is confidered as an oblique Section of it; which Section is an Ellipfis, and *B* E would pafs through its Center; confequently, the Solid would revolve regularly on *B* E, its real Axe. Whereas, on *B* C it would revolve very irregularly and unequal; but the Cone, A *B* D, would be equally poifed on B C, in a horizontal Pofition; wherefore, every Section of a Cone, through the Axe *B* C, bifects the Cone; for the Triangles, A *B* C, C *B* D, are equal, in every Section, through B C; by Prop. 18. 1. El.

THEOREM I.

The Reprefentation, or perfpective Projection, of every Circle, in a Plane to which the Picture is parallel, is a Circle.

The fcenographic Projection, or perfpective Reprefentation of an Object, is the Section of the Optic Cone, or Pyramid of Rays, by a Plane, paffing between the Eye and the Object. (See Scenography, P. 47.)

Fig. 28. Let A D G H be a Circle, in any Original Plane (Z) E is the Eye, and E A, E B, &c. Vifual Rays, from every Point in the Circumference, to the Eye, forming an oblique Cone (A E F).

DEM. Now, if the Cone of Rays be cut by a Plane (X) parallel to the Plane Z, in which is the Original Circle; a d g h, the Section of the Rays, by that Plane, is a Circle.

3 For,

For, it is the section of a Cone, by a Plane, parallel to its Base; and, whether it be a Right or Oblique Cone, every such Section is a Circle; seeing that, the Cone, a E f, cut off by the Plane X, is similar to the larger Cone (A E F).

This is otherwise demonstrable, from Theorem 9, and Corollaries; in which it is demonstrated, that the Representation of every Plane Figure, parallel to the Picture, is similar to the Original.

Every Diameter in the Original Circle (A F, D H, &c.) being equal, the Representations (a f, d h, &c.) have that proportion to their Originals, as the Distance of the Picture to the Distance of the Plane of the Original Circle. (Theo. 9.) consequently they are also equal; therefore, the Representation, a d g h, is a Circle. Q. E. D. For, all Circles are similar Figures.

COR. *Hence it is manifest, that the Representations, in Perspective, of various Circles, in the same Plane, parallel to the Picture, have all the same Ratio amongst themselves, and to each other, as the Original Circles.*

THEOREM II.

The Representation of a Circle, in a Plane not parallel to the Picture, is an Ellipsis; except in one certain Point of View, in which, its Representation is, also, a Circle.

Let A D G be an Original Circle in the Plane Z; E is the Eye, and E A, E B, &c. Visual Rays forming (as before) an Oblique Cone. C is the Center of the Circle. Fig. 28.

DEM. If the Cone of Rays be cut by a Plane (Y) which is considered as the Picture, passing through both sides of the Cone, not parallel to the Base, it is an oblique Section; and, consequently, every such Section, of a Cone, is an Ellipsis; except, as above. - Prop. 90. P. 73. Em. Con. Sec.

Wherefore, the Representation, *a d g*, on the Plane Y, (which is an oblique Section of the Cone of Rays E A, E B, E D, &c.) is an Ellipsis. Q. E. D.

Every Circle, which is visible, appears an Ellipsis, except when the Axe of the Eye is perpendicular to the Plane of the Circle, and passes through its Center; for, in that Case, only, the Visual Rays, from the Eye to every Point in the Circumference, are equal, and consequently they generate a Right Cone, the Axe of the Eye being the Axe of the Cone. In every other position, whatever, they must necessarily form an Oblique Cone, seeing that, the Axe of the Eye (which is always the Axe of the Cone) must be inclined to the Plane of the Circle, if it be not perpendicular; and consequently, the Visual Rays, forming the surface of the Cone, have various Inclinations to the Plane of the Circle, and therefore they are unequal.

Wherefore, since the section of an Oblique Cone, perpendicular to its central Axe, is an Ellipsis, and the more the Axe is inclined to the Base, the more excentric is the Section; seeing that, if the Eye be nearly in the Plane of the Circle, the Section approaches nearly to a Right Line; and, the Section, made by a Plane in that position, is the only true Appearance of the Circle; or rather, by a spherical Surface, perpendicular to the Axe, which truly measures the Optic Angle (the Radius being equal to the Distance) under which, the solid dimensions of the Cone, every way, are seen.

COR. *If the height of the Eye, or its distance from the Directing Line, be taken a mean Proportional, between the distance of the nearest convex part of the Circumference and the distance of the farthest concave part, from the Directing Line, the Representation will then be a Circle.*

Plate VII. Fig. 29. Let A B be a Diameter of the Circle, A D G, perpendicular to the Directing Line (K L) B S is the diftance of the Circle from it.
Make S E : S A : : S B : S E ; i. e. let S B : SE : : S E : S A, by Pr. 30. Geo.

At E, as the Point of View, the Reprefentation of the Circle A D G, on any Plane (X) parallel to the Plane K E L, will be a Circle, and in no other Point whatever, on that Plane, and at the diftance B S.

DEM. Draw B I parallel to S E ; and, fuppofe A E B a fection, by a Plane paffing through C E, the Axe of the Cone, and the Diameter A B, of its Bafe ; and alfo through S.

Then, becaufe B I is parallel to S E, the Triangles A I B, A E S are fimilar;
Wherefore, - - - - A B : B I : : A S : S E.
But, by Conftruction, - - A S : S E : : S E : S B.
Wherefore, - - - - - - S E : S B : : A B : B I;

† 5. 6. El. || 4. 1. and therefore, the Triangles A I B, B E S are fimilar†, for the Angle A B I is equal to E S B||; E B S is equal A I B, and I A B equal B E S; feeing that, the Sides which fubtend thofe Angles are proportional.

‡ 4. 1. El. But, the Angle B E S is equal E B I ‡, for they are alternate ; wherefore, E B I is equal I A B (of the Cone A E B) and confequently, the leffer Cone (a E b) cut off by the Plane X (parallel to B I, and the Plane K E L) is fimilar to the Cone B E A ; for, the Angle E (at the Vertex) is common to both ; the

§ C. 5. 10. 1. Angle E b a (equal E B I) is equal E A B, confequently, E a b is equal E B A§, and therefore, the Cone (A E B) is cut fub-contrary, by the Plane X.

But, if an Oblique Cone be cut fub-contrary, the Section is a Circle.

Therefore, the Reprefentation, a h b g, on the Plane X, of the Original Circle A D B F, is a Circle. Q. E. D. Prop. 89. P. 73. Em. Con. Sec.

N. B. The Center (s) of the Reprefentation, is not the Reprefentation of the Center (C) of the Original Circle ; for it is at c, where the Vifual Ray E C cuts a b, the Reprefentation of the Diameter A B. Wherefore, C E is not the real Axe ; feeing that, it cannot be the Axe of the Cone A E B, and alfo, of the leffer Cone (a E b) which is cut fub-contrary, and is therefore fimilar to A E B ; confequently, the true Axe is common to both Cones.

So likewife, d f, the Reprefentation of the Diameter D F, in the Original Circle, is not a Diameter of the Reprefentation ; but, g h, the Reprefentation of the Chord G H, (which bifects a b) is its Diameter perpendicular to a b.

Wherefore, the Reprefentation of the Segment G B H is the lower Semicircle, g b h; and, g a h, the upper Semicircle, reprefents the large Segment (G A H).

It is evident, that, if the Eye be raifed, as at E^2, the Diameter a b, in the Reprefentation, will be lengthened ; feeing, that the Vifual Rays, E^2A, E^2B, cut the Picture more oblique ; whilft the other Diameter, g h, remains of the fame

§ Cor. 3. Theo. 9. length §; and confequently, the Reprefentation of the Circle, from that Point of View, will be an Ellipfis, and a b its Tranfverfe Diameter, or Axe.

But, if the Eye be lowered, to E^3, the Diameter a b (now c e) will be fhorter than the Diameter g h (which, being parallel to the Interfection of the Picture, will have the fame length, feen from any Point in the Directing Plane, as above) and it will then be the Tranfverfe Axe; and c e (the Conjugate to it) is the fhorteft Diameter of the Ellipfis.

If the Eye be removed on either Side of the Point E, the Reprefentation will be an Ellipfis ; for, the Section will not, then, be fub-contrary (the Picture remaining as before). The Reprefentation (a b) of the Diameter A B, will ftill be a Diameter of the Ellipfis, becaufe it will pafs through its Center ; but it will be neither the tranfverfe nor the conjugate Axe, for they are always at right angles with each other.

Hence, the Point of View, from which the Reprefentation of a given Circle, on any Picture, however fituated, will be a Circle, is eafily determined.

The Original Plane (K A L) in which the Circle is situated, being produced if necessary, cuts the Picture in M N, its Intersection.

Having fixed on the Distance, or Station Point, S (at pleasure) in the Diameter A B, produced, which is perpendicular to the Intersection of the Picture; let K L be drawn, through S, parallel to M N; K L is the Directing Line. - Def. 10.

Draw S E perpendicular to K L, and parallel to the Picture.

Make S E a Mean Proportional, between S A and B S; - - Pr. 30. Geo.

E is the place of the Eye, or the Point required; from which, a Circle, a g b h, (on the Plane X) whose Center is c, and Diameter a b, will truly represent the Original Circle A D B F, on the Plane A K L.

N. B. The distance of the Picture, in this Case, is not material; for, the Point E being fixed (as above) every Section of the Cone of Rays E A, E F, E B, &c. parallel to the Plane X, or to the Directing Plane, K E L, being cut sub-contrary, will, consequently, be a Circle.

From what has been advanced, it is evident, that the Representation of every Circle, in Perspective, is some one or other of the Conic Sections; which, being a distinct Science, would not be proper to enter on here; but, from that consideration and the preceding Theorems, the two following Corollaries may be deduced.

COR. 1. *If the Circumference of the Original Circle touches the Directing Line, in a Point only, and is not cut by it, the Representation of that Circle will be a Parabola.*

For, whether the Original Circle (A D B F) touches the Directing Line, in S, (the Station Point) or in any other Point (B) it is the same; since that Point, and also the Eye (E) which is the Vertex of the optic Cone of Rays, are both in the Directing Plane (Y) it is evident, that the Directing Plane touches the Cone in the Right Line E B, from its Vertex (E) to its Base (at B). Fig. 30.

Wherefore, since the Picture (X) is always parallel to the Directing Plane ‡, the Cone, A E B, is cut by a Plane parallel to its Side (E B) the Curve produced by every such Section is a Parabola. Prop. 76. P. 223. Em. Con. Sec. ‡ Def. 4.

That part of the Original Circle (M G A N) which lies beyond the Picture, is represented perspectively, and falls above the Intersection M N (as M a N); the remainder of the Circle, lying between the Picture and the Directing Line, is projected below the Intersection; as l is the representation of the Original Point L. Every other Point (as K) is projected further from the Intersection as it lies nearer to the Directing Line (B S) and, except the Point B, in which it touches the Directing Line, may be supposed to have a Representation, on the Picture, though at an immense distance; but, the Point B can have none; because, the Line (E B) which should produce it, is parallel to the Picture, seeing, it lies in the Directing Plane, and therefore can never cut the Picture. The Representation of which Point, only, is wanting to compleat the Figure and form an Ellipsis; but, for want of that Point, in the Representation, it is kept open, and falls off in Right Lines, nearly, at a Distance, to all sense, infinite.

Hence it is plain, that, on account of the Distance, but a small portion of that part of the Curve which falls below the Intersection, can be represented, in a Picture; and the part which can, together with the part which lies beyond the Picture, differs so little, in its Representation, from an elliptic Curve, that, in delineating, it would be needless to deviate from it; and, I question that ever a distinction was made, in Practice. Nor can it ever be of use, but in delineating the inside of a large Rotunda, Circus, or circular Area; when the Spectator may be supposed to stand on the hither part of the Circumference, or in a Line which is a Tangent to it, and parallel to the Picture; in which case, only the farther, concave part of the Curve can, properly, be represented.

COR.

Plate VII. Fig. 31.

COR. 2. *If the Directing Line* (K L) *cuts the Original Circle in two Parts*; *that part* (K F A H L) *which is on the ſame ſide with the Picture, will, in its Repreſentation, form an Hyperbola, below the Vaniſhing Line* (O P) *of the Plane of the Circle*; *the other part* (K B L) *of the Circle* (*if it be repreſented on the ſame Picture*) *will be tranſprojected, and form an oppoſite Hyperbola, above the Vaniſhing Line, which is equal and ſimilar to the other.*

For, if the Directing Line paſſes through the Center of the Original Circle, and the Eye, which is the Vertex of the Cone, is in the Directing Plane; in which Caſe, the Section, made by the Picture, is parallel to the Axe of the Cone, and the oppoſite Hyperbolas are equal, and equally diſtant from the Vaniſhing Line (O P).

But, when the Circle is cut unequally, by the Directing Line (K L) the Cone (B E A F) is alſo unequally cut by the Directing Plane (K E L) the Section of which, with the Cone, is the Triangle K E L; the Picture (N O P M) being parallel to the Directing Plane, cuts the larger Segment of the Circle, in M N, and the Cone, in the Curve f g a N, which is an Hyperbola. Prop. 104. P. 160. Em. C. Sec.

If the Sides (A E, F E, B E, &c.) of the Cone B E A, are produced through the Vertex (E) forming an oppoſite Cone (a E b d) the Picture being produced, will alſo cut that Cone, produced; its Section, with it, is the oppoſite Hyperbola, or tranſprojected Repreſentation of that part of the Circumference of the Circle, (K B L) which lies on the other Side of the Directing Line, by means of the Rays B E b, Q E q, &c. produced to the Picture; which, notwithſtanding it is the Repreſentation of the leſſer Segment of the Circle, is generated by a ſimilar Cone, (a E b) and, although the Section is made at a greater diſtance from their common Vertex, it is equal and ſimilar to the perſpective and projective Repreſentation (f M g a N d) of the larger Segment.

This Curve, like the Parabola, can never be generated, or of uſe, but when the Area of the Circle is ſo large, that the Spectator is ſuppoſed to ſtand within it; and that Segment (M G A H N) which lies beyond the Picture, only, is required; and which differs ſo little from an elliptic Curve, in its Repreſentation, (M g a h N) that the diſtinction is not very obvious, and ſeldom, if ever, regarded.

Theſe are all the variety of Curves which a Circle in Perſpective can aſſume, however ſituated in reſpect of the Picture, or of the Eye; the chief of which is the Ellipſis. Every Circle, which the Eye is capable of taking, in at one View, it is manifeſt, has the Appearance of an Ellipſis; except, when the Axe of the Eye is perpendicular to the Plane of the Circle, and paſſes through its Center; in which poſition, only, it can appear, to the Eye, a true Circle; although all Repreſentations of Circles, in Planes parallel to the Picture, are Circles †; but, being ſeen oblique, when the Eye is in the true Point of View, they have the Appearance of Ellipſes.

† Theo. 1.

THEOREM III.

The Repreſentation, in Perſpective, of a Globe, or Sphere, is an Ellipſis*; except when the Center of the Sphere coincides with the Center of the Picture; in which Caſe, only, it is a Circle.

The Cone of Rays from the Eye, as its Vertex, to the apparent Circumference of a Sphere, is always a Right Cone. For, a Sphere can have but one Poſition, either to a Plane, Line or Point; and conſequently, the Diameter, every way, always preſents itſelf to the Eye, and always forms a Right Cone.

* I have found it very difficult, nay almoſt impoſſible, to convince ſome Perſons of this plain and well known truth; becauſe, as they truly obſerve, the Diameters of a Sphere always appear equal; which is as much as to ſay, that the Viſual Rays, under which a Globe is ſuppoſed to be ſeen, always form a Right Cone, i. e. whoſe Axe is perpendicular to its Baſe; which is not ſo with Circles, but in one poſition only. But, ſuch Perſons ſeem to forget, that the Repreſentation is on a Plane, and that Plane is conſidered as the Plane of the Section; which muſt, conſequently, cut every Cone oblique, but that which has its Axe perpendicular to the Picture.

Now,

DEM. It is evident, that, when a Sphere is ſo ſituated, in reſpect of the Picture and the Eye, that, the Direct Radial coincides with the Axe of the Cone, (i. e. when it paſſes through the Center of the Sphere) the Section of the Cone, made by the Picture, is parallel to every apparent Diameter of the Sphere, which is the Baſe of the Cone; and, being equal every way, it is conſequently a Circle.

But, the Direct Radial is the Axe of the Cone, which paſſes through the Center of its Baſe, and alſo through the Center of the Picture.

Conſequently, the Center of the Picture coincides with the Center of the Sphere, in this Section; and therefore, the Repreſentation is a Circle. Theo. 1ſt.

For it is the Section of a Cone, parallel to its Baſe.

2. But, if a Sphere be ſo ſituated that the Direct Radial does not paſs through its Center, the Axe of the Cone muſt be inclined to the Picture; which, being the Plane of the Section, the Cone of Rays are cut obliquely by the Picture.

But the oblique Section of a Cone, through both its Sides, is an Ellipſis. Pr. 90 Em.

Therefore, the Repreſentation of a Sphere, whoſe Center is not the Center of the Picture, is an Ellipſis. Q. E. D. For it is an oblique Section of a Cone.

EX. Let A B be a Sphere and C its Center (ſuppoſed to be ſeen). Let E be ſuppoſed the Eye of a Spectator, and E A, E B, Viſual Rays, from the Eye, to the apparent Diameter, A B *. A E B may, therefore, be ſuppoſed a Section of a Right Cone, through its Axe, EC, which is an Iſoſceles Triangle; the Sides, E A, E B, &c. of a Right Cone being equal ‡. Fig. 32. ‡ Def. 2.

Now, if F G be the Picture (which is a Plane) parallel to A H B I (the Baſe of the Cone) the Line a b, in which the Plane A E B cuts that Plane, is a Diameter of that Section; which Section, being every way equal, is, conſequently, a Circle.

But, E C, the Axe of the Cone, being perpendicular to its Baſe, is perpendicular to the Picture §, and to its Section, F G, with the Triangle A E B; § 6. 7. El. the Point, c, where the Picture is cut by the Axe of the Cone, E C, is the Repreſentation of C, the Center of the Sphere.

And, becauſe A B is a Diameter of the Baſe of the Cone, and C its Center, A B is biſected in C; conſequently, a b is alſo biſected in c; for, the Triangles C E A, C E B are congruous ||; and, becauſe a b is parallel to A B, the Triangles a E c, A E C, and c E b, C E B, are all ſimilar ‡. || 7. 1. El. ‡ 2. 6. El.

But, E is ſuppoſed the Eye of a Spectator, and, E c, being perpendicular to F G (the Picture) is the Direct Radial ¶; and the Point c, where it cuts the Picture, is the Center of the Picture §, which is alſo the Center of the Repreſentation of the Sphere, on that Picture; therefore it is a Circle. Theo. 1. ¶ Def. 15. § Def. 17.

Now, if the Cone A E B be ſuppoſed to be cut by any other Plane, paſſing through the Diameter h i, and conſequently through c, the Center of the Picture and of the Repreſentation; that Section will be an Ellipſis.

Let S D be a Section of another Plane or Picture, with the Triangle A E B, cuting the Rays E A, E B, in e and d; then is e d, in that Section, the repreſentation of the apparent Diameter A B, of the Sphere, and c is the repreſentation of its Center, as before, in the Section a b; but, e d is not biſected in c.

DEM. Let a c e be an Iſoſceles Triangle; ac=ec; and, ac=cb; th. ec=cb: Ax. 3. El.

But, becauſe b c E, in the Triangle b E c, is a Right Angle, c b E is acute †, † C. 3d and, the Angle c b d, in the Triangle d c b, is, conſequently, obtuſe. C. 2. 1. 1. 10. 1. El.

Wherefore, c d, ſubtending the obtuſe Angle, is greater than c b; 12. 1. El.

And conſequently, e c+c d, equal e d, is greater than a c+c b, equal a b.

But, the Section H E I, which is vertical to A E B, cuts both Pictures in the ſame Line, h i, equal to a b; as above.

* See the Note to Art. 4. Page 14, on Direct Viſion.

Plate VII. Therefore, the Section, e h d i, is an Ellipsis; for, the Diameter e d is larger than any other, in that Section.

But, E S, perpendicular to S D, is the Direct Radial; wherefore, S is the Center of that Picture ||; and it does not coincide with the Center of the Sphere (the Eye being at E) consequently, the Representation of a Sphere cannot be a Circle, except when the Center of the Picture coincides with the Center of the Sphere, in the Center of its Representation.

|| Def. 17.

Fig. 33. EX. 2. To illustrate this further. Let X, Y, and Z be three Globes, whose Centers are all in the same Right Line, parallel to a h, the Intersection of the Picture; whose Center is C, and E C its Distance.

Draw the Tangents E A, E B, E D, &c. to the three Globes, the Chords, A B, D F, and G H, of those Tangents, are the apparent Diameters of each; which, it may be observed, are still turned towards the Eye; and are considered as the Diameters of the Bases of the three Cones, A E B, D E F, and G E H, which are all Right Cones.

Now, since they are all cut by the same Plane (of which a h is a Section) each Cone, except the first, A E B (whose Axe, E X, coincides with the Direct Radial, E C) is cut oblique; and, consequently, one Diameter, of each Section, is larger, as the Globe is farther from the perpendicular E C; as d f, and g h, the Representations of a Diameter of each Globe (Y, and Z) d f being larger than a b, and g h still larger than d f.

But, the other Diameters, perpendicular to these, are equal, in all §, for they are supposed parallel to the Picture, and equally distant from it; consequently, the Representation of the Globe X, only, which is in the middle of the Picture, or, the Center, C, of its Representation, in the Center of the Picture, is a Circle; because, the Diameters perpendicular to each other are equal: all others (as of Y and Z) are Ellipses, because they are oblique Sections of Cones, whose Diameters are proved to be unequal.

§ Cor. 3. Th. 9.

N. B. It is the same however the Globes are situated; whether above, below, or sideways of the Center of the Picture. If they are equally removed from the Perpendicular E C, and equally distant from the Picture, the Globes being equal, their Representations are equal and perfectly similar, though differently situated; and the farther they are remote, from the Perpendicular, the more excentric is the Ellipsis.

It is obvious and demonstrable, that a b, the representation of the Diameter A B, is bisected in C, the Center of the Picture, and the representation of the Center of the Globe; for, the Triangle a E b is Isosceles. But, d f, and g h are not bisected, in i and k, the representations of the Centers of the Globes Y and Z.

For, because the Right Lines E Y and E Z (from the Eye to the Centers of the Globes) are perpendicular to the Chords, D F and G H, the Angles D E F, G E H are bisected by those Lines; because the Chords are bisected. C. 9. 1. El.

But, in the Triangles d E f and g E h, because the Angles, at E, are bisected, by the Right Lines E i, E k; d f and g h are cut, by those Lines, in the Ratio of the other sides of the Triangles; - - 3. 6. El.

and consequently, d i : i f : : E d : E f; and g k : k h : : E g : E h.

But E g is less than E h †; because, the Angle g h E is acute, and E g h is obtuse; wherefore, g k is less than k h; and also, d i than i f.

† 12. 1. El.

Therefore, d f and g h are not bisected, by the Lines E Y and E Z. And consequently, since they represent Diameters of the Spheres, Y and Z; their Representations are not Circles; consequently they are Ellipses.

SECTION

SECTION VI.

Containing a full refutation of feveral Errors and abfurd Opinions, which many Artifts entertain of Perfpective; and, therefore, look on it as an imperfect and fallacious Science.

I Shall, in this Section, in the firft place, explain the reafon why the Reprefentations of the Diameters of Columns, on a Picture which is parallel, or nearly fo, to the Columns, are continually larger the farther they are removed from the Center of the Picture, and confequently from the Eye.

As this is a particular circumftance, which many Perfons feem inclined to difpute, or, if it be admited, they look on it as an imperfection in Perfpective, I fhall endeavour, and doubt not, to make it appear confonant to reafon and Perfpective, to their entire fatisfaction. It has fo near affinity to what has been faid, in refpect of a Sphere, that the fame Diagram might have done for both; but, in order to avoid miftakes, and to keep the Ideas diftinct and feparate, I have given another.

Let V, X, Y, and Z be the Sections of four Columns, by a horizontal Plane, in which is the Eye, at E. Let L M be a Section of the Picture, parallel to the Columns, and E I, E K, E A, E B, &c. Vifual Rays, from the Eye to the apparent Diameters of the Columns, which are ftill turned towards the Eye, as Globes. Fig. 34.

It is evident, that the Vifual Rays cut the Picture more oblique, the farther the Columns are from the Perpendicular E C; and, notwithftanding the optic Angles D E F, G E H are lefs, as the Columns are continued, their reprefentative Diameters, d f, and g h, intercepted between the Vifual Rays, continually increafe; and would, if the Columns were continued, till the Interval between them was loft; the Reprefentations of their Diameters ftill increafing till they touch and cut each other. For, the Space from Center to Center, of the Columns, are equal; and are, confequently, reprefented fo on a parallel Picture, if continued infinitely; what, then, can become of the Space between the Columns, if it be not added to their Diameters, in the Reprefentations on the Picture?

I prefume, no Perfon will fay that there is any imperfection in Perfpective, in this Cafe; I do affirm there is none in Perfpective; the bufinefs, of which, is to reprefent Objects, truly, on a Plane; according to their Magnitudes, Diftances, and Situations in refpect of each other, of the Eye, and of the Picture; where, then, is the imperfection in this?

If a Perfon, not knowing how to choofe a proper Diftance, take, into the Picture, more than the Eye is capable of taking in at one View; or if, through ignorance, the Picture be abfurdly fituated, in refpect of the Object, is the fault in Perfpective, or in his Judgment? In Perfpective there is not, nor can be, on the Principles here laid down, any, the leaft error, if the Elements of Euclid are to be depended on, upon which the whole Fabric is erected; if one falls, the other falls with it.

The Art of Perfpective is to reprefent Objects on a Plane, by geometrical Rules, according to their Situations, &c. (See Perfpective, Page 47.)

It is well known (or ought to be) that no Perfpective Reprefentation can appear perfect, i. e. it cannot truly reprefent the Original Object, though ever fo accurately delineated, but when the Eye is in the true Point of View.

Suppofe, then, E to be that Point; and A E B, D E F, &c. the Optic Angles under which the Columns X, Y, Z (being equal) are feen.

It is alfo, I prefume, allowed, that Objects appear to have the fame proportion to each other, refpectively, as the Angles under which they are feen. Th. 1. S. 3. D. Vifion.

But,

Plate VII. But, the Angle DEF is lefs than the Angle AEB; becaufe their Subtenfes are equal, AB=DF (fuppofing the full Diameter of a Column to be feen) and the Vifual Rays EB, ED, EF, &c. the Sides of thofe Angles, ftill longer, the farther they are from the Perpendicular, EC†. How comes it then, that the Diameter of that Column (on the Picture) is the largeft, which is feen under the leaft Angle? The reafon is obvious; becaufe, the Picture, LM, cuts thofe Rays moft oblique, where the Angle is the leaft; in the Points g and h, &c.

† C. 14. 1 El.

DEM. If EF and EH are produced till they cut a Right Line drawn through the Centers of the Columns (which is parallel to the Picture) in *F* and *H*; ED and EG cuts that Line in *D* and *G*; all which are beyond the Circumferences of the Circles; wherefore, *DF* and *GH* are, each larger than a Diameter; and, if it was not fufficiently obvious, it would be eafy to prove that *GH* is larger than *DF*. But, AB is lefs than a Diameter; confequently, ab is lefs than df, and df than gh; for ah is parallel to A*H*. - - - 4. 6. El.

Now, if the Eye (at E) be turned towards the Column V; or, if the fituation of the Picture be changed to NM, then is S, where ES cuts NM, the Center of that Picture; on which, it is evident, that, *ik*, the reprefentative Diameter of the Column V, being neareft the Center, is the leaft; which, on the Picture LM is equal to that of Y; for, they are equally diftant from the Center of that Picture; and *g*h, on the Picture NM, the reprefentation of the Diameter of the Column Z, is confiderably larger than gh on the other. Yet, to the Eye, at E, both thefe Pictures truly reprefent the Diameters of the four Columns V, X, Y, and Z; V and Y appear equal on both, they being equally diftant from the Eye; X, the neareft, will appear the largeft, and Z, the fartheft, from the Eye, will appear the leaft.

DEM. With any radius, as ES, on E as a Center, defcribe an Ark of a Circle, cuting all the Vifual Rays EI, EK, &c. from the four Columns.

The parts, *ab*, *df*, &c. intercepted between the Rays, are the true proportions of the apparent Diameters of the Columns, and confequently, of their Reprefentations, on both Pictures.

But the Ark *ab* is the greateft, *ik* is equal to *df*, and *g h* is the leaft.

Wherefore, the Angle aEb is greater than dEf (equal IEk) and gEh is the leaft; and confequently, the apparent Magnitudes of the Columns, X, Y, and Z, are in the fame Ratio. - - - Theo. 1. Direct Vifion.

It is unneceffary to enforce this, by dwelling longer on it; as it is certain, if either of thefe Pictures be viewed from any other Station, they could not reprefent the four Columns V, X, Y, and Z, in the Pofition and Situation they are in.

Suppofe the Eye removed to E^2 (the Point of View ought always to be oppofite to the middle of the Picture) and, from that Station, to view the Picture NM.

Draw the Vifual Rays E^2a, E^2b, &c. and on E^2 defcribe an Ark of a Circle, cuting them, in 1, 2, 3, &c. The flighteft glance of the difproportion of their Appearance, from that Station, is fufficient conviction, that *ab* (which appears the largeft from the true Point of View) whofe apparent Magnitude is the Ark 3 4, does not appear half fo large as *df*; and *g*h appears larger than *df* (which ought to appear the leaft) as the Arks 3 4, 5 6, and 7 8, fufficiently evince.

That the true, apparent magnitudes of the Columns V, X, Y, and Z, from the Point of View, E, are the portions *ab*, *df*, &c. of the Ark *idh*, is manifeft, when we confider, that the Picture, on which the Columns are reprefented, is a Plane; but that, their true Appearances can only be reprefented on a fpherical Surface; i. e. on the Surface of a Sphere, the Reprefentation and the Appearance are the fame, the Eye being in its Center.

Does any one imagine that there is a real Arch in the Heavens, which has that Appearance? in which, the Stars, &c. appear, equally diftant from the Eye in its Center. The Celeftial Globe, for inftance, is a Picture of the Heavens, Planets,

4 Stars,

Stars, &c. each Reprefentation of a Star, on its Surface, if they are truly depictured, would, to an Eye in the Center of the Sphere, exactly coincide, and be in the fame Right Line with its Original in the Heavens; and, their apparent Diftances, from each other, are meafured by an Ark of the Sphere; whereas, their real Diftances from the Eye and from each other, refpectively, have not the leaft affinity to their Diftances, as reprefented on the Surface of the Globe or from its Center. Wherefore, to an Eye at E, the true Point of View for either Picture (LM or MN) each Diameter, being feen under its true Angle, and the fame as its Original, will appear lefs and lefs, the farther they are diftant from the Eye, or from the Center of the Picture; although their Reprefentations are continually larger and larger, on thofe Pictures, as the original Columns recede.

And this will ever be the cafe; on a Picture parallel to the Columns, in fome degree, at any diftance of the Picture; but, at a proper Diftance, for taking in the whole, the difference is fo little, and that ftill lefs as the Diftance is increafed, that, it is and ought to be defpenfed with, by making them equal: but, I muft obferve, it is not, then, true Perfpective.

2. Methinks I hear fome carping Critic fay, I muft allow, then, that Perfpective is fomewhat defective; by no means; I have not yet given up the Point in debate. I fay, that, although, at any Diftance, there muft and will be a difference, though fcarce perceptible at a proper Diftance, yet I would never advife a Perfon, who would reprefent a row of Columns, in full Front, to make the leaft difference in their Diameters; for, fince they fupport an Entablature, which is reprefented perfectly horizontal, their Pedeftals or Bafes the fame, confequently parallel, it would be very improper to make the Columns differ in width when they are equal in height, for this reafon; becaufe it is impoffible to confine the Eye to the true Point of View, always; from which if you deviate, ever fo little, the whole Reprefentation is diftorted and imperfect.

But, if it were poffible to confine the Eye, I would not ftep the leaft afide from Perfpective, on any account; let the Reprefentation be ever fo diftorted and prepofterous, it will, and muft, if truly reprefented (by the Rules hereafter prefcribed) appear to the Eye, in the Point of View, as the Original. If we may be allowed to take liberties, in any Cafe, where fhall we draw the line between the perfect and imperfect Reprefentation? for the whole is more or lefs diftorted; confequently, the Rules of Perfpective are not to be depended on, at all. To what, then, muft we have recourfe? the Eye is not, in many Cafes, a competent judge; we fhould, if we follow its dictates, implicitly, have as many Points of View, in a Picture, as Objects; becaufe, if every Object be reprefented exactly as it appears to the Eye, on the fame Plane or Picture, there can be no point of View for the whole; and confequently, in a long, connected, and continued Object, compofed of Planes and Right Lines (as in Buildings of any kind, of regular Architecture) it would be beyond the power of Art to connect the feveral detached pieces or projectures fo, together, as to compofe one entire and uniform Picture of the whole.

Notwithftanding an ingenious Author has treated it ludicroufly, in a fuppofed Dialogue between a Lady and an Artift, who was determined to abide by the Rules of Perfpective; his Argument has not the leaft weight, and muft be imputed to him, as not having a right notion of Perfpective. The Eye is confidered as a Point; therefore, whether we fuppofe it confined to a Pin-hole or not 'tis the fame; for he muft allow (or he was very unfit to write on Perfpective) that there can be but one Point of View for a Picture, in which it can be perfectly feen, as intended by the Artift; confequently, when the Eye is not in that Point, the Objects muft neceffarily appear more or lefs diftorted, according as they are fituated nearer to or farther from the Center of the Picture, or that Point which is oppofite to the Eye.

3. I fhall, however, for the fake of the Argument, allow each Object to be truly reprefented, as they appear to the Eye, on a Plane Picture of a tolerable length, as a h: Globes are the fitteft Subject to expatiate on, becaufe they are every way the fame. Fig. 33.

Plate VII. Let C be the Center of the Picture, the Eye being at E; and therefore, the Picture may be fuppofed to be extended equally towards a as to h.

Now it is certain, that the three Globes appear round, let the Eye be fituated where it may; but they cannot, or ought not to be fo reprefented, on the Plane a h, to be viewed by the Eye at E; for if they were, they would not appear round, but gibbous, or Egg like, ftanding erect, all but the Globe X, in the Center; and the more fo, as they are farther removed from it; the Eye muft be removed oppofite to each, and confequently, there would be as many Points of View as there are Objects, which is an abfurd Hypothefis, in one Picture.

Suppofe, from the Point of View E, I would reprefent the three Globes, X, Y, and Z, as they appear; that is, the outline of each to be a Circle; the Eye, and its Axe, E C, muft be turned towards Y and Z, as E Y, E Z; and confequently, the Picture is turned with it, into the Pofition b l, and l m. For, the Picture muft be perpendicular to the Axe of the Eye, if the Object be reprefented as it appears; in which Cafe, there are three diftinct Pictures, *viz.* a b, b l, and l m; each having a diftinct Center, and the fame Diftance, E C or Ec; on which Pictures, each Globe is reprefented by a Circle, and the fartheft from the Eye (Z) is the leaft in its Reprefentation, as they really appear.

It muft be obvious, if the Globes were reprefented fo on one Plane (of which a h is fuppofed a Section, and C its Center) that they would be under a worfe predicament than thofe reprefented ftrictly perfpectively: fuch a Picture could not appear true in any Point of View, whatever, every Object having a diftinct and feparate one.

Suppofe them reprefented on the Plane a h, as they appear to the Eye, at E; i. e. round; the Reprefentation of the Globe Y lefs than that of X, and, of Z, lefs than Y, as they are reprefented on b l and l m; would they appear in the proportion they are reprefented, at E, or in any other Point? No, certainly; for, at E, the Reprefentations would appear lefs than the Original Objects, and not round (except X, only) Y would appear more round than Z, and Z would alfo appear rounder than any other, more remote from C; but they would, all, except X, appear eliptical Spheroids, and not Globes (which is obvious, to any Perfon tolerably acquainted with Optics or Direct Vifion) the fartheft, from X, ftill more fo than the laft.

Now, let the Eye be removed to *E*, or oppofite to Z; Y, or Z, would then appear round, but not of the fame proportion, as from the Point of View E; but, at *E*, neither X nor Z would appear round; and, although equally diftant from the Eye, X would appear much the largeft; where, then, in this Cafe, muft the Eye be placed, to fee thofe Reprefentations fuch as the real Objects appear? There cannot be a Point determined, for each Reprefentation has a feparate Point of View. Can, then, the Picture a h, in this Cafe, be a true perfpective Reprefentation of the three Globes, X, Y, and Z, as they appear to the Eye? certainly no, but each Reprefentation, on the Plane a h, is as much a diftinct and feparate Picture, as the three Pictures, a b, b l, and l m; the difference is only, that the three Pictures have but one Point of View, and the Picture a h has three, equally diftant from it; by fuppofing the three Pictures placed in a Right Line, a h.

4. If what I have here advanced be not fufficient, to diveft thofe Artifts of their abfurd notions of Perfpective, I fhall give them one obfervation more, which they have not, perhaps, confidered with that attention it requires; and then leave them to purfue their own way, if it appears to them more eligible and reafonable, and will produce a better and more agreeable Reprefentation, of any Object whatever.

They quarrel and find fault with Perfpective, but without reafon; becaufe, it is an infallible and moft perfect Science. They would have all Objects reprefented, in Perfpective, exactly as they appear to the Eye; there is no fuch thing to be done; 'tis not in the power either of Art or Science to reprefent, on a Plane, any fingle Object, except a Sphere, or a plane Circle, having the Eye oppofite to its Center, and the Picture parallel to it, as it appears; and yet, Perfpective will give a true and juft Reprefentation of every regular Object.

The

The real, and the only caufe of all their errors, and falfe notions of Perfpective, is their not rightly diftinguifhing between the Reprefentation of an Object, on a Plane, and the true Appearance of it; two diftinct things, which can never be united, on a Plane Surface or Picture.

The perfpective Reprefentation of any Object is the fection of the Cone or Pyramid of Rays, by a Plane; and the Appearance of an Object is the fection of the Rays by the furface of a Sphere, only; to which, every Vifual Ray, from the Eye to the Object, muft be Perpendicular; confequently, the portions of the Arks, intercepted between the Vifual Rays, meafure the Optic Angles, under which, every part of the Reprefentation is feen (the Ark a, or *a*, *dh*, Fig. 33 and 34, may be fuppofed a fection of a Sphere, cuting the Vifual Rays E A, E B, &c). It is therefore manifeft, that the true Reprefentation, and, at the fame time, the true Appearance, can only be reprefented on a fpherical Surface, the Eye being in its Center; but that is not a perfpective Reprefentation.

5. I expect it will, again, by fome Perfons, be alledged, here, that Perfpective, then, is not fufficient, to reprefent Objects as they appear to the Eye. I affirm that it is. Let us, therefore, once more enquire, candidly, what is meant by Perfpective, and what effects it is expected to produce.

" Perfpective is a reprefentation, on a Plane, of an Object, or Objects, in a fixed and determined Pofition and Point of View. (See Perfpective, Page 47.)

I have already fhewn the bad effect of viewing a perfpective Picture, out of the true Point of View; from which, if we deviate, we cannot expect that the feveral parts of a Picture can vary their Bearings and Proportions, to each other, as the real Objects; no, certainly, that muft be a reality, not a Perfpective; which is but a Deception, a Reprefentation of a real Object, on a Plane; and, which, can never reprefent the Object, truly, from any other Point of View, but that for which it was delineated.

I have alfo fhewn, to ocular conviction (by the Apparatus) that there may be as many various Reprefentations of the fame Object, from the fame Station, or Point of View, as there can be Pofitions of the Picture; all which, are true Perfpective, and will affect the Eye alike, in the true Point of View; which, is the Vertex of the Pyramid of Rays; feeing that, every correfponding Line, on each Picture, is feen under the fame plane Angle as the Original, and every Surface, as well as the whole Object, is feen under the fame folid Angle, or Pyramid of Rays, as the correfponding original Surfaces, or as the original Object.

What, then, is it we are caviling about? would ye have a real, folid Object on a Plane, or a Reprefentation of it only, in a certain Pofition? Certainly then, if the Eye be not in the true Point of View, the Picture does not truly exhibit an appearance of the intended Object, at the fixed Station; and, although it may be a juft, perfpective Reprefentation, it may, neverthelefs, be a very diftorted and difagreeable Picture; not owing to any fault, or imperfection in Perfpective, but to the choice of the Situation of the Picture, or the Diftance and Pofition of the Object and the Picture.

Does not almoft every Object, except a Sphere, appear different from every different Point of View? and can any Perfon be fo unreafonable as to expect, that a Reprefentation of a folid Object, on a Plane Surface, can appear truly to reprefent the Original in any other Point of View, but in the Vertex of the Pyramid of Vifual Rays, under which the Object itfelf is fuppofed to be feen? the very fuppofition is abfurd to the laft degree; becaufe, no two parts of the Reprefentation can, at the fame time, be feen under the true Optic Angle, in any other Point; confequently, the Reprefentation, muft appear erroneous.

6. Now, although the true Reprefentation, and alfo the true Appearance are depictured on a fpherical Surface, yet, I affirm that fuch a Picture is fubject to much greater imperfection than a true Perfpective, on a Plane Surface; becaufe, if the Eye be the leaft removed out of the Center, the whole Appearance and Effect is deftroyed,

Plate VII. deftroyed, and exhibits a much worfe Image of the Object, than a perfpective Reprefentation, on a plane Picture, can poffibly exhibit, in any Point of View; which is fo very obvious, that it is needlefs to point out the reafon. For, fuppofe the Eye, at *E*, viewing the fpherical Picture, a *d h*; the Vifual Rays, *E* a, *E d*, &c. fhew, at firft fight, how much worfe fuch a Picture muft appear, than that on the Plane, a h, from the fame, though falfe Point of View; not one of them will appear round, and the Appearance, of all, is prepofterous.

From the circumftance I have mentioned, in refpect of the true Reprefentation and Appearance being depictured, at once, on a fpherical Surface, fome Artifts imagine, that the Reprefentation on a Plane ought to be fo delineated; it cannot be; 'tis impoffible, in the nature of things. Suppofe a true Reprefentation of a long Building, in full front, delineated on a fpherical Surface, and it were poffible, afterwards, to reduce the fpherical Surface to a Plane; is any Perfon fo weak as to fuppofe, that fuch a Reprefentation would appear like the Original, in any Point of View? he muft be weak, indeed, and have ftrange miftaken notions of Perfpective, who can; and yet I have heard this Point ftrenuoufly fupported, or rather argued for, fupported it could not be; for, any thinking Perfon (who can think with propriety about it) muft be fenfible, that, what fhould reprefent Right Lines will be curved, and, the whole, will give the Idea of a Rotunda, or externally round Building; feeing that, the extremes would fall off, not in Right Lines but curved, and they would appear lefs than the real Object; to fay nothing of the almoft impoffibility of producing or delineating fuch a Picture, at all, or by any means; I fhould be glad to be informed, how, or by what Rules.

7. There may be alledged another circumftance, which, I think, muft convince the moft obftinate, in Perfpective. I am perfuaded, that, if the Eye be fixed in a Point, at a proper Diftance from a tranfparent Plane, placed between the Eye and an Object, whilft the Hand traced, accurately, every Line of the Object, as it appeared on the tranfparent Plane; fuch a delineation, all muft allow, would be a true one. Let thofe, who are not otherwife to be convinced, try the Experiment. I will ftake all my knowledge in Perfpective, that every Reprefentation of a Right Line is a Right Line, on the Plane; that Columns or Cylinders of equal magnitude, and parallel to the Plane, will be larger as they are more remote from that Point, on the Plane, to which the Eye is oppofite; that the Reprefentation of a Circle or Sphere, feen oblique, is an Ellipfis; that Objects of equal magnitude, and equally diftant from the Picture parallel to them, however otherwife fituated or elevated will be reprefented equal; with various other circumftances; all which may be fully proved to ocular conviction, which will not admit of the leaft doubt. Surely then, if by the rules of Perfpective the fame thing be effected, which certainly will, in every refpect, it muft exhibit true Reprefentations of Objects.

It has been obferved, that the bufinefs of Perfpective is to produce the Figure of a fection of the Cone or Pyramid of Rays, by a Plane, in any determined Pofition; which, if the Rules it prefcribes be truly followed, it will moft certainly effect, without any fenfible error. For, wherever any Point, or Angle of an Object, appears on a Plane, or other Surface, between the Eye and the Object, there the Vifual Ray would cut and pafs through the Plane; but when the diftance of the Eye is fuch, that the Vifual Rays cut the Plane very oblique, or in Angles, nearly, or perhaps lefs than, half Right ones, the Reprefentation will confequently be diftorted and prepofterous, and, in other Points of View, will have a difagreeable and unnatural Appearance.

Here, then, lies the miftake, which, through ignorance or inadvertency, is attributed to Perfpective, and fuppofed to be a deficiency or imperfection in it. 'Tis, generally, in the Point of View, the Situation, &c. of the Picture or Object; which, by being too near the Eye, occafions that Diftortion and prepofterous Reprefentation we perceive, in feveral Pictures; for, if the Optic Angle, under which the whole Picture is feen, exceeds 50, or at the moft, 60 Degrees, the Diftance is not fufficient; as, the Vifual Rays will cut the Picture very oblique, near its extremes,

 and

and occafion a difagreeable diftortion of the Objects on the extreme parts of it. Yet, as I have obferved, at any Diftance, the reprefentations of the Diameters of Columns, or other cylindrical Objects, on a Picture parallel to them, will, in true Perfpective, ever be the leaft which are neareft to the Center.

8. I fhall now take notice of another great difficulty, which feems to be a ftumbling Block to many Artifts; who, one would imagine, would not hefitate one moment, to determine about it with propriety; which is, to reprefent, on a vertical Picture, the appearance of a direct Defcent; which, Mr. Kirby has affirmed impoffible, in the nature of things, to be done; that it is a ftrong inftance of the infufficiency of Perfpective, and that, we muft have recourfe to Experience, only, in fuch Cafes; intimating, that it is not poffible, by the rules of Perfpective, to give the Reprefentation of a defcending Plane; which is fo ridiculous an affertion, that, any Perfon, who underftands Geometry tolerably, will eafily be convinced of the contrary.

For, firft, we are to confider whether the Difcent (which I fhall fuppofe a Plane) is perceivable or not. If this defcending Plane can be feen, at all, from any fixed Station, it may, undoubtedly, be reprefented on the Picture, from that Station, by the ftrict Rules of Perfpective, or there is no truth in Perfpective; either it is a perfect and infallible Rule, or it is no Rule at all. If the Plane can be feen, it is a fubject of Perfpective; if it cannot be feen, it is no fubject for a Picture; which needs no Demonftration. To tell us, what defcends, and we actually know to go down-hill in Nature, will, if ever fo correctly drawn, appear to rife, upwards, on the Picture, is faying nothing to the purpofe, the expreffion is vague and nugatory; for, if the Plane defcended fo much as not to appear to rife on the Picture, it could have no place or reprefentation thereon; but if it can be feen at all, it muft, neceffarily and unavoidably, appear to rife; or rather, it muft, really, rife on the Picture, for, the Appearance is to defcend.

And yet, he fays we muft have recourfe to Experience. What kind of Experience can teach us to do, what is impoffible (as he fays) in the nature of things, to be done? Experience will teach us all that can be done; Experience, in Perfpective, will teach us to delineate regular Objects which we never faw, yea, which never exifted; but no Experience whatever, can make us perform impoffibilities; to reprefent Objects which are loft to fight in the Originals, from the determined Station. But, how can any Perfon be fo unreafonable, as to require that to be done, in a Picture, which is fo liable to falfe conftructions, or determinations, in Nature; for, a level, or horizontal Plane will appear either to afcend or defcend, by the affinity and concurence of Lines and Objects connected with them; as for Example. When we are going down a regular defcent, towards a level Plane, of Land or Water, the greater the defcent, the more the level before us will appear to rife; yea, fo ftrong is the Deception, that we are not eafily convinced of the contrary; except it be Water, before us, which, we are certain, cannot afcend. Likewife, when we are going up a gentle afcent, towards a level Plane, which is in fight, it will appear to defcend; more efpecially when we fee the boundary Lines of both, as in regular Walks, in a Garden, &c. Nor is there any thing furprifing in it, if we confider, that, in the former Cafe, it is owing to the much greater breadth which we fee, of the level Plane, and having the fame appearance as an Afcent, before us, when we ftand or move on a level. In the other Cafe, the contrary effect, in appearance, is the caufe of the Deception.

To draw two parallel and horizontal Lines acrofs the Picture, and to give an Idea, that the fpace, between them, reprefents a defcending Plane (of a certain length) without fhape, bounds or limits, fideways, or any Object fituated on the inclination, to bias the judgment, is indeed impoffible; but that is giving too great latitude to the meaning of the Expreffion. I fay, it is abfolutely impoffible, in this Cafe, to give, by Art, an Idea of a defcending Plane, fimply as fuch. For, although a proper gradation of Light and Colour may make a fmall fpace, on the Picture, appear to reprefent a great length, yet, 'tis not poffible to fay whether it defcends,

 or

Plate VII. or reprefents a level Surface; becaufe, the fame fpace, on the Picture, may reprefent an equal length, on a declining Plane, to a Spectator ftanding, as is requifite to reprefent a level Ground, to a Perfon laid along, or fiting; confequently, it is not poffible to fay, with abfolute certainty, whether it reprefents one or the other.

And yet, I queftion if a fkilful and ingenious Painter, in aerial Perfpective (who had critically obferved and copied Nature) might not, by the effect of Colour, fimply, even in this Cafe, deceive the Eye, and give the appearance of a Defcent. But, if there are Objects fituated on the inclined Plane, or, if the fhape or figure of the Plane, itfelf, is to be defcribed; whatever can be feen of fuch Objects (whether Tops or Bottoms it matters not) they may and can be reprefented, truly and exactly, by the infallible Rules of Perfpective; and that, on the fame invariable Principles, as the moft common and ordinary Cafes, whatever, are fubject to.

To illuftrate what I have advanced, by a fimple geometrical Scheme, will not be very difficult; which may be confidered as a vertical Section through the whole. Each Plane is, therefore, reprefented by a Right Line, making the true Angle of the Inclination of the Planes (as I fhall call them) with each other.

Fig. 35. Let A F reprefent the Plane of the Horizon, and A B a Defcent, as a flope Bank, or any other Declivity; making the Angle A B G with the Horizon. Let C D be a Section of the Picture, E the place of the Eye, and E F its Altitude above the Horizon, at the Diftance E C from the Picture, whofe Center is at C.

Now, if the Eye (at E) be fo fituated, that a continuation of the inclined Plane would pafs through the Eye (as B A E) it is evident that the Plane (A B) from that Station, is totally loft to fight; its whole Reprefentation being the Line of its Interfection with the Picture †. Confequently, if the Eye be moved further back (as at *E*) its appearance, or place on the Picture, would defcend; which cannot be, unlefs the Plane be confidered fimply, a Plane, without fubftance, and feen on the under fide; its apparent width from that Station is a D; but if the Eye be removed, nearer to the Picture, in any direction above B A E) the Reprefentation of the Plane rifes on the Picture. At E^2, its width on the Picture is a b, as the Rays, E^2 A, E^2 B, evince.

† Cor. 2. Theo. 2.

If the Eye be moved back again, in the direction B E^2, to E^3, it is plain that, b, the apparent height of B, remains the fame, wherever the Eye is fituated in that Line; but the reprefentative width, a b, of A B, will be increafed, towards D, as it recedes, if the Picture be at any diftance from the Defcent. Suppofe the Eye brought forward again to E^4; the width of A B from that Station is *a b*; and, if the Plane was continued infinitely, a V is its whole apparent breadth, E^4 V being parallel to A B.

9. I prefume, this is intelligible and clear, hitherto. I will, next, fhew that 'tis impoffible for the Eye to judge, merely from the width of its Reprefentation on the Picture, whether the Plane be defcending or afcending, or perfectly on a level.

From the fituation of the Eye at E^4, the reprefentative width, on the Picture, of the Slope, A B, is *a b*; whereas, if the Plane was more inclined to the Horizon, i. e. if the Defcent was more gradual, as A I, it would reprefent a lefs fpace; if it was horizontal, the fame *ab* reprefents the length A H, only; but, if the Plane afcended from the Picture, as A K, it ftill reprefents a fhorter length (A K) on that Plane (the Arks I i, H h, &c. fhew how much each is fhorter than the other) and A L, making right Angles, B L A, A L E^4, with the Vifual Ray E^4 B is the fhorteft length, from the Point A, that can be reprefented, by *ab*, from that Station, fituation, and pofition of the Picture.

Now, if any Object (as X) be fituated on the Inclination A B; it is evident, if a continuation of the Plane of its Top (M N) paffes above or through the Eye, at E^4, it cannot be feen from that Point; whereas, if the fame, or any equal and fimilar Object be fituated on the Horizontal Plane A H, the top, O P, may be feen from that Station, notwithftanding the Object is more elevated; becaufe, a continuation of the Plane of the Top, O P Q., falls below the Eye, at E^4.

The figure of the inclined Plane, itfelf, or the figure of any Object fituated upon it, is defcribed, perfpectively, in the fame manner, and on the fame Principles as on a horizontal Plane; which is exemplified in the practical part of this Work.

10. In the laſt place, I ſhall demonſtrate, that the repreſentations of Objects, which are elevated perpendicularly, above the Horizon, have the ſame proportion*, on a vertical Picture, as thoſe of the ſame Magnitude, ſituated on or near the Horizon; the Object being parallel to the Picture.

It is a miſtaken notion which ſeveral Perſons entertain, that the parts of a Building, which are elevated high above the Horizon, appear to diminiſh in a greater ratio than thoſe which are extended horizontally; ſuch an opinion may be eaſily refuted. If the ſecond Part of the 9th Theorem be well conſidered, it is ſufficient refutation.

Let A G be ſuppoſed a high Obeliſk, and A B, B D, &c. ſeveral equal Diviſions thereon. Let E S be a Spectator, E the Eye, and a g, or *a g*, a Section of the Picture; which, being vertical, is parallel to the Object A G. Fig. 36.

Now, if the Viſual Rays E A, E B, &c. be drawn, they will cut the Picture in a, b, d, &c. then is f g, the repreſentation of F G, equal to a b, the repreſentation of A B, or to b d, the repreſentation of B D†. † 4. 6. El.

For, ſince the parts A B, B D, &c. of the Original Object, A G, are equal, the repreſentations of them, on the Picture, being parallel to the Object, are alſo equal.

But, AB is an Object, direct before the Eye, at E, on the Horizon AS; and, F G is one of equal length, elevated greatly above it; therefore, the Repreſentations of equal Objects at an equal Diſtance from the Picture, and parallel to the Picture, are equal. Q. E. D.

The ſame Object may be conſidered as a Building, extended horizontally, and AEG a horizontal Plane, in which is the Eye, and alſo the Viſual Rays E A, E B, &c. ag, or *ag*, is a Section of the Picture, as before. The interſections of the Viſual Rays with the Picture, it is evident, are the ſame.

11. There is yet another point of controverſy I have ſome times been entertained with; which is, that, when we ſtand, oppoſite the middle, near a long Building, or range of Buildings in a Right Line, the horizontal Lines, in the Cornice, &c. appear to decline towards either End, yet make no Angle; and therefore, they imagine that the repreſentation of thoſe Lines, in ſuch Caſe, will be curved. What a poor Idea muſt a Perſon have of Perſpective, who advances and is really prepoſſeſſed of ſuch an Opinion.

I ſhall ſay very little on this Point, becauſe, the eighth Theorem is full and perfect Demonſtration, that the Repreſentation (on the Picture) of every Right Line, is a Right Line; and, it is ſo if extended infinitely. Becauſe, a Plane may be ſuppoſed to paſs through any Right Line and the Eye; and, the Interſection of this imaginary Plane, with the Picture, is the indefinite Repreſentation of every Line it paſſes through; for, the Eye being in a continuation of it, the whole Plane is loſt to ſight†. Therefore, the Repreſentation of every Right Line is a Right Line. † Art. 3. of a Plane.

The truth, of which, any Perſon may ſoon be convinced of, by applying a perfectly ſtreight Ruler, before his Eye, parallel, or otherwiſe, to a Right Line, in an Object, of any length, and imagine the edge of the Ruler to cut the Picture, which is a Plane; then certainly, if the Ruler coincides with the Original Line, from one end to the other (which it undoubtedly will) the Right Line, in which it is ſuppoſed to cut the Picture, is the Repreſentation of the Original Line. It is the ſame as if a tranſparent Plane was placed between the Eye and the Object, and any Right Line, in the Object, traced on it, exactly, whilſt the Eye is fixed in a Point; the Line, ſo deſcribed, will be a Right Line.

Thoſe Perſons never conſider (but 'tis plain they are not furniſhed with the means) that the Picture, being placed in the true Point of View (conſequently at its proper Diſtance) will appear the ſame as the Original; for the ratio of the Parts is always in proportion to the Diſtance. Conſequently, every part of the Picture, being ſeen under the ſame Angle as the Original, will have the ſame Appearance, in every reſpect; and conſequently, the Right Lines, on it, will appear to decline either way, or both ways, the ſame as in the Object.

* What I would ſignify by the proportion of Objects, in this place, is, ſimply, length and breadth.

12. Refpecting the appearance of parallel Right Lines direct before the Eye, there is fomething paradoxical, which is not eafily reconciled to reafon; for it is manifeft, from the foregoing, and the eighth Theorem, that every Right Line appears a Right Line, when the Eye is directed to it; and yet it is certain, that parallel Right Lines, however fituated, appear to converge, and confequently to approach each other.

Suppofe two, parallel, horizontal Lines, direct before the Eye. It is manifeft, that their Reprefentations, on a Picture parallel to them, are alfo parallel Right Lines. Notwithftanding which, it is certain, that the neareft Diftance fubtends a greater Angle, at the Eye, than any other; and that, the Angle fubtended, by their perpendicular Diftance, is continually lefs, the farther they are extended on either fide. How, then, can they appear Right Lines? feeing that, they appear wider afunder, direct before the Eye, than towards their extremes; neverthelefs, fince a Plane may pafs through each Line and the Eye, they muft neceffarily appear Right Lines, and confequently parallel; feeing they fubtend equal Angles, at equal Diftances from thofe parts which are neareft to the Eye.

This very extraordinary Phænominon may be better conceived, by imagining the Eye in the Center of a concave Sphere; and, imagine two great Circles, which are Meridians, or longitudinal Circles, making an Angle of 15 or 20 Degrees, or more. Now, the Eye being in the common Center of both Circles, each appears a Right Line, which way foever the Eye be turned, towards the Circumference. But, the Meridians interfect and crofs each other, in the Poles; which, being diametrically oppofite, cannot both be feen at the fame time; but, in looking towards either, the Circumferences of the Circles will appear like two Right Lines, converging to a Point; the Pole, in which the Circumferences interfect.

Now, fuppofe the Eye directed to the Equator of thofe Meridians; and imagine two equal Chord Lines drawn, one in each Circle, at equal Diftances from the Eye, and parallel between themfelves. Thofe Chord Lines, it is manifeft, will appear to coincide with the Circumferences of the Circles, the Eye being in their common Center; and would, if they were continued beyond the Circles, infinitely. But, it is certain, that the Circumferences, being meridian Circles, crofs each other twice; wherefore, the parallel Lines, one in each Plane of the Circles, will appear to converge to a Point towards each extreme, which are their Vanifhing Points; and, which, muft be infinite, before they can appear to coincide with the Poles of the Circles. Thus, it is evident, that Right Lines do always appear Right Lines, and are reprefented by Right Lines; notwithftanding which, parallel Right Lines appear, when direct before the Eye, to converge to a Point, towards each extreme.

I have now, I hope, fully refuted thofe truly ridiculous and abfurd Opinions, of Perfpective, which many have imbibed, and are not eafily divefted of; they, rather, obftinately perfift in them, without being able to give a folid reafon for their Opinions, and are determined not to give up their Prejudices, at any rate, right or wrong. What ftrange infatuation muft poffefs that Perfon, who, having no argument of weight, to fupport his falfe notions of Things, has recourfe only to Sophiftry; and, becaufe he cannot come at fterling Truth, himfelf, imagines there is no fuch thing to be found. In points of natural Philofophy, &c. where no certain Criterion can be obtained, to fix our affent, it is no wonder that we meet with fo many, widely different, Opinions; and, though few of the arguments advanced have the leaft foundation in Reafon, 'tis amazing with what eagernefs and warmth each Affailant attacks his Opponent. But, in Sciences, purely mathematical, all muft agree; when Truth appears, there is no refiftance can be made; we cannot with-hold our affent fo prevalent is her influence.

To fuch as are open to Conviction, and are defirous of coming at Truth, I think I have faid enough for their conviction, on the Points debated; but if they are not, all that can be faid is to no purpofe, 'tis wafte of Time and Words. I fhall, therefore, leave them to enjoy their Opinions, and pleafe themfelves with their great Sagacity; and proceed to lay down certain and infallible Rules for the Practice of Perfpective, deduced from a perfect and well founded Theory; which, if truly followed, and proper attention be given to the Leffons, contained in the Introduction, will moft certainly produce Harmony and true Effect, of any regular Object, fo far as comes within the province of linear Perfpective.

Plate VI

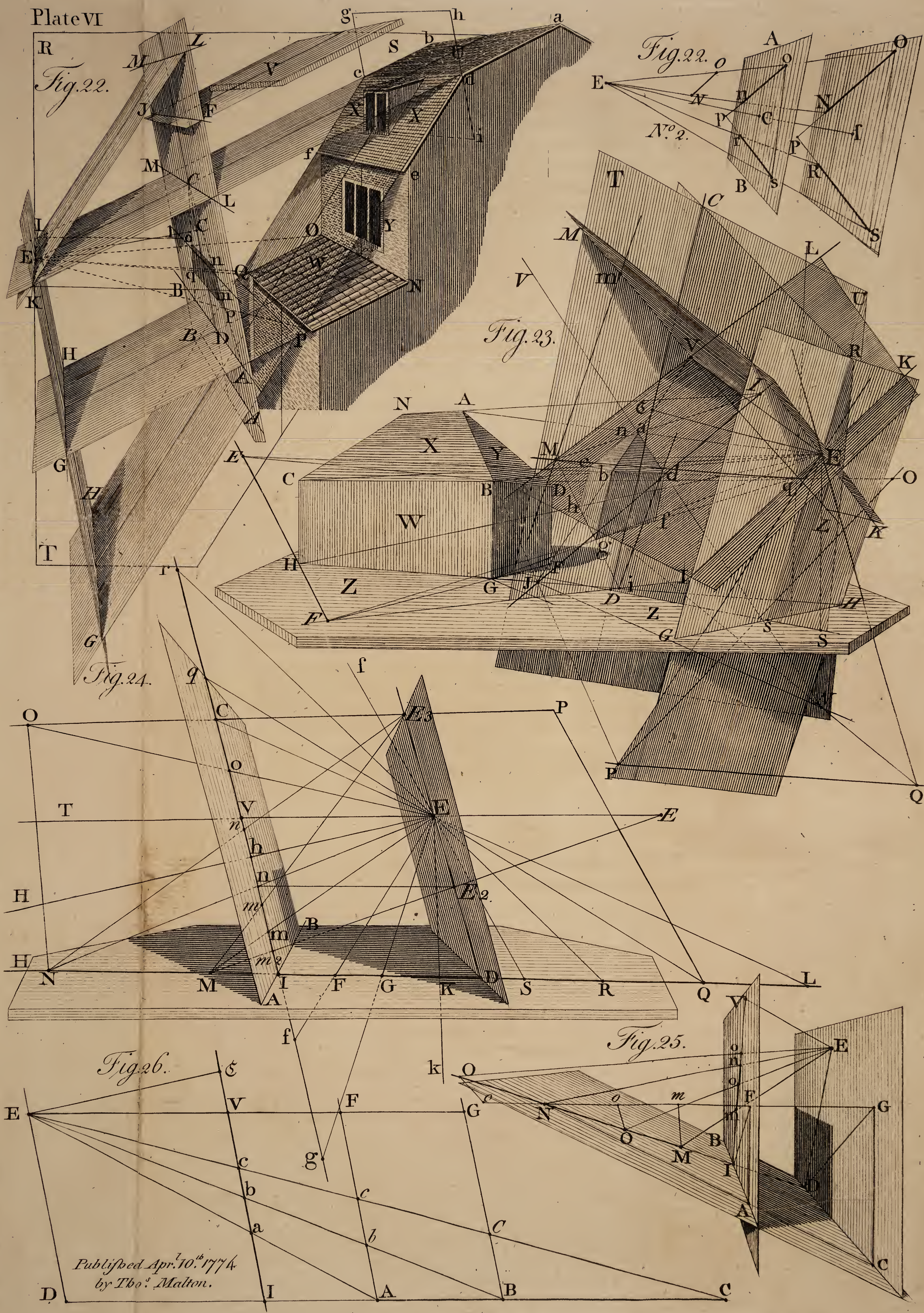

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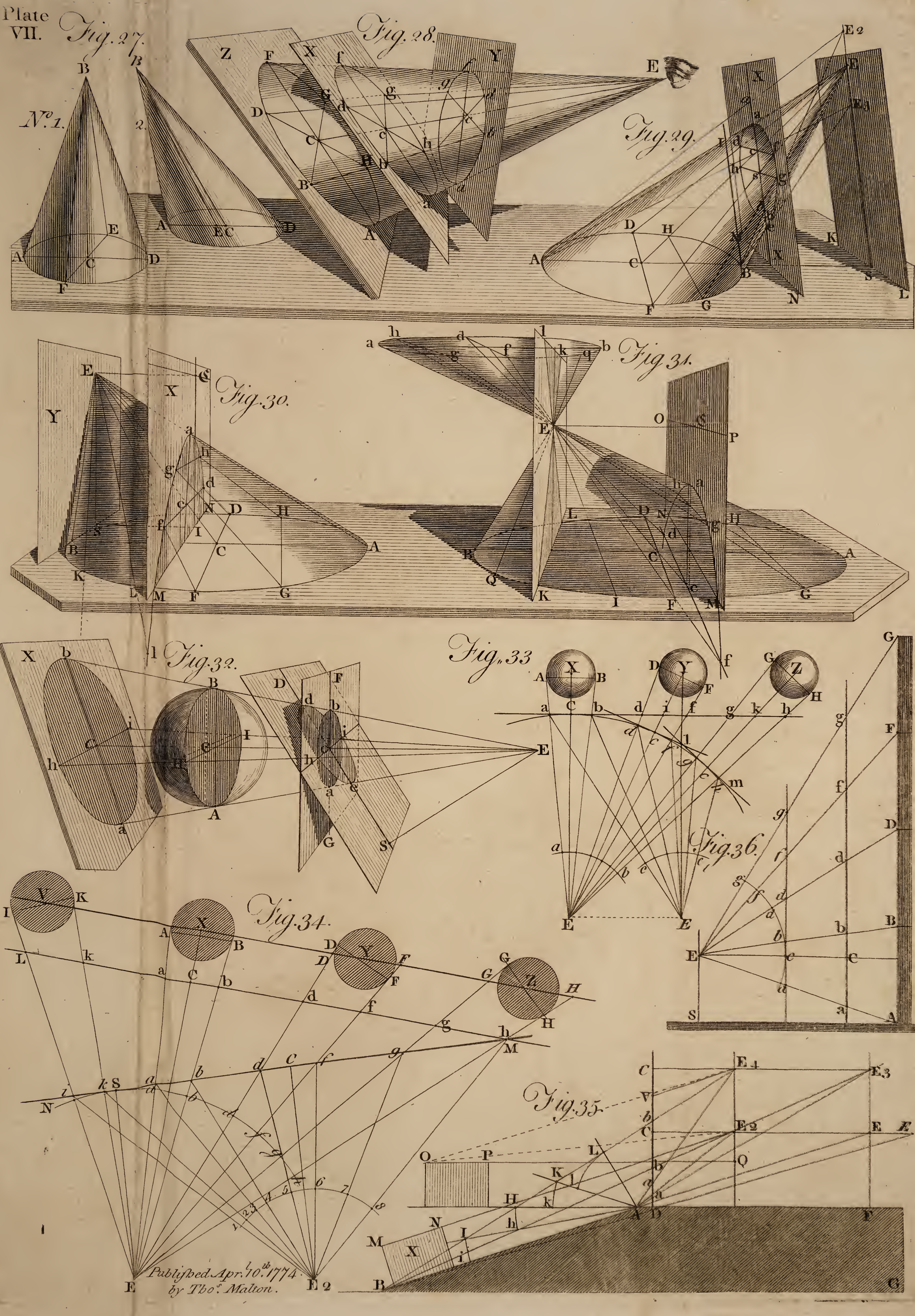
Plate VII.



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BOOK III.

Of the Practice of PERSPECTIVE.

SECTION I.

An INTRODUCTORY PREFACE.

I Come, now, to the practical, and, in that, the useful part of Perspective; to which, the foregoing Book is an Introduction, only, but a very necessary Introduction; insomuch, that, without the knowledge inculcated by it, we should proceed in ignorance and uncertainty. Neverthelefs, those Persons who have not studied Geometry, and have not, now, perhaps, either leisure or inclination to study it (though, in my Treatise of Geometry, they will find it neither so abstruse, tedious, or dry a Study, as many look on it to be, having treated that most useful branch of Science in a more familiar and intelligible manner, than has been done heretofore): I say, that, without a sufficient fund of Geometry, to perceive and be clear in the Demonstrations, if they will but treasure up, in memory, the Theorems and Corollaries, and take all for granted (as they may depend on the Truths contained in them) they will find the great advantage of it, in Practice.

Every branch of Science is in two parts, *viz.* theoretic and practical. Theory teaches the knowledge of all that is necessary for Practice, in Speculation; the other applies that Knowledge to real use. It is necessary, first, to know how, before we can perform any thing; but notwithstanding, many Persons may be said to know Perspective (as a mathematical Science) yet know not how to apply it to Practice, with succefs. So, every Art, dependant on Science, may be acquired without the Theory of it, by custom or habit; as in mechanic Trades, derived from Geometry and Mechanics; all which, require time and application, to become familiar to us. So likewise, Perspective may be practiced, without being said, properly, that we understand it; seeing, we may not be able to give a sufficient or satisfactory reason for the effects of it, in any instance. And, being well versed in the Theory, we shall neverthelefs find, that it will require time and assiduity to apply it, in all Cases that may occur, in Practice, although founded on the same invariable Principles, without familiar Lessons, in every Case, being given. But we shall, certainly, with the assistance of Theory, be able to comprehend, and to understand the various methods of applying the Rules, better than without it; as very little reflection, when we are at a loss, will set us right again, and enforce the universality of its Principles.

It is not, to be wondered at, that the best of Treatises on Perspective has been the least understood; *viz.* that, by Dr. Brook Taylor; because it is very obscure. I call it the best, on account of his general Principles, not from its real utility to Artists, in respect of practicing from its Rules. It can scarce be said, with propriety, that Perspective existed before; it certainly was not properly understood, as a Science. How insipid and imperfect are all the Books on the Subject before his, except Gravesande's and Ditton's; both which seem to touch on the same Principles, but are far short of a perfect, general System.

There are, in that small Treatise, Rules sufficient, in almost all Cases, for Plane Objects; but, various Examples requiring various ways of applying them, he has not made it so useful as it would have been, had he, instead of refering to former Problems, shewn how to apply it there. As, in Example 3rd, Page 24, Book the first, Fig. 21. We are told to make CB represent a Line, equal to that which is

reprefented by CA (by Prop. 15.) Now, I queftion that one Perfon in fifty (who underftands the whole) ever faw the leaft Affinity between the two Examples; the different fituations of the Lines makes it a very different Operation, though built on the fame Propofition of Euclid.

Refpecting plane Objects, only, Perfpective is foon acquired; knowing how to delineate Figures in horizontal Planes, it is the fame in any other; having found their Vanifhing Lines, with their Centers and Diftances; which I have firft fhewn, in Practice (as in Theory) how to find, in all ufeful Cafes whatever. But, without the embellifhments of Mouldings and other architectural Ornaments, in Buildings, &c. all the reft would be to little purpofe. That Treatife comprehends only Perfpective in Plano; but I have, in this Book, fhewn how to apply it in every neceffary Cafe that can be devifed; and doubt not, to render it, by that means, the moft ufeful work, of the kind, yet publifhed in any Language, that we know of.

Brook Taylor has, indeed, to his immortal Fame (in Perfpective) furnifhed us with new and extenfive Principles; but his Work, at beft, is imperfect, and greatly deficient. His Theory is too concife, and is not regularly digefted. It may perhaps, by fome, be objected, that I have made too much of the Theory. I could have faid much more, but know not where to curtail it; the Examples, given for Illuftration rather than Demonftration, I am perfuaded, will not be found unneceffary, to fome, or trifling to any. One Example, in each Cafe, to fome Perfons, would be fufficient; to others there can fcarce be too many, fo they are various, and not a repetition of the former. I fhall be guilty of the fame fault (if it be a fault) in Practice; I had rather fay too much than not fay enough, yet I would not be tedious; becaufe, all that can be faid, to fome, will be too little, or rather too much, feeing it will be all to no purpofe. To fteer the middle courfe is a difficulty not eafily obviated; but, it is my fixed defign to aim at it; others muft determine how I have fucceeded.

It is the opinion of many Artifts, that the whole of ufeful Perfpective may be comprifed in a little Compafs; that nothing is a greater difcouragement to the ftudy of it, than to fee a voluminous Work on the Subject. 'Tis certain, that the Principles, on which the Theory of it is founded, are contained in a very fmall Compafs; and, I would recommend Brook Taylor's Epitome for that very reafon, which contains *multum in parvo*. Yet, notwithftanding that valuable Treatife has been fo long publifhed, it is, at this time, but little known, and lefs underftood; which is a fufficient reafon, with me, to fuppofe that he has not faid enough on the Subject. The elaborate Work of Mr. Hamilton is fpun out to an immoderate length; yet to as little ufe as the other. 'Tis my Defign, to comprife the whole of ufeful Perfpective in this Book. Neither of thefe Authors, I am perfuaded, had either taught or practifed the Art of delineating; and confequently, they were not qualified for treating it in an eafy and familiar manner; one great requifite in a Work of this kind. I have had experience in both, and am well convinced, that, to make it ufeful, it cannot be comprifed in fo little compafs as many imagine; that it requires frequent repetitions of the fame Leffon, fomewhat diverfified, to familiarize the Rules, in various Cafes; without which, not one in twenty will ever be a Proficient. Dr. Taylor truly fays, it is much better for the Student to devife Examples, himfelf, in particular Cafes, than to go through thofe of others; but how few are capable of doing fo? nay, I find, many are not able to comprehend them at any rate, nor by any means; and therefore, to make fuch a Work really ufeful, variety of Cafes and Examples muft be devifed, for Practice and Experience.

In Practice, our Author has given fome Problems, containing the moft elegant and general Rules that can be, notwithftanding they are but feldom practiced; becaufe he has not fhewn, properly, how to apply them: that fhall be my care to do, where they can be applied ufefully. His Diagrams are, in general, very imperfect, and badly devifed; 'tis evident he was no practitioner or delineator, himfelf, even in Plane Objects (for he has given us no other). But, he has departed from his

own Principles, in Example V. Fig. 19. Part II. having projected the Dodecahedron, in Perspective, by means of the Ichnography and Orthography (as by the old Authors) instead of Vanishing Lines and Vanishing Points; which is much more masterly, elegant, and perfect; and is what the difference chiefly consists in. Such Subjects are indeed of little use, except to familiarize us to find Vanishing Lines and Points in all positions of Planes and Lines, to the Picture, and situations of the Object, or Picture.

Some Persons are so bigoted to the old Authors, that they cannot be reconciled to the new Terms, by Brook Taylor; nor indeed to his new Principles, till they find their Excellence by experience. It is not to be wondered at. 'Tis not easy to divest any Person of old habits and methods of practice, though ever so absurd; because it is impossible that they can see the difference, at first, and consequently cannot judge of it; but it is surprizing that they are not to be prevailed on to try; and if they do, it is with seeming reluctance, and with a fixed resolution to prefer, and persevere in their old Prejudices. I am as much against capricious innovations in Science as any Person; but, if there be an appearance of any acquisition to it, we ought, candidly and unprejudiced, to make a fair trial of their merits; without which we cannot judge of their Excellence.

In respect of the new Terms given us by Dr. Brook Taylor, such as Intersection of the Picture, Center of the Picture, Vanishing Lines and Points, &c. (together with Directing Plane and Line, which are most essential, in Theory, though but of little use, in Practice) I am of opinion that no better Terms could possibly be devised; nor any other so expressive of what is meant by them. How narrow, how limited is the Base Line and Horizontal Line (the only Vanishing Line known to the old writers on Perspective) when compared with them! What difference is there, either in Theory or Practice, between the horizontal Vanishing Line and any other, of Planes perpendicular to the Picture? None at all, seeing, they have the same Center and Distance (Th. 4.); nor indeed in any other, except in finding them; the Practice, in all, is the same, in every respect.

It was impossible for him to make the Principles of Perspective general, but by general Terms; which does not regard any Position either of the Picture or of the Original Plane, since all Planes (simply as Planes) are the same. The Intersection of the Picture includes every Intersection whatever, as well as the Base Line, and they are all of the same use. Vanishing Line is not only general, but is, at the same time, so simple and expressive, that it conveys its utility at once to the Mind.

In the second Theorem of the first Book, it is proved, that parallel Right Lines, however situated, appear to approach towards each other; and, consequently, if they are produced, infinitely, they will appear to meet, and vanish, in a Point at an infinite Distance. So likewise, parallel Planes appear to meet each other, and to vanish in a Right Line, (supposed to be infinite) or, properly, in a Point.

Now, if this Theorem be well considered and understood (together with the foregoing) it will be found to be the foundation of the new Principles of Perspective. For if a Plane be supposed to pass through the Eye, parallel to any Plane, whatever, or any number of parallel Planes, and being produced, or continued till they are lost to sight, they will all appear to unite, and to meet the Plane passing through the Eye, at an infinite Distance.

But, if the Eye be in the continuation of a Plane, the whole of that Plane is lost to sight, and appears but a Right Line (Art. 3. of a Plane, P. 41,) And, the Intersection of two Planes is a Right Line, (Ax. 3.) Wherefore, if a Plane (which may be considered as the Picture) be situated any how; the Line, in which this imaginary Plane would cut the Picture, is that in which the parallel Planes unite, and vanish; consequently it represents an infinite Distance; and consequently, the Line, so produced, is their Vanishing Line; for they cannot, if continued infinitely, appear to go beyond it.

Hence,

Hence, those Planes are, very aptly, faid to vanifh, they being loft to fight. Therefore, all Parallel Planes have the fame Vanifhing Line. (Theo. 3rd.).

Alfo, if a Right Line be fuppofed to pafs through the Eye parallel to any number of Lines, they will appear to converge towards that Line, and to meet it in one Point, at an infinite Diftance (Theo. II. Direct Vifion.)

Wherefore, if this Right Line from the Eye cuts a Plane, any how fituated, it will cut the Plane in a Point only; which reprefents a Point at any Diftance whatever, in that direction, and confequently, it reprefents the Point in which the Lines, parallel to it, converge; which is infinite. Therefore, it is their Vanifhing Point; for, they are loft to fight before they appear to reach it, feeing it is infinite.

And, fince the Line, producing that Point, paffes through the Eye, the whole Line is loft to fight, feeing that one Extreme is at the Eye; and the Extremes of Lines are Points. Therefore, the Point, in which it cuts the Picture, is its whole Reprefentation; and confequently, all Lines, parallel to it, tend to that Point.

Now I muft own, that I cannot conceive any Term fo fit to exprefs that Line, or Point, in which parallel Planes, or Lines, meet each other, as Vanifhing Line, and Vanifhing Point; becaufe they are truly faid to vanifh in them. For the fame reafon, perhaps, Mr. Noble, the laft writer on Perfpective, (except Fergufson) has made ufe of the Terms Entering Line, and Entering Point; feeing that, the Plane, or Line, begins at the one and vanifhes in the other. Had this Author been the inventor of thofe Principles, I fhould not have found fault with the Names he had given them; but, fince there were Names already given, by the Author, which are more fignificant, I muft blame even an attempt to alter them; becaufe, a multiplicity of Names, for the fame Thing, occafions a confufion of Ideas, in the Mind, of their fignification and ufe; and cannot poffibly be of advantage to the Science.

In the procefs of this Book, after fome neceffary obfervations, on the proportion of the Picture, the Height and Diftance of the Eye, &c. in the third Section, I have, firft, fhewn how to determine on the Pofition of the Picture, in refpect of the Object and the Eye, the Station being previoufly determined; then, how to prepare the Picture, for Practice, according to the Principles contained in the foregoing Theory.

The Picture being prepared, the Situation and Diftance of the Object, and the Pofition of the Picture being determined, the following Problems, in that Section, fhew how to find the Interfections and Vanifhing Lines of Planes, in all Pofitions to the Picture, if they are not parallel to it (for all fuch have no Vanifhing Line. Theo. 1ft.) with their Centers and Diftances. Then, to fix the Vanifhing Points of certain Lines in thofe Planes, and determine their Diftances; by which, the Lines, vanifhing in them, are proportioned.

The Pofition, Situation, and Diftance of the Picture (in refpect of the Object and the Eye) being determined, the Original Lines, in the Objects, are produced (if neceffary, and not parallel to the Picture) to their Interfecting Points; which are always found in the Interfection of the Plane they are in (Theo. 10th) or being much inclined to the Picture they do not, perhaps, fall within the compafs of the Picture; then, other expedients are ufed, to find the reprefentation of fome principal Point in the Line, from which Point, the indefinite Reprefentation is drawn (Theo. 12th.) And laftly, the finite Parts, which reprefent certain portions of Lines in the Original Object, are determined, by Theorem 13th; by which means, the Object is completed; proceeding from one Plane, or Face of the Object, to another; drawing all the Figures, in each, by means of their refpective Vanifhing Lines. Each two adjoining Faces, having one Line common to both, the Vanifhing Point, of that Line, is in both Vanifhing Lines (Theo. 10th) confequently, it is in their common Interfection (Cor. 2nd. Theo. 7th.) by the help of which, the Vanifhing Lines of contiguous Faces are determined.

In Section 4th, I have briefly illuftrated all the remaining, practical, Problems in Brook Taylor's Effay, refpecting the proportioning of Right Lines, perfpectively, and fhewn their great and extenfive utility; each of which, founded on the moft folid and permanent Principles, is of immenfe value. For, without knowing the whole of Perfpective, or practicing by its Rules, rigidly, an Artift, who is accuftomed to fketch, by fight, whatever he fees before him, with feeming accuracy, may, by thefe Problems, rectify any errors, in right lined, or circular, Plane Objects, from the known proportion of one part to another; the affinity of the Planes and Lines, to each other, being known, and the ratio of one Part to another; which, may frequently be obtained, when the true meafures of thofe Parts cannot.

Thefe Problems contain all the Rules neceffary for Practice. They may be compared to the five fundamental Rules in Arithmetic, by which all others are worked; and, a Perfon might, with as much propriety, imagine that he had given Arithmetic enough, for every Occafion, in thofe Rules, as Brook Taylor had of Perfpective, in his firft Effay; whereas, the Rules, he has there given, are no more than the Elements of practical Perfpective.

This Section contains, alfo, various Expedients; *viz.* for determining Vanifhing Points, when they fall beyond the limits of the Picture, geometrically and arithmetically, i. e. to determine their Diftance from the Center of the Vanifhing Line and from the Eye, both which are neceffary to be known; how to draw Lines to a Vanifhing Point which is beyond the bounds of the Picture, &c.

In Section 5th, thofe Rules are applied to real ufe, in delineating all kinds of Plane Figures; and, on Planes in various Pofitions. Firft, by means of the Figure being geometrically drawn, in the Original Plane. Secondly, without it, by their known Proportions, their pofition to the Picture, and the properties of the Figure, being regular.

In the 6th Section, from Plane Figures, I have proceeded to Solids, compofed of Planes, of various Figures; and in various Pofitions.

In the 7th, I have applied them to ftreight Mouldings, compofed of Planes and cylindrical Surfaces, in Cornices, Entablatures, &c.

Section the 8th treats of curve lined Objects, in general.

The 9th fhews how to apply the whole to compound Objects, in regular pieces of Architecture, and Buildings of various kinds.

The 10th is for internal Views, and horizontal Pictures, in Ceiling Pieces, &c.

The 11th is adapted for the particular Profeffions of Cabinet-makers, Coach-makers, &c. and for Machines, in general.

The 12th, and laft, is on inclined Pictures and Planes, in general; and, applied to Fortification, or military Architecture.

N. B. A Scale of equal Parts is always adapted, or determined on, of the Proportion we intend to delineate the Objects, on the Picture.

⁂ The methods of making Scales, for various purpofes, are given in the Appendix to my Treatife of Geometry.

Plate V.

SECTION II.

A preparatory and elementary INTRODUCTION, to the PRACTICE of PERSPECTIVE.

AS this Book is intended for a compleat practical Treatife, I have treated it in fuch a manner (in this Section) as if no Theory, or Elements, had been given. For which reafon, I have defined a few more Terms, which are fuited to Practice only; as there are feveral Terms in the theoretic Lift which may be omited here; and, there are alfo, in Practice, feveral which are not ufeful, in Theory. I have deduced from the Definitions fuch ufeful Leffons, which, if carefully attended to, will contain all the neceffary Theory for a Practitioner.

PERSPECTIVE, is the Art of delineating the true Reprefentations of Objects, on a Plane Surface, by geometrical Rules; according to the Pofition, and Diftance of the Objects, in refpect of the Picture and of the Eye. (See the Apparatus.)

Fig. 15. The PERSPECTIVE REPRESENTATION of an OBJECT, is the Section of the Pyramid of Rays, A E I, by a Plane, in any Pofition; which, is the Subject of this Third Book. For, the Picture of every Object, truly delineated in Perfpective, is fuppofed to be fo fituated, in refpect of the Object and the Eye, that if Vifual Rays, or Threads, i. e. if Right Lines were drawn from each Angle, or other Point in the Object, to the Eye, they would pafs through the correfponding Points in the Reprefentation, on the Picture.

As E A, E B, E F, &c. cut the Pictures in *a*, *b*, *f*; a, b, f, &c. which are, therefore, the Perfpective Reprefentations, of the Points, A, B, F, &c. on each Picture.

It is evident, that the Perfpective Reprefentation of an Object is nothing more than the Figure projected on a Plane, by its Interfection with the Vifual Rays from the Eye to the Object; wherefore, the whole bufinefs, of practical Perfpective, is to find the true Figure of the Section of the Rays, in all Pofitions, whatever, of the Picture (which is confidered as the interfecting Plane) and in every fituation of the Object, or of the Eye.

In order to which, it is neceffary to reconfider, well, the conftruction of the elementary Planes; as in Plate IV; or, to make them more familiar, I have given another, which is better fuited for Practice; as they are the foundation of the whole, and the origin of all the Lines and Points ufed in Practice. Alfo the general Introduction to this Work fhould be attentively perufed, and tolerably well underftood, by every one who would be a Proficient in the Practice of Perfpective.

Let B F I L be an Original Object, reprefenting a Building, and, E S a Spectator, viewing that Object, from the Point of View E (the Eye.)

It is manifeft, that whilft the Eye remains fixed, in that Point, the Object cannot vary in its Appearance. But, it is alfo obvious and demonftrable, that every different Section of the Pyramid of Rays, by a Plane in different Pofitions, will exhibit a different Picture; fo very different, indeed, they may be, that they can fcarcely be fuppofed to be Reprefentations of the fame Object, much lefs from the fame Point of View and Pofition of the Object, when viewed direct.

Compare the Pictures, M N O P and *O P*; how different are the Images of the fame Object, to each other. The one, on M N *O P*, has the Reprefentation, *b g f d c*, of the end of the Building, B F C, fimilar to the Original, in every refpect; the Reprefentation, *a h g b*, of the Front, A H G B, is diftorted, and drag'd out to a prepofterous length, when the Eye is oppofite to it; whilft the other Picture exhibits a pleafing and natural Appearance, in almoft any Point of View, but moft fo in the true one. Yet, both Pictures affect the Eye alike, in the true Point of View, and appear the fame as the Original Object, itfelf; each Line, Plane,

 or

or Figure (it is evident) being feen under the fame Optic Angle as in the Object. Confequently, the Picture being a true Perfpective Reprefentation of the Original, if it had likewife the fame degree of Colour, Light and Shade, it would not be poffible for the Eye, at E, to diftinguifh whether the Image (*a i f c*, or a i f c) delineated on a Plane, or the Object itfelf (A I F C) was prefented to view.

Having, thus clearly, explained what Perfpective is, I fhall next define all the practical Terms, by means of which, the whole procefs is performed, and the Reprefentations effected,

Let the Plane A I K B (which may be fuppofed the Picture) be raifed up into a vertical Pofition, i. e. perpendicular to the Plane it is fixed to; which is the moft natural, moft general, and convenient Pofition. Pl. VIII. Fig. 37. No. 1.

Alfo, raife up the Plane G K I H, parallel to the Picture. This Plane is called, the Directing Plane (Def. 4.) and is very ufeful, in Theory. In it, the Eye is always fuppofed to be, as at E; and if the Plane N*IK*L be turned down, parallel to A B G H, the Line *IK* meeting I K, in the Directing Plane; then, E C is the Diftance of the Eye from the Picture (equal F D); and E F, (equal C D) is its height above the Plane A B G H, on which, the Spectator (E F) is fuppofed to ftand. Therefore, at whatever Diftance the Eye is from the Picture (as E C) a Plane G K I H is fuppofed to pafs through the Eye, parallel to the Picture.

The Plane V being turned up, perpendicular to the Picture, is confidered as a part of the prime or chief Vertical Plane; and, N*IK*L is called the Horizontal Plane; both which are fuppofed to pafs through the Eye, at E.

N. B. This Conftruction of the elementary Planes, fo effential in Theory, is, alfo, very neceffary to be well confidered by every Practitioner, who may not have an inclination to go through the Theory; he will, moft certainly, find his account in it.

DEFINITIONS.

DEF. A. The GEOMETRICAL, or GROUND PLANE is that Plane on which an Original Object, intended to be delineated, is feated.

As G H Z Z; on which the Plans, A C D F and X Y Z (fuppofed to be the Seats of fome Objects) are geometrically drawn, which are to be reprefented on the Picture, A I K B.

On this Plane is always drawn, or fuppofed to be drawn, the Figures of the Seats or Plans of Objects fituated on it, in their true geometrical Proportion, according as they are fituated, in refpect of each other and of the Picture; and they are (in common Perfpective) always underftood to be beyond the Picture.

In the Apparatus, the Plane Z S Z is the Ground Plane, reprefenting the real Ground, on which the Object, B F I L, ftands. The Rectangle A B C L is its Plan, or Seat on the Ground Plane.

Note. If the Object ftands on a Floor or Table, &c. it is confidered as the Ground Plane.

DEF. B. The GROUND LINE, or BASE LINE, is a Right Line on the Picture, drawn parallel to its Top and Bottom, i. e. parallel to the Horizon.

As AB, Fig. 37. No. 1 and 3. (See Interfection. Def. 7. in the Theory.)

The Plane AIKB, being erected, reprefents the Picture, and AB Z Z, being horizontal, is confidered as the Ground Plane. Therefore, the Line AB is the Interfection of the Picture with the Ground Plane. And, fince all Objects which are to be reprefented, on the Picture, are fuppofed to be beyond it, they muft neceffarily appear above the Line of Interfection; for which reafon, it is, very properly, called the Ground Line.

Note. On the Ground Line is applied all the meafures, geometrically, of all Figures (as Plans, &c.) of Objects on the Ground Plane, to be delineated; and frequently, for other horizontal Planes, whofe Interfection with the Picture is not given or drawn.

N. B. Of thefame ufe is the Interfection of any other Original Plane with the Picture, *viz.* for proportioning Lines and Figures drawn on that Plane.

DEF. C. The HORIZONTAL LINE, is a Right Line drawn parallel to and above the Ground Line; as M L. (See Vanifhing Line Def. 8; or G.)

The Horizontal Line is called fo by way of eminence, or for diftinction from all other horizontal Lines, being the firft, or principal Vanifhing Line; from its fixed and determined Pofition and Diftance, all other Vanifhing Lines, whatever, are determined; the Vertical Line is determinable without it.

N. B.

Pl. VIII. N. B. Its Distance from the Ground Line is always equal to the height of the Eye from the Ground, being considered as a Plane, on which the Object, to be delineated, is seated. So that, whether we be siting, standing, or elevated, the height of the Eye, from the Ground Plane, being determined, and

Fig. 37. A*E*, or DC, being made equal to it (by the Scale of Proportion) NL drawn through the Point E, or C, parallel to AB, is the Horizontal Line; or Vanishing Line of horizontal Planes.

Note. It is supposed (in Theory) to be produced by a horizontal Plane (N*IK*L) passing through the Eye and cuting the Picture, in NL, their mutual Intersection.

DEF. D. The VERTICAL LINE, is a Right Line drawn at right angles with the Horizontal and Ground Line; cuting the Picture into two equal Parts, perpendicularly. As ED. (See Def. 11.)

DEF. E. CENTER of the PICTURE (or POINT of VIEW) is the Point C, in which the Horizontal and Vertical Lines cut each other. (See Def. 16 and 17.)

N. B. The real Point of View is the place of the Eye, where it ought to be fixed when viewing a perspective Picture. The Point C, on the Picture, opposite to it, where a Perpendicular from the Eye would cut the Picture, is, therefore, its Center; and is, generally, understood to be the Point of Sight; i. e. to which the Eye must be opposite.

Hence, if the Center of the Picture, C, be first determined (as it frequently is) then a Right Line drawn through C, parallel to the Horizon, as LM, is the Horizontal Line; and, another Line, drawn through C, perpendicular to it, as ED, is the Vertical Line.

DEF. F. PARALLEL of the EYE. If EC be taken, in the Vertical Line, equal to the Distance of the Picture; and, through the Point E (which is considered as the Eye) if a Right Line be drawn, parallel to the Horizontal Line, as IK, it is called the Parallel of the Eye, of horizontal Planes. (See Def. 9.)

DEF. G. VANISHING LINE, of any Original Plane, is a Right Line on the Picture, supposed to be produced by an imaginary Plane passing through the Eye, or Point of View, parallel to the Original Plane; the Line, in which such a Plane would cut the Picture, is the Vanishing Line of that Original Plane.

COR. Hence, the Horizontal Line is the Vanishing Line of the Ground Plane, and all other horizontal Planes.

For, it is produced by a horizontal Plane (N*IK*L) passing through the Eye, and cuting the Picture.

† Theo. 3. COR. 2. And hence, all Planes, which are parallel between themselves, (but, not to the Picture) have the same Vanishing Line †.

For, there can be but one Plane passing through the Eye parallel to them all; and since it can produce but one Line on the Picture, by its intersection with it, that Line is, consequently, the Vanishing Line of them all.

‡ Theo. 2. N. B. The Vanishing Line, and the Parallel of the Eye, of any Original Plane, are parallel to the Intersection of that Plane with the Picture ‡.

Therefore, LM, the Horizontal Line, and IK, the Parallel of the Eye, of horizontal Planes, are both parallel to AB, the Ground Line; which is the Intersection of the Ground Plane, with the Picture.

§ Theo. 1. N. B. 2. Original Planes which are parallel to the Picture have no Vanishing Line, nor Intersection §.

DEF. H. VISUAL RAY is a Right Line supposed to be drawn from the Eye to any Point in an Original Object.

Let the Picture, AIKB, be turned up into a vertical Position; and, let the Plane W be turned over, till *C* coincides with the Center of the Picture; then will *E* coincide with the Eye, or Point of View (E) if the Directing Plane, GEH † be turned up parallel to the Picture.

† Def. 4. Let the Plane AFDC, on the other Side of the Picture, be also turned over, till it falls into the same Plane with W. Then, the Right Lines EA, EB, EC, are Visual Rays from the Eye (E) to the Original Points, A, B, and C, on the other side of the Picture; which, by their Intersections with the Picture, give the Representations a, b, and c, of the Original Points, A, B, and C, on the Picture.

See the Apparatus; in which, the Threads EA, EB, EF, EG, &c. are Visual Rays; producing, by their Intersections with the Picture, the Representations, a, b, f, g, and *a*, *b*, *f*, *g*, &c. of the Original Points A, B, F, G, &c. in the Original Object.

PM and *P*M are the Ground Lines of those Pictures; VY, and CX are the Horizontal Lines; and, CL, and *O*P are the Vertical Lines. C, their Intersection, is the Center, or Point of View of each Picture; for, the Eye, considered as a Point, is opposite to either.

DEF. I. DISTANCE of the PICTURE is the length of the Perpendicular, or the shortest Line that can be drawn from the Eye of a Spectator, in the true Point of View, to the Picture; the Distance between the Eye and the Center of the Picture. As E C. (See Def. 15 and 18.)

DEF. K. INTERSECTING POINT, of an Original Line, is that, in which any Original Line (being produced) would cut the Picture.

B, P, and S, are the intersecting Points of XY, ZY, and ZX of the Three Sides of the Triangle XYZ.

In the Apparatus, *F*, *G*, and *H*, are the Intersecting Points of the Original Lines, IF, HG, and F G.

N. B. The Intersecting Point, of every Line, is in the Intersection of the Plane the Line is in, with the Picture (Theo. 10.) Therefore, *F G H*, drawn through the Intersecting Points *F*, *G*, and *H*, is the Intersection of the Plane F G H I. Also, *B G*, drawn through the intersecting Points, *B* and *G*, of the Lines A B and H G, is the Intersection of the Plane A B G H.

DEF. L. VANISHING POINT, of an Original Line, is that Point, on the Picture, in which, a Right Line, from the Eye or Point of View, parallel to any Original Line, would cut the Picture.

N, O, and L, are the Vanishing Points of the three sides of the Triangle X Y Z. C is the Vanishing Point of the Lines AC and F D; and, M and L of the Diagonals AE, BF, &c.

N. B. The Vanishing Point, of every Line, is in the Vanishing Line of the Plane it is in. (Th. 10.)

Therefore, a Right Line drawn through V and W, the Vanishing Points of I F and G F, is the Vanishing Line of the Plane F G H I. Fig. 15.

Also, R W, drawn through the Vanishing Points, Y and W, is the Vanishing Line of the Plane B F C; in which, the Lines B C, or G D, and G F, are situated.

COR. All Lines which are parallel amongst themselves have the same Vanishing Point ‡; except they are parallel to the Picture; in which Case they have no Vanishing Point (Cor. to Theo. 1.)

‡ Cor. 1. Theo. 3.

For, E V, producing the Vanishing Points, C and V, on both Pictures, being parallel to A B, is consequently parallel to G H, F I, &c §. and, since it can produce but one Point on each Picture, that Point is, consequently, the Vanishing Point of them all, by the Definition. § 4. 7. El.

Also, E W, being parallel to F G and H I, produces their Vanishing Point, W.

In the Apparatus, V is the Vanishing Point of all the Lines A B, G H, and F I: Y is the Vanishing Point of B C, and G D, &c. and W, of the Lines G F and H I; on the Picture M N O P.
The Vanishing Point of F D is out of the Picture, below the Ground Line.

In the Picture M N *O P*, C, its Center, is the only Vanishing Point; *viz.* of the Lines A B, G H, &c. to which E C is parallel. For they are perpendicular to that Picture.
All the other Lines in the Original Object, in the Plane B F C, are parallel to that Picture; and therefore, they have no Vanishing Point.
Also, B G, A H, &c. being parallel to both Pictures, have no Vanishing Point, on either.

COR. 2. Hence, the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture.

For E C, the Direct Radial, produces the Center of the Picture (Def. 17.) and, because it is perpendicular to the Picture, it is parallel to all Lines that are perpendicular to the Picture, and consequently it produces their Vanishing Point; by this Definition.

DEF. M. STATION POINT is at the foot of the Spectator; or, it is that Point, in which a perpendicular from the Eye would cut the Ground Plane.

If the Directing Plane, G H I K, be turned up, on G H the Directing Line, perpendicular to the Ground Plane; E being the Eye, E F perpendicular to G H, cuts it in F, the Station Point; and E F is the height of the Eye above the Ground Plane. Fig. 37.

DEF. N. STATION LINE is a Right Line drawn from the Station Point, in the Ground Plane, perpendicular to the Ground Line; or, it is the Intersection of the Vertical Plane †, with the Ground Plane. As F D, produced.

† Def. 5.

SECTION III.

Containing ſome preliminary obſervations, concerning the Proportion and Poſition of the Picture; of the Height and Diſtance of the Eye, &c. Secondly, How to prepare the Picture, for Practice. Thirdly, How to find Vaniſhing Lines, in all common Caſes; and to fix the Vaniſhing Points, of Lines in all Poſitions, in reſpect of each other and of the Picture.

IT may appear ſomewhat ſtrange, and unaccountable to many, of what uſe are all the imaginary and elementary Planes in delineating Objects; or how ſo many Planes, cuting each other, can poſſibly be applied, in Practice; ſeeing that, the Plane or Picture, on which the Object is delineated, is the only real Plane made uſe of. It may alſo be a matter of wonder, how a Viſual Ray can be drawn from an Object, which is beyond the Picture, to the Eye, on this Side; by means of which, it is very obvious (from the Apparatus) the Repreſentations of the ſeveral Angles, &c. in the Object, are projected on the Picture ; but how this can be effected in delineating, muſt appear ſtrange to a Novice in Perſpective. All which, are, I preſume, in the following Pages, accounted for, very ſatisfactorily.

The Art of drawing in Perſpective has this advantage of all other. It is no a random ſketch, depending on the Hand and Eye; but, every Line (Right Line at leaſt) may be drawn to the utmoſt exactneſs; and the Points, where the Viſual Rays would paſs through the Picture, are determined, mathematically true; yet may be done by a Perſon entirely unacquainted with Mathematics, who ſhall adhere to the Rules contained in the following Sections.

It is evident, ſeeing that Viſion is conveyed (from the Object to the Eye) in Right Lines, that every Section of the Pyramid of Rays will appear the ſame, to an Eye in the Vertex. Conſequently, whether the Section be made nearer to the Eye or to the Object, whether by a Plane or other Surface, whether it be direct or an oblique Section, the effect, to the Eye, is the ſame; but it is obvious, that it can be ſo only in that Point of View; for, every different Section has a different Repreſentation ; but, all parallel Sections are ſimilar Repreſentations. So that, whether the Picture be drawn by a larger or a ſmaller Scale of Proportion, the Repreſentation is the very ſame, except in Dimenſions; and ſuppoſes the Section to be made farther from, or nearer to the Eye, the Vertex of the Pyramid of Rays.

It may alſo be obſerved, that, if the Viſual Rays were produced or continued, beyond the Model, they would diverge, ſo, as to take in a real Building, ſituated exactly as the Model, of any Dimenſions: Which being premiſed, it is manifeſt, that a Picture, delineated, truly, by a ſcale of equal Parts, of any Proportion, from a real or imaginary Model, of a Building, &c. will as truly repreſent the Original, as if the full meaſures of it were applied; which could not poſſibly be done.

This conſideration may account for the Meaſures applied in Practice; which muſt always be in the ſame Ratio or Proportion, to each other, as the real Meaſures of the correſponding Parts in the Original Object.

PRELIMINARY OBSERVATIONS.

IN reſpect of the Shape and dimenſions of the Picture, no Rules can be preſcribed, it is always at the diſcretion of the Artiſt; unleſs it be proportioned to ſome particular Place which determines its Figure and Dimenſions. The oblong Rectangle is, in general, a more agreeable and convenient ſhape than a Square; about the Proportion of 3 to 2, i. e. if the length be three feet, the width may be two, or thereabout; as conveniency, for taking in the Objects, may require. Some Objects requiring it upright; others, and more generally, length-ways.

2. Neither

2. Neither can a certain and invariable Rule be given, for fixing the height of the Eye, and, confequently, of the Horizontal Line. To fix it to half or a third part of the height, abfolutely, would be ridiculous; it muft ever be at Difcretion, in proportion to the Scale of the Drawing; a Landfcape View, from an Eminence, may raife the Horizon to the middle of the Picture, or higher, yet may be very natural. In general, five feet, or five feet fix inches, the natural height of the Eye, is the moft agreeable, being moft accuftomed to fee Objects at that height; altho' it may not be, perhaps, above one fourth or fifth part of the height of the Picture. Too low, in a general View, is not agreeable, becaufe the recedings of the parts of Objects, on the Ground Plane, are not fo diftinguifhable, as, they approach nearly to Right Lines, with each other.

3. Refpecting the Diftance, fomething may be afcertained. The Diftance of the Picture is a material Circumftance which ought to be well attended to; otherwife the whole performance may be a difagreeable Diftortion, inftead of a pleafing and natural Reprefentation. The Diftance ought always to be confiderably more than the height of the Eye; although, Brook Taylor and others, in their Diagrams, have made it much lefs, which produces a very bad effect; the Reprefentations of the receding parts of the Object, on the Ground Plane, are, by that means, drag'd out to an immoderate length. To illuftrate it.

EX. Let AFEB be fuppofed the prime Vertical Plane, in which the Eye is fituated, at E. Let EC, be its Diftance from the Picture; of which, CD may be fuppofed a Section. Fig. 37. No. 2.

This, I prefume, is eafy to conceive; for, fuppofe two Planes cuting each other at right angles, one of which is confidered as the Picture; the other, a Plane paffing through the Eye, perpendicular to it. The Right Line CD being confidered as their common Section; and the Planes turned around on it, as an Axis (fee Fig. 8. Pl. 3. in the general Introduction) in which revolution, either Plane will become direct before the Eye, and the other pafs through the Eye, cuting the former in CD.

Now, E is fuppofed the Eye of a Spectator, and, C the Center of the Picture; EC is, therefore, its Diftance. Let an Object be fuppofed beyond the Picture, whofe Seat on the Ground is AD.

If a Right Line, EA, be drawn, cuting CD, the common Section of the Picture and the Vertical Plane, in a; it is obvious that, to the Eye at E, the reprefentation of the Point A will be at a, on the Picture; and the length of Ground, between the Picture and the Point A (equal DA) is reprefented by Da. For, EA is a Vifual Ray, from the Eye to the Point A; in which Direction, that Point is feen; and confequently, a is the Point in which it would pafs through the Picture.

As it may be obferved in the Apparatus, by placing the Eye in a continuation of either Picture and the Ground Plane; i. e. in the Line of their common Section, PM.

Then (confidering the Eye of the Spectator as the Point E; and, A, or any other Point, on the Ground Plane, anfwering to A, in the Figure) EA reprefents the Vifual Ray, and a, the Point where it paffes through the Picture; which is reprefented by the Right Line CD.

Now, Da reprefents the length of Ground between the Picture and the Point A, the Eye being at E; but, being moved to *E*, the Appearance of it is at *a*; and, the Point a reprefents, at that Station, the length DG only, which is prepofterous.

For, fince DC is the whole indefinite Reprefentation of DA, produced (Theo. 12.) and Da, a*a* reprefent the finite, fmall Portions DG, GA, only; confequently, the remainder, *a*C, of the indefinite Reprefentation, DC, reprefents the whole of DA, produced infinitely beyond A.

It is evident, that the Point, a or *a*, from either Station, is mathematically determined; agreeable to Theorem 13th. For, DC being the whole indefinite Reprefentation of DA (Th. 12.) the reprefentation of any Point, A, is found; by making Da (or D*a*) to DC, as AD to AB (or AD added to CE) i. e. Da : DC : : AD : AB, as it is demonftrated in that Theorem.

The Triangles AaD, AEB, and CEa being fimilar; Da : aC : : AD : CE; confequently, Da : Da+aC (i. e. DC, equal BE) : : AD : AD+CE, equal DB (i. e. AB.)

N. B. While the Eye moves in the Direction EC, the Indefinite Reprefentation DC remains the fame, and the finite Parts (Da or D*a*) of AD is continually varied †.

Alfo, whilft it moves in the Direction EA the finite part Da remains the fame, and the indefinite Reprefentation is varied. As, at e, the whole indefinite Reprefentation is D*a*, and the finite part is ftill *D*a.

† Cor. 5. Theo. 12.

Thus, it is evident, that the Height of the Eye is productive of as great Diftortion as the Diftance; but, to determine, abfolutely, in what proportion one fhall be to the other, is not poffible, as various circumftances may render all fuch Rules exceptionable. In general, the Diftance ought not to be lefs than twice the Height of the Eye (as at E) or, at the leaft, as three to two (as at E^2) but there may be a neceffity, in fome Cafes, to make it equal, or perhaps higher, for the conveniency of fhewing fome particular parts of the Object.

Pl. VIII. Mr. Kirby makes it a general Rule, for the Diſtance not to be greater than the Perpendicular of an equilateral Triangle, whoſe Side is the length of the Picture; nor leſs than half the Diagonal; which (when the Center of the Picture is in the middle) if rightly underſtood and applied, will not produce great Diſtortion, in the Repreſentations of Objects thereon.

No. 3. For, if S be the Center, or Point of View, of the Picture A N O B, the Diſtance S N, or S O, may be ſufficient, if there be no Objects near the Extremes, at A or B; but if there are, they will be diſtorted; becauſe, it is evident that the Optic Angle, under which they are ſeen, is a Right Angle, or 90 Degrees.

But, this Rule, for fixing the Diſtance, is ſometimes injudiciouſly adhered to; when the Center, or Point of View, is near either Extreme, as at M or L; in which Caſe, eſpecially if the View be internal, or have Objects ſituated near the other Extremes A or B, they will be greatly diſtorted.

Let A S or S N be taken for Radius, and, on M, deſcribe an Ark of a Circle, from E to F. It is manifeſt, that all the part beyond that Circle, towards L, being ſeen under an obtuſe Angle (which the Eye cannot poſſibly take in) will be prepoſterous.

Now, this is ſo very obvious, to a Perſon of any diſcernment, or a ſmall ſhare of knowledge in Geometry, that 'tis needleſs to expatiate longer on it; yet, this is one principal reaſon, why many Artiſts quarrel with Perſpective, and pronounce it deficient. For, not being acquainted with the Theory, they would have, on every part of the Picture, Objects repreſented as they appear; which cannot poſſibly be, if they are remote from the Center; for the reaſon given above.

Wherefore, if the Center of the Picture be judiciouſly fixed (as at C) in the middle of the Picture (reſpecting its length, only, not height) as it ought always, except in particular Caſes, to be, and C N or C O be taken for the Diſtance, there can no great inconvenience accrue, eſpecially in external Views; as there will, very probably, be nothing but Clouds repreſented, at N, or O.

The reaſon of all this is evident. For, wherever the Center of the Picture is fixed, every part of the Picture, ought, at the moſt, to come within a Circle, whoſe Radius, or Semi-diameter, is its Diſtance.

I ſhall however give one general Rule; which is, to make the Diſtance, at leaſt, equal to the length of the Picture, *EF*, the Center, C, being in the middle; but, if it be on either Side, as at M, or L, it muſt be equal twice M*F*, or *E*L, incluſive; ſeeing that, the Eye is always ſuppoſed to take into the Optic Angle, as much on one ſide of the Center as the other, every way.

4. The Poſition of the Picture, in reſpect of the Object and the Eye, is another eſſential Point to be well conſidered, and determined on. Without due regard to that, the other Preliminaries are to little purpoſe; as all the imagined caution, in the Height and Diſtance, may be rendered abortive, by that means.

The Diſtance of the Picture ought always to be regulated and governed by the real Diſtance of the Object, if it be a ſingle one; or, if there are a multiplicity of Objects, it muſt be calculated from the neareſt, intended to be repreſented.

Fig. 38. Let A B C, be the Seat or Plan of a Building, which is intended to be delineated from the Station E; the Diſtance from the neareſt part is E B.

Let D F be the Interſection of the Picture with the Ground Plane, or Horizontal Plane; and E the Station Point, or the Eye; then is E S the Diſtance of the Picture, applied cloſe to the neareſt part of the Object, A C, which is ſeen under the Optic Angle A E C.

The Diſtance E S (equal D F) is ſufficient for that Picture, i. e. for the Object A C. But, if there are more Objects, as X and Z, extended to the full length of the Picture, on either ſide, it is little enough; they being ſeen under the Angle D E F, which the whole length of the Picture ſubtends, viz. 55 Degrees, on the Ark *df*; and is the leaſt Diſtance I would ever uſe, in ſuch Caſe, when the Objects extend the full width of the Picture; although it is more than Mr. Kirby's greateſt.

ES

E S is therefore the Diftance of the Picture D F, and alfo of the Objects, or of a Plane paffing through the neareft Planes of X and Z; in which Cafe, it is evident that the Picture muft be as large as the Objects themfelves, being applied clofe to them.

If they are fuppofed real Buildings, and the Scale of proportion be determined on, *viz.* a 10th or 12th part, or any other; take E f (one third part of E S) for the Diftance, and draw d f parallel to D F; then is E f equal d f; for E f : d f :: E S : D F; i. e. they are equal, and confequently, the Objects x, y, and z, being reduced to the fame Proportion will all fubtend the fame Angles, refpectively, as the Originals. By which means the Picture may be delineated of any Proportion.

Now, if the Station (E) be determined, from which the Object A C is to be drawn, the Pofition of the Picture is alfo determined. For, E S, the Station Line, bifecting the Optic Angle, A E C or D E F, ought always to be perpendicular to the Picture; confequently, the Pofition of the Picture, D F or d f, is determined; i. e. it muft be perpendicular to E S.

For, when the Station, or Point of View, is fixed, from which we are determined to delineate the Object, is it not moft rational to fuppofe the Picture, on which the Object is to be reprefented, placed direct before the Eye? (as M N O P in the Apparatus) or is it more eligible to place it parallel to either Plane, A B G H or B F C (as M N O *P*) i. e. to A B or B C, as D *C* or B *G*, whofe Center, or Point of View, is at *C* or G. The very fuppofition of it is abfurd to the laft degree; and yet, this abfurdity is commited by every Artift who places the Point of View at either Extreme, or perhaps entirely off the Picture, as is frequent. Fig. 15. Fig. 38.

The difference muft be obvious to every Perfon who confiders it. For, the Picture being placed direct, according to D F, the Optic Angle is no more than A E C, or *a* E *c*, about 23 or 4 Degrees; but, being placed in the Pofition D *C*, the Optic Angle is, *a* E *g*, 102 or 3 Degrees; for, E *C*, perpendicular to D *C*, bifects the Optic Angle, on that Picture; as E S bifects the Angle A E C, on the Picture D F; notwithftanding, the Object, A B C, does not occupy one fourth part of the Angle *a* E *g*.

In the Apparatus, S *P*, on the Ground Plane, bifects the Optic Angle, p S t, under which the Object is feen, according to the Pofition of the Picture M N *O P*; for, the Angle, at the Station Point (S) is the fame as in the Horizontal Plane, at the Eye.

How to prepare the PICTURE for PRACTICE.

Let A I K B (when raifed perpendicular to the Ground Plane) be fuppofed the Picture; alfo, let the Directing Plane, G H I K, and Horizontal Plane, *I K* L M, be placed parallel to the Picture, and the Ground Plane. Fig. 37. No. 1.

Suppofe the Figures A D, and X Y Z, on the other fide, are to be reprefented on the Picture; and the Points a, b, c, &c. where the Vifual Rays E A, E B, &c. would pafs through the Picture, to be geometrically determined thereon.

In order to which, feveral preparatory Lines are neceffary to be drawn on the Picture; the Center and Diftance muft always be known or determined, together with the pofition or fituation of the Object, in refpect of the Picture and of the Eye, and alfo its height, above the Ground Plane.

Now, E is the place of the Eye, E C is the Diftance of the Picture; and E F is the height of the Eye; the Point C, where the Perpendicular E C cuts the Picture, is its Center. M L, the Line in which the Horizontal Plane cuts the Picture, is the Horizontal Line (Def. C) or the Vanifhing Line of horizontal Planes; and A B, in which the Picture cuts the Ground Plane, is the Ground Line (Def. B.) confequently parallel to the Horizontal Line. Thefe two are the firft and moft ufeful Lines; the Center of the Picture (C) in this Cafe, is in the Horizontal Line, the Picture being vertical.

Pl. VIII. The next Line to be confidered, and of ufe, is the Vertical Line, ED (Def. D.) If the Plane V be turned up, perpendicular to the Picture, it will pafs through the Eye, at E, and confequently through EC, cuting the Picture in EC, the Vertical Line; which, being produced will pafs through D.

This Line always paffes through the Center of the Picture, and is of the fame ufe for all vertical Planes, which are perpendicular to the Picture, as the Horizontal Line for horizontal Planes; i. e. it is their Vanifhing Line, which has, confequently, the fame Center (C) and Diftance (EC).

The conftruction of thefe five Planes fhould be well attended to, and the Lines they generate, by their Interfections; three of which, *viz.* the Ground Line, the Horizontal Line, and the Vertical Line, are already produced on the Picture; and the Parallel of the Eye, IK (whofe real place is out of the Picture) is tranfpofed to the Picture, by turning up the Horizontal Plane on ML, its Interfection, till it coincides with, or falls into the Picture; and with it the Direct Radial, or Diftance, (EC) together with the Eye (E) which falls into the Vertical Line, at E.

When the perpendicular pofition of the Planes has been confidered attentively, let them be pufhed either from or towards you, keeping the Directing and Horizontal Planes, joined in IK; the Vertical Plane (V) may ftill be fuppofed to cut them all at Right Angles, and generates ftill the fame Line on each. In all Pofitions, i. e. let the Angles they make with each other be what it may, the Planes are ftill parallel to each other refpectively; and their Interfections are ftill parallel amongft themfelves. (Theo. 2nd.)

Now, let the Picture be turned down, on AB, its Interfection with the Ground Plane, till they coincide; and let the Horizontal Plane, with the Eye (E) and the Parallel of the Eye (*IK*) be alfo turned down into the Picture (as it is reprefented on the Picture); alfo, let the Vertical Plane (V) be turned down on either fide, into the Picture, and with it the Direct Radial (EC) i. e. the Diftance of the Picture, falling into the Horizontal Line, with the Eye at *E*. And, laftly, let that part of the Ground Plane, which lies beyond the Picture (on which the Seats of Objects are geometrically drawn out) be fuppofed to be turned, on AB (its Interfection with the Picture) quite over to the other fide; and imagine it, fimply, a Plane, without thicknefs; fo that, all the Figures, defcribed on it, are feen on the other Side, inverted; as *ABEF*, and XYZ; in which Cafe, it is obvious, that they have the fame Pofition, or Situation to the Picture, as before; and the Lines (which are not parallel to the Picture) being produced, cut the Picture, in the Interfection AB, in the fame Points, as before.

Thus, is the Picture prepared for Practice, in common Perfpective; and all the elementary Planes are reduced to one Plane, viz. the Picture*.

All the fame Lines and Points, may be feen in No. 3. which is divefted of that apparent intricacy of Planes on Planes, confequently it is more fimple and intelligible; having only the preparatory Lines, anfwering to the Interfections of the elementary Planes with the Picture, on it. The Figures below AB, the Ground Line, are Plans of Objects, on the Ground Plane, intended to be delineated.

Fig. 37. No. 3. To prepare which; let the Ground Line (AB) be firft drawn, at fuch a convenient Diftance from the bottom of the Picture, and parallel to it, as to allow room, below it, for drawing the Plans of Objects (as X, Y, Z) intended to be delineated, in their true geometrical Proportion, Place, and determined Pofition to the Picture, if neceffary. The fpace, below AB (as AFB) is not confidered as a part of the Picture; but of the Ground Plane, whofe real place is beyond the Picture.

* The Directing Plane (GHIK) not being of ufe in common Perfpective is fuppofed to be turned down, or removed out of the way.

Through

Through the middle of the Picture, draw ED, perpendicular to the Ground Line, dividing the Picture into two equal Parts. This Line is the Prime vertical Vanishing Line, in which, the Center of the Picture, or Point of View, is always (though not always in the Horizontal Vanishing Line, but when the Picture is vertical, as it is now supposed to be); make CD equal to the determined height of the Eye, C is the Center; through which, draw ML, the Horizontal Line, parallel to AB, the Ground Line.

Then, with the Radius EC (the determined Distance) describe a Semicircle, cuting the Horizontal and Vertical Lines in *E*, E, and *F*; i. e. make C*E*, &c. equal to the Distance of the Picture, which are considered as the Eye, transposed to the Picture * (generally understood by the Points of Distance) and, through E, draw IK, the Parallel of the Eye, parallel to the Horizontal Line; then is the Picture prepared, having all the fixed Lines and Points determined thereon. It only remains, now, to find other Intersections, Vanishing Lines, and Vanishing Points, necessary for delineating the Objects intended.

The Ground Line, or Intersection of the Picture with the Ground Plane, is the first and principal Intersection; and the only one, in general, made use of as such. But, the Intersections of other Planes are often wanted, and particularly vertical Intersections; on which, the measures of the heights of Objects are applied; although they are frequently made use of, yet very few consider them as Intersections; and consequently, they do not see the generality of the Principles and Rules by which the Work is performed; as they look on the operation in Planes which are vertical or inclined, in a different light from such as are horizontal; whereas, if it be well considered, they will find it is the very same; for, wherever the geometrical measures are applied, in proportioning any Line, it is considered as an Intersection of some Plane, in which that Line is or may be situated. And, since the Intersection of any Plane determines the Position of its Vanishing Line, I have shewn how to find the Intersections of Original Planes, in all necessary and generally useful Cases. Inclined Planes, in general, I have reserved for the last Section; but, as various Planes, which are perpendicular to the Picture, are inclined to the Horizon, and vertical Planes are frequently inclined to the Picture, I shall not consider them, in either Case, as inclined Planes, but such only as are inclined to both.

As the Horizontal and Vertical Lines both pass through the Center of the Picture, they are, therefore, the Vanishing Lines of Planes perpendicular to the Picture † (being vertical) which, from their fixed and invariable Position, determine all other Vanishing Lines whatever.

Therefore, in the following Problems, I have, in the first place, shewn how, by them, to determine the Vanishing Lines of all Planes that are perpendicular to the Picture, either with or without the Intersection; which are subject to one general Rule.

Secondly, how to find the Vanishing Lines of vertical Planes, in all Positions to the Picture (except parallel) and also, of certain Planes inclined to the Horizon, which are subject to the same invariable Rule, as vertical Planes.

The fifth Problem shews how to find the Vanishing Lines of Planes any how inclined, both to the Horizon and to the Picture (being vertical) from their known inclination to the Horizon, &c. as specified in the Problem. All which, are frequently necessary in common Practice.

† Theo. 4.

* The Eye, or Point of Distance, may be any where in the Circumference of a Circle, whose Radius is EC, as occasion requires; or, according to the Position of original Planes, which are perpendicular to the Picture, and their Vanishing Lines; all which, have the same Center and Distance (Theo. 4th.)

 PROBLEM

Plate IX.

PROBLEM I.

The Interfecting Point of any Line, in a Plane which is perpendicular to the Picture, being given, together with the Angle of its Inclination to the Horizon; to determine its Interfection and Vanifhing Line; the Center of the Picture being given.

Fig. 39. Let I be the Interfecting Point of fome Line, given; C is the Center of the Picture; and X is the Angle of Inclination of the Plane to the Horizon.

Through C, draw AB, the Horizontal Line, parallel to the Bottom of the Picture; and, through I, draw ID parallel to AB.

Make the Angle DIF equal to the given Angle X; and, IF is the Interfection required.

Through C, the Center, draw GH parallel to IF; GH is the Vanifhing Line.

Note. The Vanifhing Line may be determined, by making the Angle ACG or BCH equal to the Inclination known; without the Interfection.

DEM. Let ID be fuppofed the Ground Line, and I the Interfecting Point of the common Interfection of the Ground Plane and the other Plane; which, feeing both Planes are perpendicular to the Picture, is alfo perpendicular to the Picture †.

† Cor. 1. 9. 7. El. And, fince I is the Interfecting Point, of a Line in the Plane, the Interfection of the Plane muft neceffarily pafs through that Point ‡; and confequently, it will make the fame Angle with the Ground Line and Horizontal Line, as the Planes make with each other (DIF, equal BLF ||) equal to the given Angle X. (See Article 4th, of Planes and their Pofitions. Page 42.)

‡ Th. 10.

|| 4. 1. El.

Therefore, IF is the Interfecting Line of that Plane.

§ Theo. 4. But, the Vanifhing Lines of all Planes, which are perpendicular to the Picture, pafs through the Center of the Picture §. And, the Vanifhing Line of every Plane is parallel to its Interfection. (Theo. 2.)

Therefore, GH is the Vanifhing Line of a Plane perpendicular to the Picture, and inclined to the Horizon in the Angle HCB equal FID, equal X.

N. B. If F be the Interfecting Point of any other Line in that Plane, and EF be drawn parallel to the Horizontal Line; the Angle EFI, equal FID (equal X) produces the fame Interfection.

Alfo, if K be the Interfecting Point of any Line in another Plane, parallel to the former; then, AK, parallel to IF, is its Interfection.

For, all parallel Planes have parallel Interfections. But they have the fame Vanifhing Line (GH).

If *E*C, or EC, be equal to the Diftance of the Picture; AK, or EF, may be confidered as the parallel of the Eye; and *E*C, or EC, (perpendicular to GH, or AB) is the Vertical Line.

SCHOL. If the Angle of Inclination be greater or lefs, the procefs is the fame. But, if the Angle be increafed or enlarged to a Right one, the Plane is no longer inclined, but vertical; and, if the Interfecting Point, J, be in a Line, paffing through C, perpendicular to AB, the Interfection and Vanifhing Line are the fame, *viz.* the Vertical Line EJ; which is the Vanifhing Line of all vertical Planes that are perpendicular to the Picture. See this Problem illuftrated by moveable Planes; Fig. 15. No. 2.

Fig. 15. No. 2. *IKLM* is the Original Plane, TU is its Interfection with the Ground Plane, or other horizontal Plane; *I* is its Interfecting Point, and *IN* is the Interfection of that Plane with the Picture; making the Angle *NI*B equal to the Angle PQR, of the Original Plane with the Ground Plane.

And, if IKLM (the Vanifhing Plane of all Planes which are perpendicular to the Picture) be placed parallel to the Original Plane, *IKLM*, it will cut the Picture in ON, the Vanifhing Line of *IKLM*, in that Pofition.

† 9. 7. El. For, it paffes through the Eye (at E) and the Direct Radial (EC); confequently it is perpendicular to the Picture †, and paffes through C, its Center.

Hence; all Vanifhing Lines, which pafs through the Center of the Picture, have the fame Center and Diftance, *viz.* the Center and Diftance of the Picture.

Note. If the Plane inclined on the other Side of its Interfection TU, the Angle of its Inclination is made on the other fide of the Vertical Line; to which, particular regard muft always be had.

PROBLEM II.

The Vaniſhing Line of a Plane being given, with its Center and Diſtance; and the Angle of inclination which any Original Line, in that Plane, makes with the Interſection, of the Plane it is in, with the Picture; to find the Vaniſhing Point of that Line, and to determine its Diſtance.

AB is the given Vaniſhing Line, and C is its Center. Fig. 40.

Draw CE perpendicular to AB, and equal to the Diſtance of the Vaniſhing Line given or found.

Through E draw GH (the Parallel of the Eye) parallel to the Vaniſhing Line. Make the Agle GEA equal to the Angle which the Original Line makes with the Interſection, cuting the Vaniſhing Line in A, the Vaniſhing Point ſought; and EA is its Diſtance, E being conſidered as the Eye.

Note. Regard muſt always be had, on which ſide the Original Line inclines, and the Angle GEA, or HEB muſt be made accordingly.

DEM. Imagine the Plane AGHB turned over, on AB, till E coincides with the Eye; then is GH in its true Place (ſtill parallel to the Picture); and EA will be parallel to the Original Line, producing its Vaniſhing Point†; and making the ſame Angle with the Vaniſhing Line (AB) and Parallel of the Eye, as the Original Line makes with the Interſection and Directing Line, of the Plane it is in||.

† Def. 22.
|| Theo. 11.

EX. The Picture, AIKB, being erected, turn over the Vaniſhing Plane, *IK*NL, parallel to the Original Plane. Fig. 37. No. 1.

Then, the Radials EN, EO, EL, &c. are reſpectively parallel to their Originals, XY, YZ, and XZ, in the Triangle XYZ; and conſequently, they make equal Angles, reſpectively, with NL, and *IK*, as the Originals make with AB, the Interſection of the Plane they are in.

PROBLEM III.

The Interſecting Point of any Line, in a vertical Plane, being given, and the Angle of Inclination of the Plane to the Picture, to find its Interſection and Vaniſhing Line; the Center and Diſtance of the Picture being given, and the Picture ſuppoſed vertical.

Let I be the interſecting Point given, and C the Center of the Picture. Fig. 40.

Through C, draw AB, the Horizontal Line, and ED the Vertical Line at right angles; and through I, the Interſecting Point given, draw IH, parallel to ED; which is the Interſection required.

Make CE equal to the Diſtance of the Picture; and make the Angle CEA equal to the Complement of the given Angle of Inclination; cuting AB in A.

Or (having drawn EG parallel to AB) make GEA equal to the given Angle; and through A, draw FG parallel to IH, which is the Vaniſhing Line required.

DEM. Becauſe the Picture and the Original Plane are both vertical, their Interſection is perpendicular to the Horizon‡; and conſequently, ſince I is the Interſecting Point of ſome Line in that Plane, IH perpendicular to FI, the Ground Line, is the Interſection; by Theorem 10th.

‡ Cor. to 9. 7. El.

And, becauſe EAC (equal AEG) is equal to the Angle of Inclination, EA is the Radial, or Parallel of the common Interſection of the vertical Plane with horizontal Planes, producing its Vaniſhing Point. (By Prob. 2.) Therefore, FG, drawn through A, parallel to IH, or ED (i. e. perpendicular to FI) is the Vaniſhing Line (Theo. 2nd).

For,

Plate IX. For, imagine the Picture FGHI turned up, vertical, and the Triangle AEC vertical to it, i. e. horizontal; reprefenting a part of the Horizontal Plane; E is the Eye.

Then, a vertical Plane, paffing through the Eye and the Line EA, will cut the Picture in FG, the Vanifhing Line, which is parallel to IH, the Interfection; by Theorem 2nd.

For, they are the Sections of parallel Planes with the Picture (Def. 8).

N. B. If the Interfecting Point of any other Line, in the Plane, had been given, as B or H, the Interfection IH would be the fame.

And confequently, if the Vanifhing Point of that Line, or any other, as F, or G, in any vertical Plane, be given or found, the Vanifhing Line is determined.

For, it paffes through that Point (Th. 10.) perpendicular to the Ground Line, from its Pofition.

Example, by the Apparatus.

Fig. 15. If the Planes ABGH and BFC were produced, they would cut the Picture MNOP, in *BK* and *BE*, the Interfections of thofe Planes; which are parallel between themfelves, becaufe their common Section, BG, is parallel to the Picture †; and, becaufe BG is vertical, they are, alfo confequently, vertical, being parallel to BG refpectively ‡.

† Theo. 5.

‡ 4. 7. El.

And, if a Plane (RSTU) be fuppofed to pafs through E (the Eye of the Spectator) parallel to BFC, cuting the Picture MNOP, produced, in RU, it is the Vanifhing Line of that Plane, parallel to its Interfection, *BE*.

Alfo, a Plane (STOP) parallel to ABGH, cuts both Pictures, in OP and *OP*, the Vanifhing Lines of that Plane, on both; parallel to RU or W, i.e. to *BK*.

When the Original Plane is inclined both to the Horizon and the Picture, having its common Section with horizontal Planes, parallel to the Picture; it is a fimilar Cafe to this; and by turning the Picture fideways, on IH, (Fig. 40.) 'tis the very fame in every refpect; as it is fully illuftrated in Fig. 15. No. 3.

No. 3. EX. *ADFB* is the Original Plane; *AB*, its Interfection with the Ground Plane, is parallel to the Picture; i.e. to AB. Let it be raifed up, making the Angle PQR, with the Horizon. AONB is the Picture, which being raifed up vertical, the Plane *K*ON*I*, paffing through the Eye, at E, will be parallel to the original Plane, *ADFB*; and cut the Picture in ON, its Vanifhing Line, parallel to the Horizontal Line, LM.

And, if the Original Plane was produced, it would alfo cut the Picture in a Line parallel to LM, or AB (i. e. to ON) below AB, the Ground Line.

But, if the Original Plane inclined towards the Picture, on this fide of *AB*; then, its Vanifhing Line would fall below the Horizontal Line, and the Interfection above it.

SCHOL. If the Interfection of an Original Plane, in any pofition whatever, be given, and the Inclination of that Plane to the Picture known, its Vanifhing Line is determined as by this Problem; feeing that, the Vanifhing Line of a Plane is always parallel to its Interfection with the Picture. Confequently, if the Interfection, be parallel either to the Horizontal or Vertical Line, the Vanifhing Line fought is alfo parallel to them.

But, when the Interfection of the Original Plane is not given; and which, by reafon of the great Diftance of the Plane, from the Picture, or Inclination to it, cannot be had, nor its Pofition afcertained; then other Expedients are ufed, to find the Vanifhing Lines of fuch Planes; *viz.* by finding the Vanifhing Points of two Lines in the Original Plane. (See Prob. 5.)

N. B. The Center of every vertical Vanifhing Line is the Point in which it is cut by the Horizontal Line; and the Center of every Vanifhing Line, which is horizontal, is the Point in which it is cut by the Vertical Line (by the 7th Theorem) the Picture being vertical.

Fig. 40. The Diftance of every Vanifhing Line which does not pafs through the Center of the Picture, is the Hypothenufe of a Right angled Triangle (as AEC) whofe Bafe and Perpendicular are the Diftance of the Picture, and the Diftance, AC or EC, between the Center of the Picture and the Vanifhing Line.

For, in refpect of the Vanifhing Line, FG, of vertical Planes, if CE be the Diftance of the Picture, EA is the Diftance of the Vanifhing Line, and A is its Center.

But, if GH be a Vanifhing Line of a Plane inclined to the Horizon, in the Angle GEA, and to the Picture, in AEC, having the fame Diftance, AE, E is its Center, and AC is the Diftance of the Picture.

PROBLEM IV.

The Vanishing Line of a Plane, its Center and Distance, being given, and the Vanishing Point of some Line in that Plane; to find the Vanishing Point of other Lines, making a given Angle with that Line, whose Vanishing Point is given.

A B is the Vanishing Line, C is its Center, and A the given Vanishing Point. Fig. 41.

Draw C E perpendicular to A B, and equal to the Distance of the Vanishing Line, given or found.

Join A E; and make A E B, or A E D, equal to the Angle, which the Original Lines make with each other, cuting the Vanishing Line in B or D, the Vanishing Point sought.

DEM. Imagine the Triangle A E B turned up, on A B, perpendicular to the Picture (if A B be considered as the Vanishing Line of a Plane perpendicular to the Picture) or, making the same Angle with the Picture, as the Plane, of which, A B is the Vanishing Line.

Then is A E B the Parallel of whatever Plane the Original Lines are in, producing its Vanishing Line A B ‡; E coincides with the Eye, and E A, E B, &c. are respectively parallel to the Original Lines, seeing they pass through the Eye and the Vanishing Points of those Lines (Def. 22.) ‡ Def. 8.

But, the Radials of two Lines producing their Vanishing Points, make the same Angle, at the Eye, as the Original Lines make with each other †. Therefore, B, or D is the Vanishing Point required. † Theo. 11.

N. B. If the Angle A E B be obtuse, it is not the Angle of Inclination of the Lines. In which Case, regard ought particularly to be had to the Position of the Original Lines, in respect of each other, and of the Picture.

For, suppose F G to be the Representation of a Line, whose Vanishing Point is A; then, if F B be drawn, A F B represents an Angle equal to A E B; consequently, the obtuse Angle is towards the Picture. In which Case, let the Angle of Inclination be made on the other Side; that is, produce A E to I, and make I E B equal to the given Angle; for if the Angle A E B (i. e. A E D) was made equal to the Angle of Inclination, the Point D (in that Case) would not be the Vanishing Point required.

PROBLEM V.

The Angle of the Inclination of a Plane to the Horizon, together with the Angle which its Intersection, with any horizontal Plane, makes with the Picture, being given, to find its Vanishing Line; the Intersection of the Plane not being given, nor its Position known. The Center and Distance of the Picture are given.

Let A B be the Horizontal Line, and C the Center of the Picture.

Fig. 42.

Draw C E perpendicular to A B, and equal to the Distance of the Picture; and, through E, draw D E parallel to A B.

Draw E A, making the Angle D E A equal to the Angle which the Intersection of the inclined Plane (with horizontal Planes) makes with the Picture, cuting the Horizontal Line in A, the Vanishing Point of the common Intersection.

Draw E B, perpendicular to A E, cuting A B in B; and, through B, draw B G perpendicular to A B, indefinite.

Make B F equal B E; and make the Angle B F G equal to the Inclination of the Original Plane to the Horizon, cuting B G in G, and draw A G, the Vanishing Line sought.

This Problem, for finding the Vanishing Lines of Planes, casually inclined to the Horizon and to the Picture, is universal, and applicable in all Cases, when the Angles are determinable; as in the Roofs of Buildings, Pediments, &c. which, being frequently necessary, in common Subjects, could not be dispensed with

Plate IX. Fig. 42. with here; otherwife, I fhould have omitted it till the laft Section, which treats more fully on fuch Subjects. I fhall therefore referve the Demonftration of it till then; where, every Cafe and Circumftance, refpecting inclined Planes, are fully demonftrated, and exemplified by moveable Planes.

N. B. The Center of the Vanifhing Line, AG, is determined by drawing CH perpendicular to the Vanifhing Line, cuting it in H, its Center†, nor does it differ, in that refpect, from any other, except in Pofition; for, CH is a part of the Vertical Line of the Original Plane.

† Th. 7.

And, if CI be drawn, parallel to the Vanifhing Line, and equal to the Diftance of the Picture, the Line IH, to the Center, is the Diftance of the Vanifhing Line AG; which, Diftance, is as applicable to that Vanifhing Line, as the Diftance of the Picture (IC) to all Vanifhing Lines which pafs through its Center, i. e. of Planes perpendicular to the Picture.

Fig. 15. EX. In the Apparatus, VW, the Vanifhing Line of the Plane of the Roof, HIFG, is determined by this Problem; V being the Vanifhing Point of the common Section, GH, with a Horizontal Plane, DGHK; and EW, i. e. the Radial or Right Line from the Eye, parallel to GF determines W, in RU produced, the Vanifhing Point of GF and HI, on the Picture MNOP.

For, *E*, in the Horizontal Line, reprefents the Eye tranfpofed to the Picture; *E*Y being equal to the Diftance of the Eye from the Point Y; and Y*E*W is equal to the Angle DGF of the Inclination of the Roof to horizontal Planes.

Wherefore, fince V is the Vanifhing Point of one Line (GH or IF) in the inclined Plane, and W is the Vanifhing Point of another Line (GF or HI) in the fame Plane; confequently, VW (a Right Line drawn through thofe Points) is the Vanifhing Line of the Plane GFIH (Theo. 10th. Cor 1.)

For, a Plane paffing through the Eye, and thofe Vanifhing Points, would be parallel to GFIH, and would cut the Picture in the Line VW; which is, therefore, the Vanifhing Line of that Plane. Def. G.

SECTION IV.

CONTAINING

The ELEMENTS of PRACTICAL PERSPECTIVE.

HAVING, in the foregoing Section, fhewn how to find and determine the Interfecting and Vanifhing Lines, of Planes, in all common Cafes; and alfo the Vanifhing Points of Lines, in any Plane whofe Vanifhing Line is given or found; by means of which Vanifhing Points, all original Right Lines (not parallel to the Picture) have their indefinite Reprefentations, on the Picture, truly and accurately defcribed. In this Section I have fhewn how to cut off certain portions, from the indefinite Reprefentation, which are the perfpective Reprefentations of certain portions, or fegments, of Lines in the Original Object.

Having well confidered, that moft regular Objects are bounded by Planes, and the bounds of Planes are Lines*; it is evident, that to find the Reprefentations of Lines, in all Pofitions, is to find the Reprefentation of the Figure, or Object, bounded or circumfcribed by thofe Lines. And, fince the extremes of Lines are Points, it follows, that, if the two Extremes, of a Right Line, be found, the whole Line is determined; and, by finding fundry Points in curved Lines, the Reprefentation of the Curve is determinable. Wherefore, the whole, of practical Perfpective, confifts in finding the Reprefention of a Point, any how fituated.

But, fince Points are the interfections of Lines, and, to find the Reprefentation of a Point, in Perfpective, it muft, neceffarily, be fuppofed in fome Line; hence it follows, that, to determine the Reprefentations of Lines, in all Cafes, is the whole fum and fubftance of Practical Perfpective.

* See Page 50 and 51. Book II. Section II.

Now, Right Lines can have but three Pofitions, in refpect of themfelves and of the Picture, *viz.* they muft be either parallel, perpendicular, or inclined; and, having learnt how to manage Lines in all thefe Cafes, by the following Problems, there remains little more to be done; for, by conftructing a number of Lines together, properly, an Object is formed.

By Theorem 12. the Indefinite Reprefentation of a Right Line, not parallel to the Picture, is a Line drawn through its Interfecting and Vanifhing Points. But, fince the Interfecting Point is not always wanted (nor is it always attainable) if any Point in the indefinite Reprefentation be determined, a Line drawn through that Point, to the Vanifhing Point, is the fame; for, it would, if produced, pafs through the Interfecting Point of the Original Line.

In this Section, which contains the whole Subftance of practical, rectilinear Perfpective, I have fhewn how the indefinite Reprefentations of Lines, in all Pofitions, are determined; and then, how to proportion them, in any given or known ratio to the Original; and afterwards, how to manage them when the Vanifhing Point is not within the limits of the Picture, by various Expedients.

Let the Reader take particular notice, that I fhall, always (to fave repetition) in the following Problems, fuppofe the Center of the Picture to be given, and its Diftance known; except, in particular Cafes, when it is otherwife expreffed.

The Diftance of the Picture (being determined) is applied, in Practice, by the fame Scale of Proportion to which the Picture is delineated.

Let it alfo be obferved, that I fhall always (in the Diagrams) make ufe of the initial Letters of the following Terms, *viz.* C for the Center of the Picture, E for the Eye, in its firft or principal place on the Picture, and *E* for its firft tranfpofed place, in any Vanifhing Line, &c. and E^2 for the next tranfpofition, &c. and V L for any particular Vanifhing Line. But, feeing that the Ground Line, the Horizontal, and Vertical Lines never vary their Places, and are always ftronger drawn than the operative Lines, I think it needlefs to particularize them otherwife.

P R O B L E M VI.

How to find the Reprefentation of a Point whofe Seat on the Picture is given, and its Diftance from the Picture known.

Let C be the Center of the Picture, and S the Seat of the Original Point. Fig. 43.

Draw a Right Line CS, through the Center of the Picture, and the given Seat, indefinitely beyond S. Draw C A, at pleafure; and SB parallel to CA.

Make AC equal to the diftance of the Picture, and SB equal to the diftance of the Original Point from its Seat. Draw AB, which will cut CS in b, the Point fought.

Or, if the Original Point be between the Eye and the Picture, make S*B* equal to its diftance as before, and draw A*B*, which produce to the Picture, cuting it in *b*. Then is *b* the projected Reprefentation, of the Point *B*, on the Picture.

Compleat the Parallelogram ACSD.

DEM. AC is equal to the Diftance of the Picture, SB to the Diftance of the Original Point from its Seat, and AC is parallel to SB (Con.) Confequently, the Triangles ACb, and bBS are fimilar. Wherefore, Sb : bC : : SB : AC; and, confequently, Sb : Sb + bC (i.e. SC) : : SB : SB + AC, equal SD (i. e. BD.) that is, Sb : SC : : SB : BD. Alfo S*b* : SC : : S*B* : *B*D. - Theo. 13.

Now, becaufe SB is the Diftance of the Original Point, from the Picture, and AC is equal to the Diftance of the Picture; draw CE and SF, both perpendicular to CS, confequently parallel; make CE equal CA, and SF to SB, and draw EF; which will cut SC in the fame Point, b.

For, the Triangles CEb, bFS are fimilar. Wherefore, Sb : bC : : SF : EC; i.e. as SB : AC.

Plate IX. Fig. 43.

Hence it is evident, that the projection of the Point b does not, in the least, depend on the situation of the Lines C A and SB, in respect of C S, but on their parallelism and proportion to each other; wherefore, if the true Distances are not known, but only their Ratio, the Point b, will be projected the same. e. g.

Take C I at pleasure; and make SG to CI, as SB to AC, and draw I G; the Point b will be projected the same, on the Picture.

For, since S G : CI : : SB : C A, and Sb : bC : : SB : C A; consequently, Sb : bC : : S G : CI, and consequently, the Point b is the same.

Hence may be seen the universality of the 13th Theorem. For, conceiving E to be the Eye, and E C the Direct Radial, i. e. the Distance of the Picture, and SF the Distance of the Original Point from its Seat, i. e. from the Picture, imagine the Triangle E C b to be turned up, on bC, till EC is perpendicular to the Picture, and suppose b F S turned back, on Sb, till S F is also perpendicular to the Picture, on the other Side; then is E in the true place of the Eye, and F is in the Place of the Original Point, and consequently, E F is a Visual Ray from the Eye to the Point; which it is evident will pass through the Picture, in the Point b; and, since Vision is conveyed in Right Lines to the Eye, the Point F will appear, on the Picture, at b; which is, therefore, the perspective Representation of the Point F. (See App.)

Again, because EC is parallel to S F and cuts the Picture in C, its Center, C is the Vanishing Point of S F; (Cor. 2. Def. L.) for, the Line S F is perpendicular to the Picture; and S is its interfecting Point; (Def. K.) wherefore, S C is its indefinite Representation †; and E F is a Visual Ray from the Eye to the Original Point; (Def. H.)

And, because EC is parallel to SF, and EF cuts them both, they are, therefore, all in the same Plane ‡, consequently, the Visual Ray, E F, will cut the Picture somewhere in S C, the Intersection of that Radial Plane with the Picture, and consequently in b; making Sb to bC, as SF (or SB) to E C (or AC) or, Sb to SC, as SF or SB, added to EC, or AC; i. e. to BD, or F H §; for, D is the Directing Point of B S, and H of F S.

† Theo. 12. Def. 25.

‡ Ax. 7.

§ Theo. 13. and Th. 11. 8. of El.

SCHOL. Thus, the whole business of practical Perspective will be found (when well understood) to consist in finding the representation or projection of a Point on the Picture, any how situated; i. e. to determine that Point, in which a Visual Ray, from the Original Point to the Eye, would cut and pass through the Picture.

For, if the two Extremes of a Right Line are found, the whole Line is determined; and curved Lines can only be represented by finding the representations of several Points in the Original Curve, and joining them carefully, by hand. I would therefore advise the young Student, to bestow the utmost attention on this and the following Problems, as they really contain the whole essence of Practical Perspective.

PROBLEM VII.

The Seats, on the Picture, of any two Points in an Original Line being given, and the Distance of the Points from the Picture; to find the Inclination of the Line to the Picture, its Interfecting and Vanishing Points; to draw its indefinite Representation, and to find the Representation of each Point.

Fig. 44.

A and B are the Seats of the Points given, and C is the Center of the Picture.

Draw A B, indefinite; draw A F and B G perpendicular to AB, and equal to the Distance of the Original Points from their Seats, respectively; and draw F G, meeting A B in the Point I.

Then is I the Interfecting Point of the Line, in which the Original Points are situated; and A I F is the Angle of its Inclination to the Picture.

Through C, draw C V parallel to A I, indefinite; draw C E perpendicular to C V, and equal to the Distance of the Picture; and, draw E V parallel to I F, cuting C V in V, the Vanishing Point.

Draw I V, the indefinite Representation of the Original Line; and lastly, draw E F and E G, or A C and B C, cuting I V in a and b, the Representations of the Original Points, F and G.

DEM. Becaufe A and B are the Seats of the Original Points, A B is the Seat of the Line they are in, and confequently, it is the Interfection of a Plane paffing through the Line perpendicular to the Picture. (See N. B. Art. 7. General Introduction, Page 44.)

Now, fince A F and B G are perpendicular to A B, i. e. to the Picture, and meafure the Diftance of each Point, refpectively, from its Seat; if the Triangle A F I was turned back, behind the Picture and perpendicular to it, F and G would be in the true places of the Original Points, in refpect of the Picture, and F G would reprefent the Original Line; which, being produced to the Picture, would cut it in I, in the Line A B, produced. — — — — 11. 7. El.

For, A F and B G are parallel Lines, and confequently, all other Right Lines which cut them both are in the fame Plane †.

Therefore, I is the Interfecting Point of the Line F G, and A I F is the Angle of its Inclination to the Picture, i. e. to its Seat, A B. † Ax. 7.

Again. Suppofe the Triangle C E V turned up, perpendicular to the Picture, it will be parallel to A F I; for C V is parallel to I A, and, A F I and C E V are both perpendicular to the fame Plane.

Now C E is perpendicular to the Picture, and, fince it is equal to the Diftance, E is the true place of the Eye; and becaufe E V is parallel to I F, and cuts the Picture in V, V is the Vanifhing Point of I F *; feeing they make equal Angles with the Picture; (E V C equal A I F; by Theorem 11th.) * Def. 22.

But, I is the Interfecting Point of F G, and V is its Vanifhing Point; wherefore, I V is its Indefinite Reprefentation; by Theorem 12th.

And, E F, E G, are Vifual Rays, from the Eye (E) on this Side, to the Points F and G, on the other fide of the Picture, which would cut and pafs through the Picture at a and b.

For, E V is parallel to I F, and they are both cut by I V and E F; wherefore, they are all in the fame Plane ‡; and I V is the Section of that Plane with the Picture, becaufe, the Points I and V are both in the Picture; the Line F G cuting it in I, and E V in V, (Def. K and L.) ‡ Ax. 7 & 8.

Now, fince E F and E G are alfo in the fame Plane, they muft cut the Picture, fomewhere, in the Interfection (I V) of the Plane they are in, confequently in a and b.

For, the Triangles V E a and a I F, V E b and b I G are fimilar; and therefore, I a : a V : : I F : E V as F a : a E; and, I b : b V : : I G : E V; as by Theorem 13th.

Secondly. A C and B C alfo determine the fame Points, a and b.

For, A F and B G are Lines perpendicular to the Picture; therefore, the Center of the Picture is their Vanifhing Point §; E C being parallel to them ||; alfo, A and B are their Interfecting Points; therefore, A C and B C are the indefinite Reprefentations of A F and B G; by Theorem 12th. § Cor. to Theo. 4.

But, the Reprefentation of the common Section of two Lines, is the Point in which the Indefinite Reprefentations cut each other. (Cor. 7. Theo. 12.) || Def. 22.

Therefore, a, the Point in which I V and A C cut each other, is the Reprefentation of F; and b of G, in which the Lines B C and I V cut each other.

Alfo, becaufe C E is parallel to A F, the Triangles C E a, a F A are fimilar; wherefore, A a : a C : : F a : a E : : A F : E C, i. e. : : I a or I F : a V, or E V. Q. E. D.

Or, if the Diftance of the Points F and G, from the Interfecting Point, are transfered to D and H; and the Diftance of the Vanifhing Point, E V, be fet off, from V to *E*; *E* D and *E* H, being drawn, will give the fame Points; that is, they will cut I V the fame, in a and b.

For, the Ratio of I a or I b, to a V or b V, is ftill as I F or I G, to E V; i. e. as I D or I H, to *E* V.

Alfo; if A C, or B C, be confidered as the indefinite Reprefentation of a Line (B G) in which the Point G is fituated; make B K equal B G, and C E^2 (on the contrary fide) equal C E; K E^2 will determine the fame Point, b, &c.

N. B. The Seat of a Point, on any Plane, is the Point where a Perpendicular from the Original Point cuts the Plane, and confequently meafures its Diftance. See Ichnog. and Orthog. Sect. II. Page 46.

SCHOL. The Diftance of any Point, in an Original Object, from the Picture, may be obtained when the Interfecting Point, of the Line it is in, and its Diftance from the Interfecting Point, cannot, for various reafons; and, fince it is demonftrated (Theo. 13.) that the Diftances of feveral Points in a Line, refpectively, from the Picture, are in the fame Ratio as their Diftances from the Interfecting Point of that Line; confequently, having their Seats on the Picture, and Diftance from their Seats, refpectively, their Reprefentations, on the Picture, may be determined without the Interfecting Point. (Cor. 1. Th. 13.)

SCHOL. 2. If the Original Points are in a Plane which is perpendicular to the Picture, the Seats of thofe Points are in the Interfection of that Plane with the Picture.

Wherefore,

Plate IX. Fig. 44. Wherefore, if F and G are two Points in a Line on the Ground Plane, the Seats of thoſe Points are in the Ground Line (I D) where the Perpendiculars F A and G B cut it. A and B are the Seats; and their Diſtances A F to B G, are as the Diſtance of the Point F is to the Diſtance of G, from the Interſecting Point of the Line F G, where it meets A B produced.

COR. I D being the Interſection of a Plane in which the Line FG is or may be ſituated, V C is its Vaniſhing Line (Theo. 2. and 10.) and, becauſe it paſſes through C, the Center of the Picture, it is, conſequently, the Vaniſhing Line of a Plane perpendicular to the Picture (Theo. 4.) whoſe Diſtance is E C.

COR. 2. If the Interſection, (I D) and Vaniſhing Line (C V) of any Plane, with its Center, (C) and Diſtance, (E C) be given, and any Line (F G) in the Original Plane; its Interſecting Point (I) is found by producing the Line; and its Vaniſhing Point (V) by drawing E V parallel to FG; or, by making the Angle E V C equal to A I F. (Theorem 11.)

PROBLEM VIII.

The Repreſentation of a Line and its Vaniſhing Point being given or found, to divide the Line, perſpectively, in any known Proportion.

Fig. 45. A B is the Line given, and V is its Vaniſhing Point.

Draw A H, at pleaſure, and EV parallel to A H.

Make A F, F G, and G H in the ratio, or known proportion, of the Original.

Draw B H, and produce it, through B, cuting V E, in E; and draw E F, and E G, cuting A B in C and D.

Then is A B divided, in C and D, perſpectively, as A H is divided; in F and G.

DEM. If V E be conſidered as the Vaniſhing Line of a Plane in which the Original of A B is ſituated, A H may be conſidered as its Interſection; and E F, E G and E H repreſent parallel Lines †; for they are all in the ſame Plane, whoſe Vaniſhing Line is E V, and E is their Vaniſhing Point.

† Cor. to Theo. 3.

Wherefore, ſince A B H is a Triangle, and C F and D G are ſuppoſed parallel to B H; conſequently, A B and A H are divided in the ſame Ratio, in the Points C, D, and F, G‡.

‡ 2. 6. El.

N. B. If a B had been drawn through the other Extreme (B) inſtead of A H, and a B divided, in the ſame Ratio as AH, that is, in the given Proportion; A a being drawn, cuting E V in E, and E f, E d, produced, will cut A B in the ſame Points, C and D.

Any other Line (A *H*) being divided in the ſame Ratio as A H, and *E* V being drawn parallel to A*H*; *E H* drawn, through B, and *E F*, *E G* alſo drawn, will give the ſame Points C and D, which may be demonſtrated in the ſame manner.

SCHOL. By this Problem, any Right Line in any Object, ſketched by hand and depending on the Eye, may be divided, with certainty, in the proportion of the Original, being known, no regard being had to what Plane the Line is in, nor its Vaniſhing Line.

By the ſame means, a Line drawn in Perſpective may be biſected, as A D, or C B, by making A F, F G, or F G, G H equal; conſequently, if the Parts A F, F G, and G H, are all equal, A B is triſected, perſpectively.

COR. If A V be the indefinite Repreſentation of a Line, and A C repreſents ſome certain portion of the Original; and it is required to cut off another Portion, as C D or C B, in a known Ratio to that repreſented by A C.

Draw A H, and V E parallel to it, at diſcretion; aſſume any Point (E) in E V, and draw E F, through C, cuting A H in F.

Make F G, or F H, to A F in the known proportion of the Original, and draw E G, or E H; which will cut off C D, or C B, in the proportion required.

2 COR.

COR. 2. If A C reprefents a certain portion, and it is required to cut off, from any other Point, as D, a part (D B) which fhall be to A C in a certain Ratio.

Proceed as before, and, from any Point (E) in E V, draw E F, E G through C and D. Make G H to A F in the known proportion, and draw E H cuting A V in B, the Point fought.

PROBLEM IX.

The Vanifhing Line of a Plane being drawn, and the Reprefentations of two Lines in that Plane; if one of the Lines be divided any how, how to divide the other in the fame Ratio.

Let A B and C D be the given Reprefentations, of which, let A B be divided, any how, in E; it is required to divide C D in the fame Proportion. Fig. 46.

G H is the Vanifhing Line of the Plane they are in.

At any Diftance, at difcretion, draw a d parallel to G H; and, from any Point (G) in the Vanifhing Line, draw G A, G E, and G B, and produce them till they cut a d, in a, e, and b.

From the fame Point (G) or any other, in the Vanifhing Line, as H, draw H c, H d, through C and D.

Divide c d, in f; as a b is divided, in e; i. e. make c f to c d, as a e to a b †.

Draw H f, which will divide C D, in F, in the fame Ratio as A B is divided.

† Prob. 32. Geometry.

This Problem may be performed as well by one Point (O) as by two; if the Line, *C D*, be fo fituated, as to come within compafs. From which conftruction it is evident, that A B and *C D* reprefent equal Lines, as well as being equally divided, *a b* and c d being equal, and the two Lines having the fame Vanifhing Point.

For, they reprefent parallel Lines, and they are both cut by parallel Lines, *a* O, *b* O, c O, &c. at equal Diftances, *a b* equal c d, &c.

Or, it may be done by drawing two Lines, A I and C K, which are parallel to G H, through either Extremes (A and C) of the given Lines; by which Conftruction, the Demonftration evidently appears from the laft; feeing that, A I, and a b, or *a b*; C K and c d are all divided in the fame Ratio.

After the fame manner, C D may be divided, perfpectively, into as many parts as A B is divided into, and in the fame ratio to each other, refpectively.

The general utility of this Problem is much the fame as the other; the former being, how to divide a Line, perfpectively, in the proportion of the Original; and this fhews, how to find the Proportion of the Original from the perfpective Reprefentation; by which means, any other Line may be divided the fame.

In the former, the Vanifhing Line was not neceffary, only the Vanifhing Point of the Line, which may be in any Plane whofe Vanifhing Line would pafs through V, its Vanifhing Point, as V E or V *E*. But, in this Cafe (the Lines having different Vanifhing Points) the Vanifhing Line, of the Plane they are in, muft be had; and, it muft alfo be obferved, that both Lines are in the fame Plane, or the Operation cannot be performed by one Vanifhing Line.

PROBLEM X.

The Vanifhing Line of a Plane being given, and the Reprefentations of two Lines in that Plane, having different Vanifhing Points, to cut off, from any Point in one of them, a portion equal to that reprefented by the other, or in any known Ratio.

The different pofitions or fituations of the Lines, to each other, may make this Operation appear very different; for which reafon I fhall give it varioufly, of which, the firft is according to Brook Taylor.

Plate X. Fig. 47. Let A B be a given Reprefentation of a Line, and F G the indefinite Reprefentation of another Line. N M is the given Vanifhing Line, and C its Center.

It is required to cut off (from the Point F) a portion, which fhall be to that reprefented by A B in a known Ratio.

Produce A B to its Vanifhing Point, D; and, through F, draw A H, to the Vanifhing Line, cuting it in H.

Draw C E perpendicular to the Vanifhing Line and equal to its Diftance, and join E D and E G, by Right Lines.

Make E K and E L in the Ratio required; i. e make E K to E L as the Original-Line, reprefented by A B, is to the other; and draw E M, or E N, parallel to K L.

Draw B H and F D, interfecting in I; draw I M, or N I, cuting F G, in O or P.

Then, F O, or F P, reprefents a Line having that Proportion, to the Line which A B reprefents, as E L to E K.

DEM. If N M be confidered as the Vanifhing Line of a Plane perpendicular to the Picture; imagine the Triangle N E M turned up perpendicular to the Picture; or, if the Plane in which the Original Lines are fituated, be inclined to the Picture, let N E M be fuppofed parallel to it; then, E is in the true place of the Eye.

Now, E D and E G are the Radials of A D and F G, producing their Vanifhing Points, confequently they are parallel to them, refpectively; † and make the fame Angle (D E G) at the Eye, as the Original Lines make with each other. (Cor. 1. Theo. 6.)

† Def. 22.

And, becaufe E M is parallel to K L, D, G, and M are the Vanifhing Points of the three Sides of a Triangle (E K L) the Radials, E D, E G, and E M being parallel to them, refpectively.

But, the three Lines, F I, F O, and I O, vanifh in thofe Points, refpectively; confequently, F I O reprefents a Triangle fimilar to E K L; the Angle D F G reprefents the Angle D E G (Prob. 4.) and confequently, I M reprefents a Line parallel to E M; i. e. to K L.

Therefore, the other two fides, F I and F O, or F P, have that Proportion to each other, perfpectively, as E K to E L.

But, becaufe A B and F I, A F and B I have the fame Vanifhing Points, D and H, refpectively, A B I F reprefents a Parallelogram (Cor. 1. Theo. 3.) confequently, F I reprefents a Line equal to the Original of A B; ‡ and therefore, F O, or F P, reprefents a Line which has that Proportion to the Original of A B, as E K to E L.

‡ 15. 1. El.

N. B. If the fituation of the Line A B was fuch, that a Right Line, joining A and F, was parallel to the Vanifhing Line N M, then B I muft be drawn parallel alfo.

But, if a b, the given Line, be fo fituated, that the Line a F is fo much inclined to the Vanifhing Line as not to reach it within the compafs of the Picture, take any Point (J) in the Vanifhing Line, and draw J a, J b, indefinite; draw A D, at difcretion, cuting them in A and B, and proceed as before.

For, A B and a b having the fame Vanifhing Point (D) alfo, A a and B b have the fame Vanifhing Point (J) a b B A reprefents a Parallelogram; and confequently, A B, a b, reprefent equal Lines.

Fig. 48. Cafe 2nd. Let the given Lines (A B and F P) be fo fituated as to crofs each other; and, it is required to cut off, from the Point (I) of their Interfection (or any other) a part (I F or I P) equal, or in any Ratio to the Original of A B.

Draw A c parallel to the Vanifhing Line (M D) indefinite; and, from the Point H, or any other, at difcretion, in the Vanifhing Line, draw H I and H B, cuting A c in c and b; then, A b is to b c in the Ratio of the Original of A B to B I §.

§ Prob. 8.

Make a c equal to A b and draw a H, cuting A B in d; d I reprefents the fame meafure as A B, *viz.* a c equal to A b.

Produce A B to its Vanifhing Point (D) and F I, to H; if it be not already done.

C being the Center of the Vanifhing Line, draw C E perpendicular to H D, and equal to its Diftance; and draw E D, E H.

Make E K, E L, equal, or in the Ratio required, and join K L; to which, draw E G parallel, giving the Vanifhing Point G.

Draw G d, and produce it, cuting I F in F; then, if E K be equal E L, I F reprefents a Line equal to the Original of d I, that is, of A B, as it was required; or, whatever Ratio E K has to E L, F I reprefents the fame to A B.

Again. Draw m n, through I, parallel to A c, cuting a H in m.

Make I n equal I m; draw n H cuting A D in O; and draw O G, cuting F H in P.

I P reprefents a Line equal to I F, equal A B; i. e. to their Originals; or in the Ratio of E K to E L, whether equal or otherwife.

DEM.

DEM. Becaufe a m and F I have the fame Vanifhing Point (H) they reprefent parallel Lines; and, becaufe m n is parallel to a c, and to the Vanifhing Line H D, a m l c reprefents a Parallelogram; confequently, the Originals of m I and a c are equal.

But, I n was made equal I m, and n H reprefents a Line parallel to a H; confequently, I O and I d reprefent equal Lines. (See Problem 8th.)

And, fince O P and F d have the fame Vanifhing Point (G) they alfo reprefent parallel Lines §; and confequently, I O P, I d F reprefent equal Angles †; wherefore, they are fimilar Triangles, for the Angles P I O, d I F, are equal ‡. § C. 1. T. 3. † 4. 1. El. ‡ 2. 1. El.

But, they alfo reprefent congruous Triangles, for I P : I F :: I O : I d :: P O : d F; and I O reprefents a Line equal to I d; confequently, I P reprefents a Line equal to I F, and P O to d F.

If it had been required to cut off, from the Point J, in F H, a portion, in a certain Ratio to A B; draw B J.

If B J be parallel to H D, draw D J cuting Ab in a; then J a reprefents an equal Line as A B.

But, if B J be not parallel to the Vanifhing Line, produce it to its Vanifhing Point, and proceed as by Fig. 47, ufing J a for A B, as I d for A B, before.

N. B. If K L was parallel to the Vanifhing Line, then P O and d F would alfo be parallel; for E G would be parallel; and confequently, could not produce a Vanifhing Point.

Suppofe it was required to cut off, from the Point a (Fig. 47.) in the Line a F, a part, equal, or in any known Ratio, to the Original of A B.

Draw A a till it cuts the Vanifhing Line in J, and draw J B; alfo draw a D to the Vanifhing Point of A B. Then a b reprefents an equal meafure as A B, and a F may be cut in any Ratio to a b, as in the next Cafe; which is the 3rd Example, Prop. 16, of Brook Taylor, mentioned in the Preface to this Book.

Cafe 3rd, Let A C or C B be the given Line. It is required to cut off from the Point C, a part, which reprefents an equal meafure as the other. Fig. 49.

D and G are the Vanifhing Points of the two Lines, and E is the Eye.

Draw E D and E G, and bifect the Angle D E G, by the Line E H.

Draw A H, which will cut C G in the Point B, as required; or C A in A.

This Cafe, or this pofition of the Lines, is but another Example of this Problem, which Brook Taylor calls, finding the Reprefentation of a Circle from the reprefentation of one Radius given. It is certain that the whole Circle may be completed, from a Radius given, though not by this Cafe only, but by both, as I fhall exemplify.

Let A D and F G be drawn indefinite; C, their Interfecting Point, is the reprefentation of the Center, and C B is the Radius given, in the Pofition by Brook Taylor.

He fays, "make C A to reprefent a Line equal to that reprefented by C B (by the 15th)" *viz.* by the firft Cafe of this; "i. e. bifect the Angle E O D;" and, to make it ftill lefs intelligible, in his Diagram, the Angle D E G (i. e. E O D, in his Treatife) is not bifected, nor trifected, but is cut, very nearly, in the Ratio of 2 to 5; which negligence, in a Perfon of his fagacity, is to me furprizing.

It is demonftrated in the 3rd of the 6th of El. that, if any Side of a Triangle be divided in the Ratio of the other two Sides, a Right Line joining the oppofite Angle and the point of Section will bifect that Angle. But, there may, very juftly, be an exception to this; for, it is neceffary that the greater Segment be contiguous to the greater Side, and confequently the leffer Segment to the leaft Side.

In the Triangle D E G, if D G be divided, in H, in the Ratio of D E to E G, then, E H bifects the Angle D E G; becaufe G H, the greater Segment, is contiguous to the greater Side, E G; which, it is obvious, could not be otherwife.

The Conftruction, for proof of the Affertion, is to produce either Side, as G E, and make E J equal to E D, and join J D; which is proved to be parallel to E H.

For, becaufe E D = E J the Angle E J D = J D E (9. 1. El.) and, becaufe D E G is equal to them both (10. 1.) and D E H = H E G (Con.) J D E = D E H (Ax. 4.) therefore, E H is parallel to J D (4. 1.)

But, J E is the Radial of C B, produced, and E D of A C; and E J is made equal to E D; alfo, E H is a Right Line from the Eye, parallel to J D (i. e. to K L, as in the former Cafe) producing the Vanifhing Point of the Line A B (i. e. of I O, and F d, in the two former Cafes); by means of which Vanifhing Point (H) C A and C B are cut, reprefenting equal portions of thofe Lines.

Hence may be feen the affinity between the two Cafes, in bifecting the Angle D E G.

For

Plate X. Fig. 49. For E J=E D, i. e. E K to E L, and E H is parallel to K L; consequently, A C B represents a Triangle similar to K E L, or J E D; and consequently, D, G, and H are the Vanishing Points of the three Sides A C, C B and A B; which is Isosceles, by Construction.

And, since D C G represents an Angle equal to D E G (Prob. 4.) consequently, A C G represents an equal Angle to D E J †; therefore, the Sides, containing the external Angle D E J, are, in this Case, made in the ratio (E K to E L) of the Originals, which A C and C B represent; and since they are equal, consequently, A C and C B represent equal Lines.

† 1. 1. El.

Now, C A and C B represent equal Lines, from the same Point C; wherefore, if C be considered as the Center of a Circle, in Perspective, C A and C B are Radii of the Circle; and this is all that Brook Taylor has done towards finding the whole Representation; which is far from being sufficient.

In B C produced, make C F represent a part equal to what C B represents, and C I to C A, or C B; then, B F, and A I represent Diameters of the Circle.

Proceed as in Case 1st. by making E *K* equal to E L, and draw E N parallel to *K* L; draw A N and B N, cuting B F and A D in F and I, the Points sought.

Or, having drawn A N, only, cuting B F in F; draw F H, cuting A D in I.

The four Points, A, B, I, and F, are all in the Circumference of a Circle, whose Center is C; for, C A, C B, &c. represent equal Lines.

But, these four Points are not sufficient, for compleating a Circle in Perspective.

Draw H C and N C indefinite, and cut off, from the Center (C) C a, C b, C i, and C f, representing also equal measures, to those represented by C A, &c. e. g.

Make E M equal to E L; join M L, and draw E O parallel to M L.

Also, make E *L* equal E *K*; and draw E P parallel to *K L*.

Draw A O, O I, B P, and P F, cuting the indefinite Lines in a, i, b, and f.

The Angle N E H being bisected, by E Q, shews the Affinity to the first Case.

Through the eight Points, A, a, B, b, I, i, F, and f, an Ellipsis may be described, which will be the representation of a Circle in Perspective; from the given representation of one Radius, A C or C B.

PROBLEM XI.

From three Points given, in the Circumference of a Circle, to find the Representation of the Circle; having the Vanishing Line of the Plane the Circle is in, and the place of the Eye.

Fig. 50. A, B, and C are the three given Points, L M is the Vanishing Line, and E is the place of the Eye. E S is the Distance of the Vanishing Line.

Produce A B and C B, cuting the Vanishing Line in I and K; draw E I and E K.

Make the Angles K E L and I E M each equal I E K, producing the Points L and M, in the Vanishing Line.

Draw A K and A M, C L and C I, intersecting in D and F; the Points D and F are in the Circumference, which passes through A, B, and C.

DEM. Because the Angles K E L, K E I, and I E M are equal (Con.) and E is the Eye, the Angles L D K, K B I, and I F M represent equal Angles (Prob. 4.) and consequently A D C, A B C, and A F C also represent equal Angles ‡.

Therefore, those Angles touch the Circumference §; for they are in the same Segment, or stand on the same Ark, A b C.

‡ 2. 1. El.
§ 10. 3. El.

Now, here are five Points, A, D, B, F, and C, in the Circumference; but they are not sufficient for describing the true Curve of the Ellipsis.

Therefore, draw A H at pleasure, cuting the Vanishing Line, in H.

Make the Angle H E J equal I E K, and draw C J, cuting A H in G, which is another Point in the Circumference of a Circle.

Thus may as many Points be found in the Circumference as are neceffary to defcribe the whole Curve; which, paffing through all the Points, A, G, D, B, &c. will be an Ellipfis; for, it is the Reprefentation of a Circle in Perfpective, feeing that thofe Points are all in its Circumference.

If there be too much Space between any two Points, to defcribe the Curve with greater accuracy, another Point may readily be found; as a, or b, between A and C.

Draw a B, or b B, at pleafure, cuting the Vanifhing Line in S, or O. Join S E, or O E, and make the Angles S E H and S E N, or O E P, equal to I E K, cuting the Vanifhing Line in H, N, or P, refpectively.

Draw H D, or N F, cuting S a in a; or P F, cuting O b in b; which Points, a and b, are alfo in the Circumference.

For, becaufe A D and C B have the fame Vanifhing Point (K) alfo, A B and C F having the fame Vanifhing Point I, the Originals of the Arks, D B and B F, are each equal A C †; for A D and C B reprefent parallel Lines ‡; confequently, B a D or F, and B A D or F, or B b F, reprefent equal Angles §, each being equal to the Original of A B C; i. e. I E K.

† Cor. to 10. 3. El.
‡ Cor. 1. Theo. 3.
§ Cor. 2. 9. 3. El.

Othewife; draw M C, indefinite; produce M E, and make the Angle Q E R equal to I E K; E R, being produced, will cut the Vanifhing Line, fomewhere, if it be not parallel to it.

If E R be parallel to the Vanifhing Line, draw A a alfo parallel; or to the Point R, in which E R would cut the Vanifhing Line, cuting M C in a.

For, the Angle Q E R + R E M = two Right Angles †. And, the Angle M a R (i. e. A a C) reprefents the obtufe Angle M E R; and, A B C reprefents an Angle equal to Q E R.

But, the oppofite Angles of every Quadrilateral, infcribed in a Circle, are equal to two Right Angles ‡.

Confequently, A a C reprefents an Angle in the oppofite Segment; for, A a C and A B C are oppofite Angles, in the Quadrilateral A B C a.

† 1. 1. El.
‡ 11. 3.

There is not, perhaps, in the whole Book, a more elegant Problem than this, which induced me to give it a place, and to perfect it. Its utility is not fo great; yet it may frequently be applied, by thofe who do not care to be confined ftrictly to the Rules of Perfpective, in every refpect. For, having, by any means, obtained the reprefentations of three or more Points, which are known to be in the Circumference of a Circle, the whole may, readily and accurately, be determined; if they know the Diftance of the Picture, and the Vanifhing Line of the Plane of the Circle.

Brook Taylor has made a lame affair of this elegant Problem, notwithftanding his Principles are the fame; by reafon of the fhort Diftance he has taken for the Eye, and the prodigious Dimenfions of the Circle, it is the moft diftorted and prepofterous Diagram in the whole Book. He finds no more than one Point, on each fide, and leaves his Readers to find out the reft; nor does he go about it in a proper manner, there being no occafion (if the Diftance was fufficient) to draw another Line (d O) at pleafure, in order to make another Angle, equal to D O E; but, to make equal Angles with E O and D O, on either fide, continually. One thing I am much furprized at; he fays, "make the Angle d O e equal to D O E; or, having made an Inftrument, containing the Angle D E O, turn it round the Center, O, till it comes into the Pofition d O e," &c. which, is fo ungeometrical, that I could fcarce conceive it to be the expreffion of fo great a Man.

Having had occafion, in this Problem, for a Vanifhing Point (J, or R) which was not within the compafs of the Picture, the next Problem fhews how to determine the Diftance of fuch Vanifhing Points from the Center of the Picture, or Vanifhing Line, and alfo from the Eye.

PROBLEM XII.

The Vanifhing Line of a Plane being given, and the place of the Eye, with a Line, from the Eye, inclining to the Vanifhing Line; to find the Center of the Vanifhing Line, and to determine the Diftance of that Point, in which the inclined Line would cut the Vanifhing Line, from the Center, and from the Eye.

Plate X. Fig. 51. AB is the Vaniſhing Line given, and ED is the inclined Line from the Eye, at E. Db is conſidered as the bounds, or limits of the Picture, and B is the Point in which ED, produced, would cut the Vaniſhing Line, AF, produced.

Draw EC perpendicular to the Vaniſhing Line, cuting it in C, its Center.

Take CG any part of CE, a half, a third, or fourth, as ED is leſs or more inclined to AB, and draw GF parallel to ED.

† 2. and 4. of 6. El. Then, as CG : CE : : CF : CB †; that is, if CG be a third part of CE then CF is a third part of CB, and GF of EB; or, in whatever Ratio CG is of CE.

Or, drawn EA perpendicular to ED. cuting the Vaniſhing Line in A.

Then, find a third Proportional to AC and CE (Prob. 31. Geo.) it will be CB.

For, AEB is a Right Angle (by Conſtruction) and EC is perpendicular to AB;

‡ 7. 6. El. therefore, as AC : CE : : CE : CB ‡;

Alſo as AC : AE : : CE : EB; i e. EB is a fourth Proportional to AC, CE, & AE.

In Numbers, they are thus determined, by a Scale of equal Parts.

§ 9. 6. El. Firſt. Take AC and CE by the Scale; ſquare CE, and divide the Product by AC, which, will give CB §; for AC, CE, and CB are three Proportionals.

† ſame. Or, if the Triangle CFG be uſed, multiply CF by CE, and divide the Product by CG; the Quotient will be CB †. For, CG, CF, CE, and CB are four Proportionals.

Secondly. Since AC : CE : : AE : EB, conſequently they are four Proportionals. Wherefore, multiply CE into AE, and divide the Product by AC, the Quotient will be EB, the Diſtance of the Vaniſhing Point, B, from the Eye.

If the Diſtance of the Picture be known (equal CE) and the Angle of the inclination to the Picture, of one Side of a right angled Object, be determined; the Vaniſhing Points, and their Diſtances, are determinable.

Let AB be the Horizontal vaniſhing Line, and C the Center of the Picture.

Draw CE perpendicular to AB, and equal to the Diſtance given, or determined.

‡ Theo. 11. Make the Angle CEA equal to the Angle given (or to its Complement, according to which ſide the Line inclines) cuting the Vaniſhing Line in A, the Vaniſhing Point of one ſide ‡; from which all the reſt are determinable.

§ 4. 6. El. Or, if the Diſtance, CE, be ſo great, that it cannot be laid down on the Picture (as is frequently the Caſe) take CG half, a third, a fourth, or any portion of CE, and proceed as before; making the Angle CGH equal to CEA, the given Angle. Then will CH be half, a third, &c. of AC, or whatever portion CG was taken of CE §; by which means, the diſtance of the Vaniſhing Point A, from the Center, C, is aſcertained.

Make HGF a Right Angle, i. e. draw GF perpendicular to GH, cuting the Vaniſhing Line in F; then will CF be alſo half, a third, &c. of CB, the diſtance of the other Vaniſhing Point (B) from the Center (C.) Alſo, GF will be the ſame portion of EB.

☞ Thus may the real Diſtances, be found, and the place of the Eye tranſpoſed to the Picture, as it will be exemplified in Practice, when their real places cannot be had thereon; all which may be found arithmetically, as follows.

Let the Diſtance of the Picture, CE, be given, equal 6, 5 (Feet, or what you pleaſe) and let the Vaniſhing Point A be determined as above, or at diſcretion, on the Picture; *viz.* AC equal 2, 6, from which all the reſt may be determined.

First; to find the Diſtance of the Vaniſhing Point, A, from the Eye.

† Prob. 3. 7. 6. El. ‡ 20. 1. Square AC and CE, which being added together, the ſquare Root of that Product will be AE †; for AE ſquare is equal to AC added to CE ſquare ‡.

§ Prob. 3. 7. 6. El. Secondly; to find the Diſtance of the other Vaniſhing Point B, from the Center.

Square CE, and divide that Product by AC, the Quotient will be CB §.

For, AC : CE : : CE : CB. Alſo, AC : AE : : AE : AB.

Thirdly; to find the Diſtance of the Point B from the Eye.

Having obtained AE, by the firſt, multiply CE by AE, and divide that Product by AC, the Quotient will be EB; for AC : CE : : AE : EB.

The Diſtance of each Vaniſhing Point, A and B, from the Eye (E) are as neceſſary to be had as their places on the Picture, the uſe of which I ſhall exemplify and explain in its proper place.

Firſt. A C and C E being given, to find A E; the Diſtance of A.

Square A C, *viz.* 2,6;	alſo, ſquare C E, *viz.* 6,5	A E = 7
multiplied by 2,6	multiplied by 6,5	7
156	325	ſquared 49
52	390	
6,76 Square of A C.	42,25	
	Square of A C added 6,76	
	Sum of both)49,01(7, ſq. Root; = A E.	

Secondly. To find C B; A C and C E being given.

Square C E; 6,5
6,5

divide by A C = 2,6)42,25(16,25 = C B

Thirdly. To find E B.

Multiply C E = 6,5
by A E = 7

divide by A C = 2,6)45,5(17,5 = E B.

Or; having found C B (by the 2nd) add the two Squares, of C E and C B, into one Sum; the ſquare Root, of which, will be E B. (By 20. 1. El.)

When the two Vaniſhing Points, A and B, are known, to find the Diſtance of the Picture, C E; the Center being given.

Multiply C B by A C, and extract the ſquare Root of that Product, it will give C E the Diſtance of the Picture; by Prob. 4. 7. 6. El.

To find the Diſtance, C E, geometrically; A and B being given, the vaniſhing Points of Lines at right Angles with each other, and C, the Center.

Draw C E, perpendicular to A B; and, having biſected A B, on F (the point of biſection) with the Radius A F, deſcribe the Ark A E, cuting C E in E. C E is the Diſtance required, of the Picture, or of the Vaniſhing Line, A B.

For, A C : C E : : C E : C B; conſequently, A E B is a Right Angle; by 7. 6. El.

PROBLEM XIII.

To draw a Line to a Vaniſhing Point which is not on the Picture,* its Diſtance from the Center being given, or ſome Line tending to the Point; the Vaniſhing Line of the Plane it is in, being alſo given.

Let A B be the given Vaniſhing Line, and C its Center.

Let I be the Interſecting Point of ſome Line; or the Repreſentation of any other Point in the Line, given or found. Fig. 51. No. 1.

It is required to draw a Right Line, from the Point I, tending to B, which is out of the Picture; by the known Diſtance of the Vaniſhing Point B, from C, the Center of the Vaniſhing Line. D b is the limits of the Picture.

* By the Vaniſhing Point not being on the Picture is meant, only, that it does not fall within its preſcribed Bounds or Limits. But the Picture may be imagined to be produced, as occaſion may require, ſo that, the Vaniſhing Points are always ſuppoſed to be on the Plane of the Picture.

Take

Plate X. Fig. 51. No. 1.

Take C F equal half, a third, or any other equal part of C B.

Draw C I, and F J parallel to C I. Make F J : C I : : F B : C B; i. e. make J F equal half, two thirds, or the ſame Complement of I C, as F B is of C B.

Draw I J which will tend to the Point B.

Or; make C K to C I, as C F is to C B; i. e. if C F be a third part of C B, make C K one third of C I. Join K F, and draw I J parallel to K F.

DEM. Firſt. In the Triangle C B I, becauſe J F is parallel to I C, C F : F B : : I J : J B. - 2. 6. El. Conſequently, C F : C F + F B (equal C B) : : I J : I J + I B (equal I B) i. e. as J F is to I C. 2. and 4. 6. Therefore, I J B is a Right Line; for C B I is a right lined Triangle.
2nd. In the Triangle C B I, becauſe C K : K I : : C F : F B (Con.) K F is parallel to I B. Q. E. D.

2ndly; by a Right Line tending to the Point B; the Diſtance not being known.

Let I J be a Line tending to the Point B, in the Vaniſhing Line, A B.
It is required to draw, from the Point a, a Line tending to the ſame Point.

Draw a C and a I, at pleaſure, making an Angle I a C, cuting the Vaniſhing Line in any Point (C) and the given Line at I. Draw I C.

Take any Point (K) in I C, and draw K F parallel to I J, cuting the Vaniſhing Line in F. Draw K L parallel to I a, and join L F.

A Right Line, a b, drawn through a, parallel to L F, will tend to the Point B.

Note. The Point is given at a and *a*, both without and between the given Lines; and the proceſs would be the ſame if the Point was given on the other Side of the Vaniſhing Line; or any other Line, inſtead of the Vaniſhing Line, which tended to the ſame Vaniſhing Point.

DEM. In the Triangles I a C, I *a* C, becauſe K L is parallel to I a, C L : L a : : C K : K I. 2. 6. El
But, C K : K I : : C F : F B; for K F is parallel to I B, by Conſtruction.
Conſequently, C L : L a : : C F : F B; therefore, L F is parallel to a B. Q. E. D.

Or, the ſame thing may be done in another manner.

No. 2. Let A B and C D be two given Lines; E is a Point given, through which it is required to draw a Line tending to that Point, in which A B and C D would interſect.

Draw A C, at pleaſure (through E) and B D, parallel to A C, at diſcretion.

Draw either Diagonal, as A D; draw E F parallel to C D, and F G to A B, cuting B D in G; a Right Line, drawn through E and G, will tend to the ſame Point with A B and C D.

DEM. In the Triangle A D C, becauſe E F is parallel to C D, the Sides A C and A D, are cut proportionally, in E and F ‡; i. e. AE : EC : : AF : FD; and becauſe F G is par. to A B, AF : FD : : BG : GD; wherefore, BG : GD : : AE : EC; and conſequently, A B, E G, and C D will tend to the ſame Point.
2ndly. When *E* falls without the two Lines. Becauſe *EF* is parallel to C D, A C : C*E* : : A D : D*F*; and becauſe *FG* is parallel to A B, B D : D*G* : : A D : D*F*, i. e. as A C is to C*E*; therefore, &c.

‡ 2. 6. El.

Hence it appears, that nothing more is required than to find a fourth Proportional. For, having drawn a Right Line (A C) at pleaſure, through the given Point, E, cuting any two Lines which tend to the ſame Vaniſhing Point; and, at a proper Diſtance, another (B D) parallel to the former (the farther off the better) B D (or L M) being divided, in G, as A C is divided in E; then, if a Right Line be drawn through the Points E and G (or g) it muſt neceſſarily tend to the ſame Point; for Right Lines, proceeding from the ſame Point, cut parallel Lines proportionally §. This method is the moſt eligible, becauſe it is the readieſt.

§ Cor. to 6. 6. El.

There are various ways of performing this; of which, thoſe by Brook Taylor are very ingenious, and do not require any parallel Lines to be drawn.

No. 3 & 4. Let A B and C D be two given Lines, tending to one Vaniſhing Point (P.)

It is required to draw a Line, through E, which ſhall repreſent a Line parallel to the Originals of A B and C D; and conſequently, it muſt tend to the ſame Vaniſhing Point; ſeeing they are not parallel.

First, when the Point (E) is between the two Lines.

Through the Point E draw two Lines, at pleasure, cuting the two given Lines, in A, G. C, and F; and, through A, C, and F, G, draw two Lines, meeting at a. No. 3.
Draw a H and a B, at pleasure, cuting the given Lines in I, H, B, and D; join B I and DH, interfecting at K.
A Right Line drawn through E and K will tend to the same Point, with AB & CD.

2nd. When the Point *E* is situated without the two given Lines, A B and C D. No. 4.
Draw *EA* and *EF* at pleasure, cuting the two Lines, B*F* and *CI*, in *C*, *A*, *F*, and *G*; join *AG* and *CF*, interfecting at a.
Draw any other Line, B*K*, indefinite; and, at *D*, where it cuts *CI*, draw *DH*, through a; also, draw B*I*, through a.
Lastly, through *H* and *I*, draw *HK*, cuting B*D* produced, at *K*.
A Right Line, drawn through *E* and *K*, will represent a Line parallel to the Originals of *AF* and *CI*; and consequently it will tend to the same Van. Point, P.

Although Dr. Taylor has given Demonstration of all the Problems, previous to this, he has not favoured us with a Demonstration of it. But a little consideration will make it very obvious, on inspection of the Figure.

In No. 3, because AB and CD have the same Vanishing Point, they represent parallel Lines; and if a be considered as the Vanishing Point of AC, FG, &c. they, consequently, represent parallel Lines.
Wherefore, AFGC and IHBD represent Parallelograms; and consequently, E and K, the Interfections of their Diagonals AG, GF, &c. represent the Centers of those Parallelograms †. † 16. 1. E.
And since the Parallelograms are between the same Parallels, their Centers are, consequently, equally distant from each; therefore, EK represents a Line parallel to the Originals of AB and CD.

In No. 4; because B*F* and *CI* represent parallel Lines, the Lines *AG*, *CF*, &c. which pass through the Point a are all cut proportionally in that Point; wherefore, the Originals of B*H* and *ID*, of *AF* and *CG* are in the same Ratio respectively, i. e. the Originals of B*H* : *DI* : : *AF* : *CG*; consequently, *EA* : *EC* : : *EF* : *EG*, i. e. as *KH* : *KI*; for, *KH* : *KI* : : *HB* : *ID*; i. e. as *AF* : *CG*; *viz.* as *EA* is to *EC*. N.B. Cor. 1. 2. 6. El.
Wherefore, since *EA* : *EC* : : *KH* : *KI*, consequently, *EC* : *CA* : : *KI* : *IH*; or, as any other Line drawn from *K*, whose Original is parallel to *EA*, would be cut, by B*F* and *CI*.
But, if *KH* be supposed parallel to *EA*, and *AH* to *CI*, *CAHI* represents a Parallelogram; wherefore *IH* is equal *CA*; consequently, *IK* represents a Line equal to *EC*; and consequently, *EK* represents a Line parallel to *CI*. For, *ECIK* also represents a Parallelogram.

5. If it be required to draw several Lines to a Vanishing Point which is out of the Picture, from various Points, in a given Line.
Let A, B, and C, be three Points in the Line, AC; let DE and FG be two Lines which tend to the same Vanishing Point, P. No. 5.

At any Distance, at discretion, draw HL parallel to AC, and draw AH, DI, BK, and CL parallel to either Line (as FG) cuting HL, in the Points H, I, K, & L.
Assume any Point (G) in the same Line, FG, and draw GI, cuting DE in E; and, through E, draw ac parallel to AC and HL; lastly, draw GH, GK, and GL, cuting ac, in the Points a, b, and c.
Then, if Right Lines Aa, Bb, and Cc are drawn, they will tend to the same Point, P.

For, considering the Originals of AC and HL to be parallel to the Picture, and FG as the Vanishing Line of some Plane; because AH, DI, &c. are parallel to FG, and Ha, IE, &c. have the same Vanishing Point (G) they are all in parallel Planes, of which FG is the Vanishing Line; and, because ac is parallel to AC and HL, the Original of ac is parallel to their Originals, and consequently may be in the same Plane with either (Ax. 5.) wherefore, Aa, DE, &c. represent parallel Lines, in the same Plane, AacC, or in parallel Planes AHa, DIE, &c. of which FG is considered as the Vanishing Line; and consequently, they vanish in the same Point in that Line.

There are various other Expedients might be given in this Section; but, I shall omit them, till they occasionally occur in the Work; when they will be more intelligible and better understood; and, being immediately applicable, in Practice, they will, at the same time that we acquire them, shew their use; by which means, they will be deeper rooted in the Mind, and the application of them, in future Examples, will be more familiar.

SECTION V.

Of the PRACTICE of PERSPECTIVE, refpecting PLANE FIGURES.

IN the laft Section, I have exemplified and illuftrated the Elements of Practical Perfpective, according to Brook Taylor; which, notwithftanding they are fo excellent in themfelves, and of great utility in Practice, yet they do not lay a foundation whereon to begin; but teach, only, how to proportion one Part by another, either given or found, on the Picture; fo that, a Novice, in thefe matters, cannot poffibly apply them to real ufe. Nor, in my Opinion, would any Perfon ever be made a Practitioner, from that Treatife, unlefs he was endowed with an extraordinary Talent, and a very comprehenfive Capacity; being quite converfant in Geometry, and particularly acquainted with the Doctrine of Proportion; having a clear Idea of Planes and their Interfections with each other, and of Right Lines cut by Planes.

Before I proceed to Figures, I have fhewn in three Problems, how to find the perfpective Reprefentations of Right Lines, in the three Pofitions, parallel, perpendicular, and inclined to the Picture; having the Interfection and Vanifhing Line of the Plane they are in (their Diftance from the Picture being known and the Pofition of the inclined Line; that is, the Angle of its inclination to the Picture, or to the Interfection, and its interfecting Point, or the feat of fome Point in it; or the Reprefentation of fome Point in the Line.) How to proportion them, that is, to cut off fuch Portions, or Parts, as are the true perfpective Reprefentations of certain Parts in the Originals, (in which the whole foundation of Practical Perfpective confifts) is contained in the laft Section (Prob. 8th, 9th and 10th) and are exemplified in this.

Prob. 6th fhews how to find the reprefentation of a Point on the Picture, any how fituated; its Seat on the Picture and its Diftance from its Seat being given; which, in reality, contains the whole; as it will be found hereafter. I fhall, in the next Problem, give a more familiar and introductory Leffon, how to find the reprefentation of a Point, fituated on the Ground Plane; which is, undoubtedly, the firft Plane to be confidered. At the fame time, let it be obferved, that there is not the leaft difference, in the Operation, between the Ground Plane and any other, whofe Interfection and Vanifhing Line is given, and its Diftance known; as it will be fhewn.

Having, therefore, in the preceding, and in the next four, Problems, given the Elements of the whole, and fully demonftrated it; I fhall not trouble the Reader with the Demonftration of every Operation, in the following Work; but only in particular Cafes, which may not readily be deduced from the foregoing; as it would only fwell the Work to an enormous bulk, and would be of no ufe to the Practitioner. Therefore, where I fee occafion, I fhall refer to the elementary Problems, or Theorems, for Demonftration, and to fhew how each particular Problem is applicable in Practice, in various Cafes, in the courfe of the Work.

Let it, here, again, be obferved, that whenever any Point, Line, or Figure, is given in the Original Plane (the Interfection of the Plane being alfo given) it is fuppofed to be fo fituated on the other fide of the Interfection, as it would be, if the Original Plane was turned over, on the Interfection, to the other Side of the Picture, making the fame Angle with it; and (if it be not perpendicular) inclined towards the fame Part.

See Fig. 37; the Triangle X Y Z, on the other Side of the Picture, is inverted, on this Side, for conveniency in Practice.

PROBLEM

PROBLEM XIV.

To find the Reprefentation of a Point, fituate on the Ground Plane, or other horizontal Plane*, its real Place being given thereon; the Interfection of the Plane it is in being given.

There are feveral ways to find the reprefentation of a Point.

Let A be an Original Point in the Geometrical Plane, of which, BD is the Interfection; C is the Center of the Picture, and CE its Diftance; E is the Eye. Plate XI. Fig. 52. No. 1.

Through C, draw *E*F parallel to BD, the Interfection; then is *E*F the Vanifhing Line of the Original Plane; which being horizontal, confequently, *E*F is the Horizontal Line, or Vanifhing Line of the Ground Plane.

Now, fince BD is the Interfection of the Original Plane, and *E*F is its Vanifhing Line, if the Original Plane was produced infinitely, beyond the Picture, its whole perfpective Reprefentation would be between its Interfection and Vanifhing Line; confequently, the Reprefentation of any Point, Line, or Figure, in that Plane (beyond the Picture) muft be fomewhere between thofe Lines; to find which obferve the following Rules. (See Introductory Preface; Page 104.)

Find S, the Seat of the given Point; i. e. draw AS perpendicular to the Ground Line or Interfection, cuting it in S; which is the Seat of the Point A; becaufe it is in a Plane perpendicular to the Picture.

Draw SC, and AE cuting it, in a, the Point fought. (See Problem 6.)

For, EA is a Vifual Ray from the Eye, at E, to the Point A, which would pafs through the Picture at a, making Sa to aC, as AS to CE†; therefore, a is the Reprefentation of the Point A. † Theo. 13.

METHOD 2nd. On C defcribe a Semicircle, with the Radius CE, cuting the Horizontal Line in *E* and F; which are the tranfpofed places of the Eye (E) to the Vanifhing Line. (See Fig. 37, No. 1, the Vertical Plane (V) being turned down, on either Side, into the Picture.)

Therefore, having made C*E* or CF, on either Side, equal to the Diftance of the Picture, make SB, or SD, equal to SA, and draw D*E* or BF, both which will cut SC in the fame Point, a, as before.

For C*E* is parallel to SB or D, and, C*E* is to SB or SD, as CE to SA, *viz.* as the Diftance of the Picture, to the Diftance of the Original Point from its Seat, i. e. from the Picture. (See Prob. 6.)
If CE^2, and S*F* be drawn perpendicular to SC, or any how, parallel between themfelves, it is ftill the fame. For *F*a is to aE^2, as Sa is to aC, as before.

Note. This Method is the moft eligible of all other; for, the Diftance of the Picture being known (as it is always fuppofed to be) and being placed on either Side of the Center, and SB, or SD, equal to the Diftance of the Point, on the other Side, *E*D or B*F*, is a Vifual Ray from the Eye to the original Point, as EA before.

N. B. If the Original Point be fituated in the Station Line, as at *A*, the firft method cannot be applied; becaufe its Seat, *D*, being alfo in the fame Line, *D*C and *EA* are one Line, and cannot interfect; in which Cafe, the Eye being tranfpofed to *E*, on either Side, and *A* to *B*, then is *EB* a Vifual Ray, from the Eye, on one fide of the Picture, to the Point *A* (i. e. *B*) on the other Side (fuppofing the Vertical Plane, in which the Vifual Ray muft be, turned on C*D*, its Interfection with the Picture, direct before the Eye) in its true, geometrical proportion, or length.

* If the Interfection and Vanifhing Line, with its Center and Diftance, be given or found, of any Plane whatever, whether perpendicular or inclined to the Picture, the Procefs is the fame, in every refpect.

If

Plate XI. Fig. 52. No. 1. If f be the Seat, on the Picture, of a Point in any other Plane, (suppose Vertical) then fG being drawn, parallel to CE, is its Intersection; and, having drawn fC, make fG equal to its Distance, and draw GE, cuting fC in g, the Representation of G; i. e. of a Point situated beyond the Picture at the Distance fG.

Thus the Representation of any Point may be obtained, knowing its situation and Distance from the Picture.

No. 2. METHOD 3rd. Through the given Point, A, draw AB and AD, at pleasure, cuting the Intersection in B and D.

From E, the Eye, draw EF and EG, parallel, respectively, to AB and AD, cuting the Vanishing Line in F and G, the Vanishing Points of AB and AD. (Def. 1.)

Draw BF and DG, intersecting in a, the Representation of A.

For, BF is the indefinite Representation of BA, and DG of DA; consequently, the Point, in which they intersect, is the Representation of the Point in which the Lines BA and AD intersect †.

† Cor. 7. Theo. 12.

N. B. The 2nd Method will be found, on inspection, to be the same as this; for, if E*E* and AD be drawn, they will be parallel, and EC is parallel to AS; also C*E* is parallel to SD; consequently, C and *E* are the Vanishing Points of AS and AD respectively. So that, to find the Representation of a Point is to find the Representation of a Line; seeing, it must be supposed in some Line.

No. 3. METHOD 4th is rather a matter of curiosity than real use, being seldom, if ever, practised; it may be performed without either Intersection or Vanishing Line; nothing more being required than the Place of the Eye, and the Original Point; their Distances from the Picture being known, or the Ratio of their Distances.

On A the given Point, with the Radius AS, its Distance fram the Intersection, or from the Picture, describe an Ark, BSD; and, on E, with the Radius EC, the Distance of the Vanishing Line, or of the Picture, describe the Ark FCG.

Draw the Tangents, BF and DG, interfecting in a, the Representation of A.

Or, if the Radius be taken, in the ratio of the Distances, respectively, it will be the same, i. e. the Tangents will cut each other in the same Point, a.

DEM. Draw AB and AD, EF and EG perpendicular to the Tangents, BF and DG.

Then, the Trapezium ABaD is similar to aFEG; for aB=aD and aF=aG; (C. 2. 16. 3. El.) also AB=AD, and EF=EG; the Angles at B, D, F, and G are Right, and BaD=FaG (2. 1. El.) consequently, BAD=FEG. (See Th. 1. 10 1. El.)

Wherefore, Ba:aF::AB:EF; i. e. as AS.EC; or, as Aa is to aE (which are corresponding Diagonals) as in the first Method (No. 1.) for, AE is a visual Ray, in both, and a is the Point in which it cuts the Picture.

PROBLEM XV.

To find the Representation of a Line perpendicular to the Picture; its Place being given in the Original Plane, the Intersection and Vanishing Line of the Plane it is in, and the Place of the Eye.

Fig. 53. Let AB be the Original Line; AD is the Intersection of the Plane it is in, and *E*C*E* is the Vanishing Line; E is the place of the Eye.

Draw EC perpendicular to the Vanishing Line, cuting it in C, the Center; and, because AB cuts the Intersection, A is its Intersecting Point.

Draw AC, and BE cuting it in b; then is Ab the Representation of AB.

Or (having drawn AC,) make C*E* equal to the Distance of the Picture, and A*B* equal to the Original Line (AB) and draw *BE*, cuting AC, in the same Point, b. In which process, the measures, it is obvious, must always be placed on contrary Sides of the indefinite Representation, or the Visual Ray cannot cut it.

3 DEM.

Plate VIII.

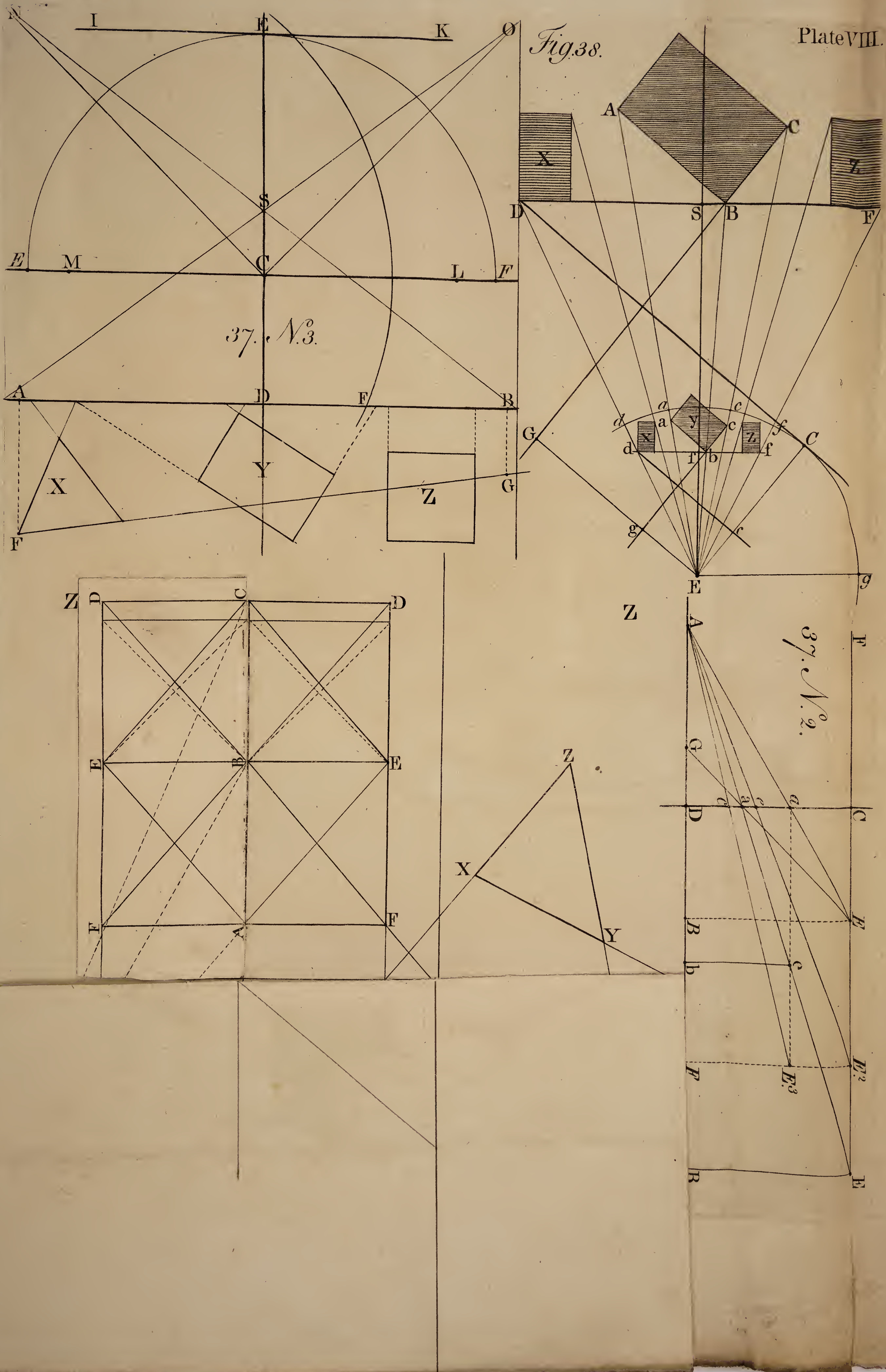


Plate IX.

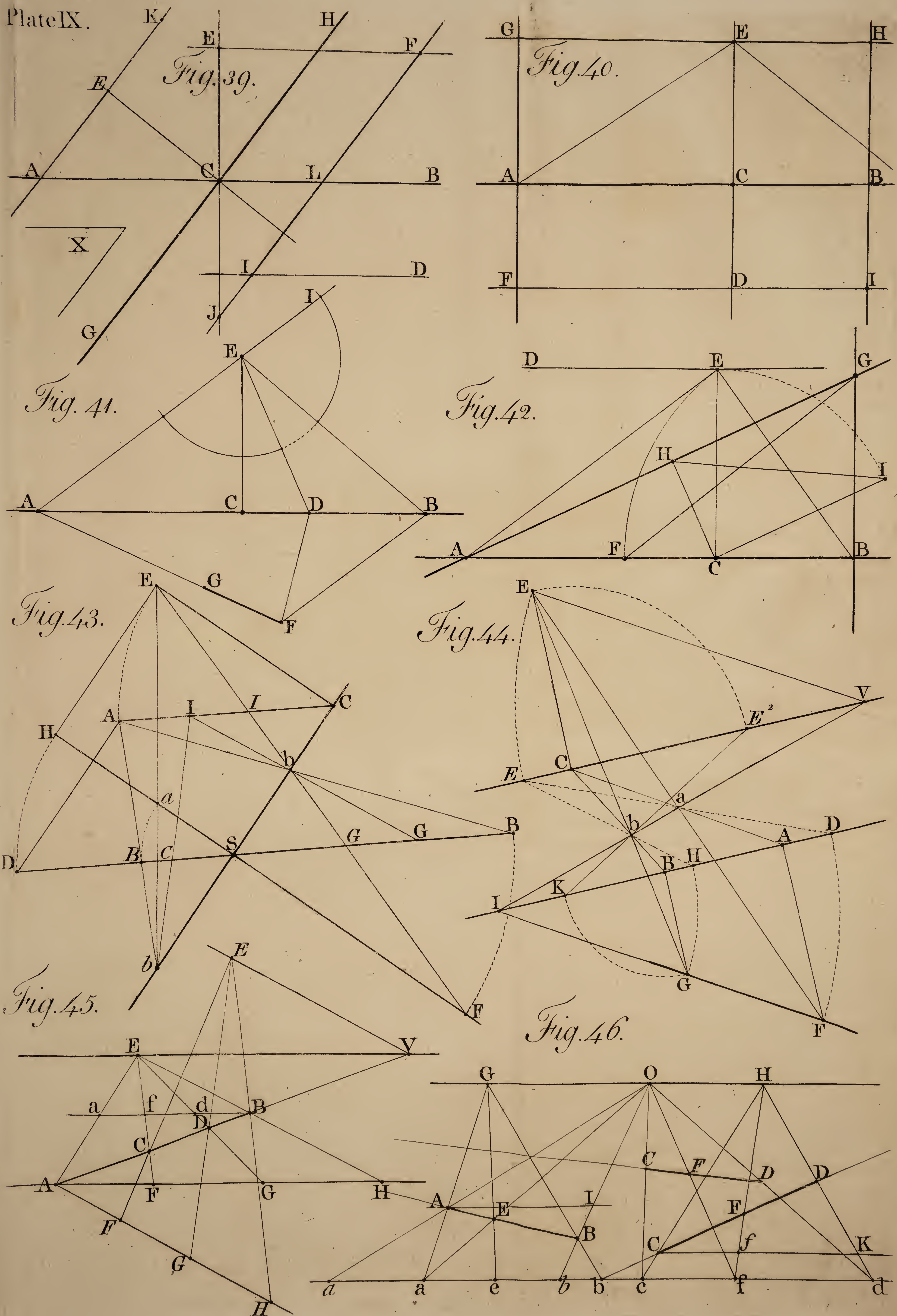


Plate X.

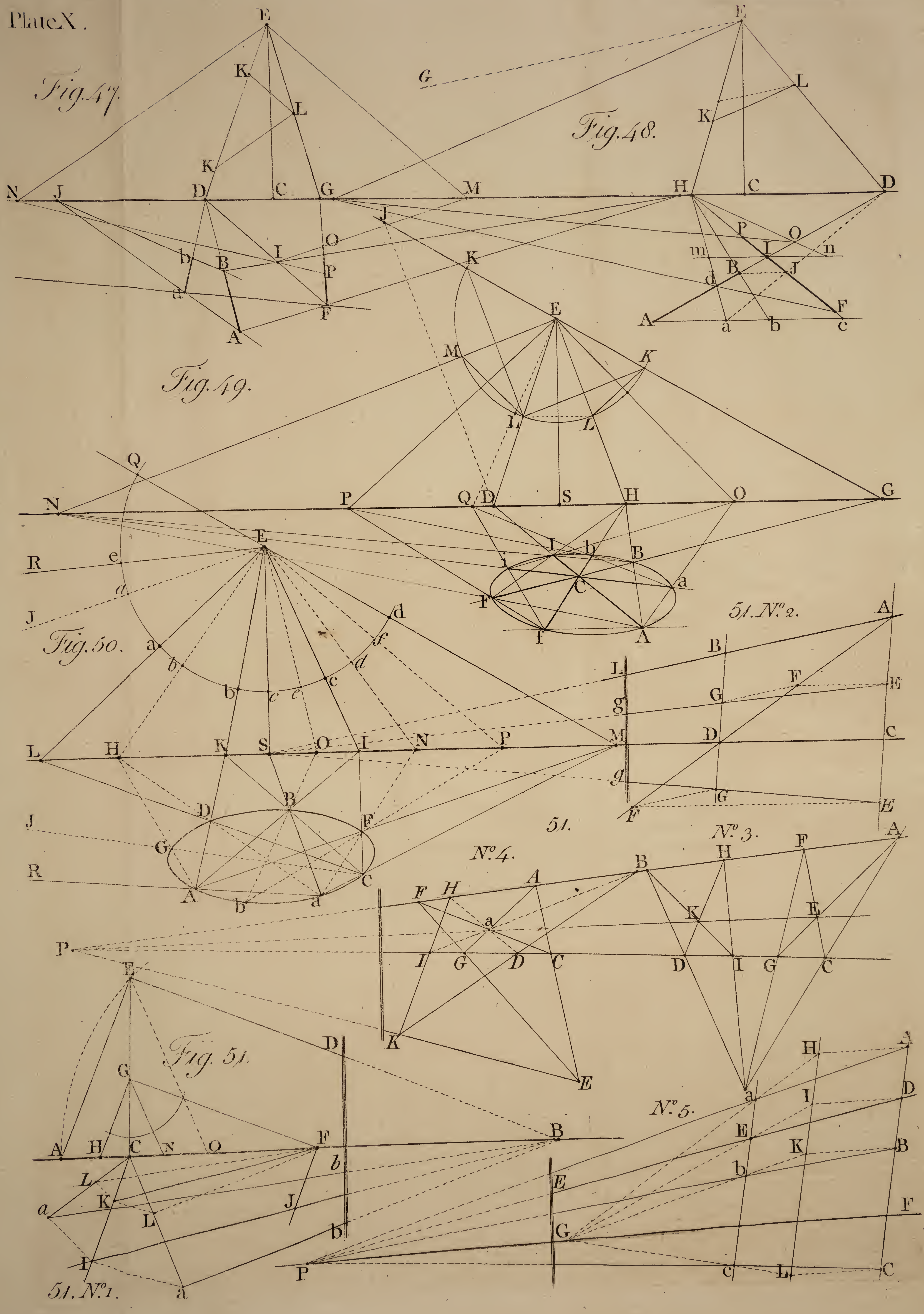
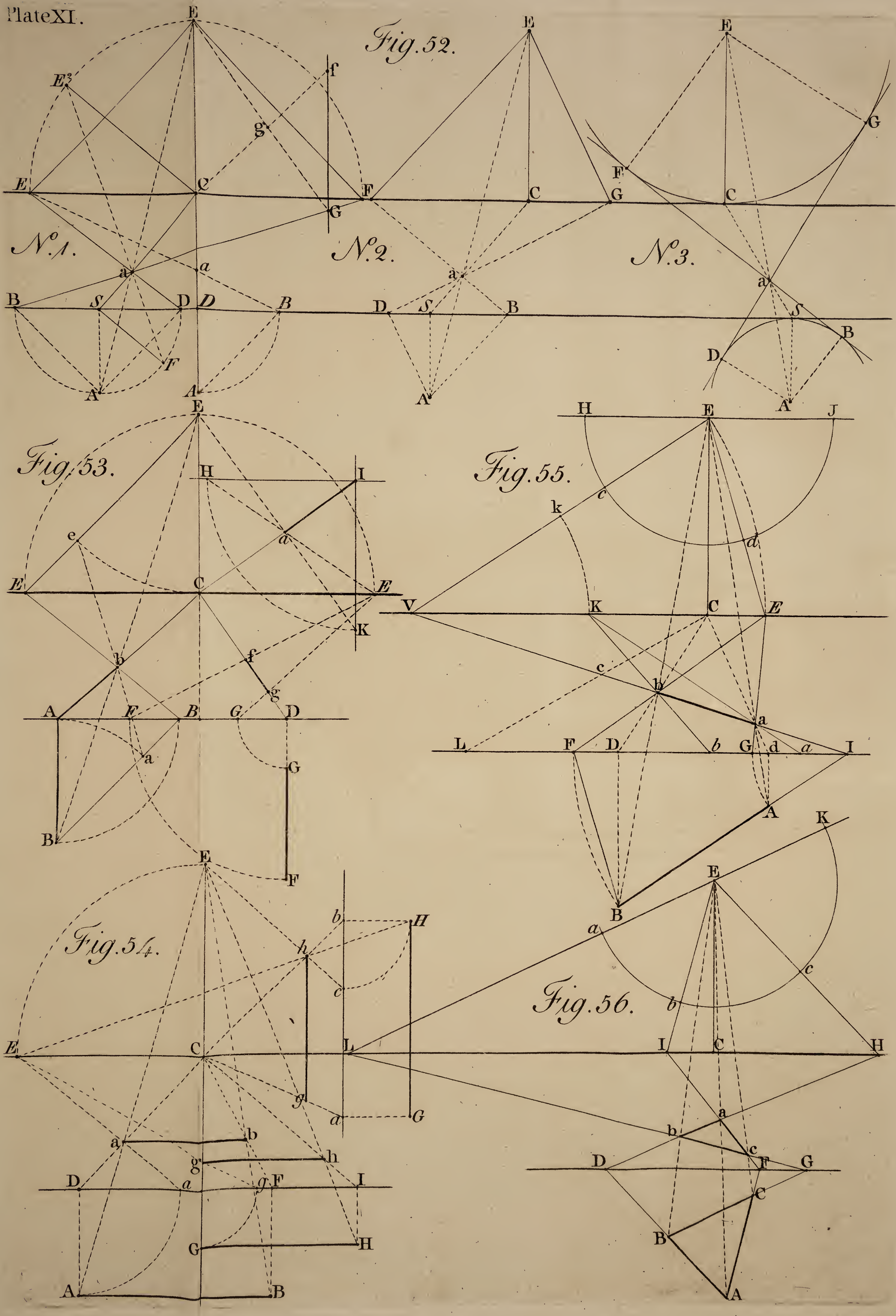


Plate XI.



DEM. Becaufe AB, the Original Line, is perpendicular to the Picture. EC, the Direct Radial, is its Radial; wherefore, C, the Center of the Picture, is its Vanifhing Point †; and, becaufe A is its Interfecting Point, AC is the indefinite Reprefentation of AB, infinitely produced (by Theorem 12th) † Def. 22.

But, EB (or *EB*) is a Vifual Ray, from the Eye (E) to fome Point (B) in the Line; which muft cut the Picture fomewhere in the Line AC; which is the Section of a Radial Plane, paffing through the Original Line and the Eye ‡; and confequently it will cut it in b; making Ab:bC:: AB:CE. ‡ Theo. 8.

Therefore, fince A, one extreme of the Line AB, is in the Picture, and b is the Reprefentation of B, another Point in the Line, Ab is the perfpective Reprefentation of AB. Q. E. D.

To illuftrate this, ocularly (See Fig. 37, No. 1.) turn up the Picture AIKB, vertical; alfo turn over the Plane W, till *C* coincides with the Center of the Picture, then *E* will be in the true place of the Eye, and *EC* is perpendicular to the Picture, confequently parallel to all other Lines, which are perpendicular to the Picture; therefore, to AC and FD, &c. on the other fide of the Picture; confequently, C is their Vanifhing Point. (Def. L.)

Turn over the Plane AFDC, or X, till it coincides with W; then, the Plane WX is a Radial Plane, paffing through the Original Line, AC, and the Direct Radial, *EC*, its Parallel, confequently through the Eye; producing, by its Section with the Picture, the Right Line RC; which is the indefinite Reprefentation of AC; (Theo. 12) for, it paffes through R, the interfecting Point of AB, and alfo through C, its Vanifhing Point.

Now, EA, EB, and EC, are Vifual Rays, from the Eye to the Points A, B, and C; and fince they are Right Lines (Def. H) and the two Extremes, E, A, &c. are in the Radial Plane, WX, confequently, the whole Line, EA, &c. are in that Plane (Ax. 1.) and confequently, they will cut the Picture, fomewhere in RC, the Interfection of the Plane they are in.

Therefore, in a, b, and c; as by Theorem 13th.

N. B. Any Point (B) in the Line AB, may be found, by drawing B*B*, at pleafure, and E*E* parallel to it; cuting the Interfection and Vanifhing Line, in *B* and *E*, and drawing *EB*; as in Prob. 14.

Or, having drawn B*B*, at pleafure, and E*E* parallel to B*B*; make Ba to Ee, as AB to EC (equal or otherwife) and draw ae; which will cut AC in the fame Point, b.

If the given Line does not cut the Interfection, as FG, it muft be produced till it does, as at D; and then proceed the fame as before; making D*G* equal DG, and D*F* equal DF; draw DC; and *EF*, *EG*, cuting DC in f and g: fg is the Reprefentation of FG.

If I be the Interfecting Point of a Line, perpendicular to the Picture, whofe Reprefentation is required; draw IC; and IH, or IK, parallel to either the Horizontal or Vertical Line; make IH, or IK, equal to the Diftance of any Point, in the Original Line, from its interfecting Point, and draw H*E* or KE, cuting IC in *a*. I*a* is the Reprefentation of a Line, perpendicular to the Picture; from its interfecting Point, I, equal to IH or IK.

PROBLEM XVI.

To find the Reprefentation of a Line, parallel to the Picture; in a Plane which is perpendicular to the Picture.

Let AB be an Original Line, in the Ground Plane, *E*CL is the Vanifhing Line of the Ground Plane, and DI is its Interfection with the Picture. Fig. 54.

If the Original Line was applied clofe to the Picture (as DF) it would have its full Dimenfions delineated thereon, and be its own Reprefentation.

But, if AB be fituated at fome Diftance from the Picture (as AD) draw AD and BF, perpendicular to the Interfection, cuting it in D and F, the Seats of the extreme Points, A and B. Draw DC and FC.

 Having

Plate XI. Fig. 54. Having made C *E*, or C E, equal to the Diftance of the Eye, and D *a* equal D A, draw A E or *a E*, cuting D C in a, the reprefentation of A.

A Right Line, a b, being drawn, parallel to the Interfection, or Vanifhing Line, till it cuts F C, is the Reprefentation of A B.

DEM. Becaufe AD and BF are Lines perpendicular to the Picture, C, the Center, is their Vanifhing Point (Prob. 15.) and, D and F are their Interfecting Points†; confequently, DC and FC are the indefinite Reprefentations of AD and BF‡.

† Theo 10.
‡ Theo. 12.

But EA or *Ea* is a Vifual Ray, from the Eye to the Original Point A; which, it has been proved, would cut the Picture, fomewhere, in the Indefinite Reprefentations, DC, making Da:aC::AD:CE, &c. by the thirteenth Theorem.

§ Theo. 9. But, the Reprefentation of a Line, parallel to the Picture, is parallel to its Original§; and confequently to the Vanifhing Line of the Plane it is in; and has that proportion to the Original, as the Diftance of the Picture, to the Diftance of a Plane paffing through the Original Line parallel to the Picture; which fhall be proved here.

‖ 15. 1. El. Now, DF=AB‖; and, in the Triangle DCF, becaufe ab is parallel to DF, the Triangle aCb is fimilar to DCF; wherefore, Ca:CD::ab:DF, i. e. as ab is to AB, equal DF. (2&4. 6. El.)

But Ca:aD::CE:AD; confequently, Ca:CD (i. e. Ca+aD)::CE:CE+AD.

But, CE is equal to the Diftance of the Picture, and AD is equal to the Diftance of the Original Line, beyond the Picture; wherefore, AD+EC is equal to the Diftance of a Plane, paffing through the Original Line, parallel to the Picture.

Therefore, ab, the Reprefentation, is to AB, the Original Line, as the Diftance of the Picture, CE, is to CE+AD. Q. E. D.

N. B. The Reprefentations, a and b, of the Extremes, A and B, of any Line, may be found, by any of the Methods for finding the Reprefentation of a Point, in Problem 14th.

Any other Line, as G H, parallel to the Picture, is found by the fame means.

Or, being fituated in any other Plane, as *G H*, whofe Interfection, *ab*, and Vanifhing Line, E C G, is given, or found; whether the Plane it is in be perpendicular to the Picture, or inclined, having the Center and Diftance of the Vanifhing Line, or of the Picture, the Procefs is the fame.

a and *b* are the Seats of *G* and *H*, and *E H* or E *c* are Vifual Rays; the reft is obvious, on infpection of the Figure. But, if the Original Plane be inclined to the Picture, *a* and *b* are not the Seats of *G* and *H*.

PROBLEM XVII.

To find the Reprefentation of a Line, any how inclined to the Picture, fituated in a Plane perpendicular to the Picture; having the Interfection and Vanifhing Line given.

Fig. 55. Let A B be an Original Line in the Ground Plane, whofe Interfection is LI, and Vanifhing Line, V C; C is the Center of the Picture.

Draw CE, perpendicular to the Vanifhing Line, and equal to the Diftance of the Picture. Produce B A, to its interfecting Point, I; and draw E V, parallel to A B, cuting the Vanifhing Line in V, the Vanifhing Point of A B.

Draw IV, the indefinite Reprefentation of A B; and the Vifual Rays, A E, B E, being drawn, will cut it in a and b, the Reprefentations of the two Extremes, A and B. a b is the finite perfpective Reprefentation of A B.

Or, the Seats of the extreme Points being found, by drawing A d and B D, perpendicular to the Interfection; d C and D C, being drawn, project the fame Points a and b, as before.

Or, make V *E* equal to V E, I G equal I A, and I F equal to I B.
Draw G *E* and F *E*, cuting I V in the fame Points, a and b.

DEM.

DEM. Becaufe E V is a Right Line, from the Eye, parallel to the Original Line, and cuts the Picture in V, V is the Vanifhing Point of the Line A B †; and I being its interfecting Point, I V is its indefinite Reprefentation ‡; a and b are the Interfections of the Vifual Rays, with the Picture, as proved in the former Problems. Therefore, a b is the Reprefentation of A B.

† D f. 22.
‡ Theo. 12.

G*E* and F*E* alfo project the fame Points, a and b.

For V*E* is parallel to F I, and equal to E V, the Diftance of the Vanifhing Point; alfo, I G is equal to I A, and I F to I B; by Conftruction.

Wherefore, if E*E* and B F be drawn, they will be parallel; for the Triangles, E V *E* and B I F, are fimilar Ifofceles; and fince I F, I B are refpectively parallel to E V and V*E*, conf. B F is parallel to E*E*.

Therefore, *E* is the Vanifhing Point of B F; and conf. F b : b*E* : : B b : b E, i. e. as I b : b V, &c.

The laft Method of proportioning inclined Lines will be found the moft convenient, in Practice, of all other; and it may alfo be obferved, that if the Reprefentation, a, of any Point, A, in the Original Line, be given or found, and the Interfection of the Plane it is in; the Reprefentation of any other Point (B) in that Line, may be determined, its Diftance from A being known, without the Interfecting Point I.

Alfo, the Inclination of A B being known (equal B I F) the Vanifhing Point, V, may be found; by making the Angle H E V, with the Parallel of the Eye (H J) equal B I F; or C E V equal to its Complement; or, if E *E* be the Radial of any other Line whofe Vanifhing Point is *E*, make the Angle *E* E V equal to the Angle which the two Lines make with each other; all, which, produce the fame Point V.

Then, having made V *E* equal V E, the Diftance of the Vanifhing Point from the Eye; and, if a be the known Reprefentation of fome Point A, in the Original Line (A B) and F I the Interfection, of the Plane it is in, draw the indefinite Reprefentation, a V, from that Point, and draw *E* a cuting the Interfection, in G; make F G equal to the Diftance of the Point B from A, and draw *E* F, cuting the indefinite Reprefentation, a V, in b, the Reprefentation of B.

If the Vanifhing Point, V, be out of the Picture, having found its Diftance from the Center, C, and from the Eye, E (by Prob. 12) C*E* will be equal to the Difference; i. e. make C *E* equal V E, lefs by V C.

Or, when the Vanifhing Point is in the Picture, and the Point *E* cannot be had thereon (as is frequently the Cafe, when the Vanifhing Point falls near the Center of the Picture, as *E*) take any Portion of V E, *viz.* V K half V E; draw K a, cuting the Interfection in *a*; make *a b* equal half A B; or the fame Portion V K was taken of V E, draw K *b*, which will project the fame Point, b, for the Reprefentation of B, as before. This Expedient will be found, frequently, neceffary in Practice.

If the Reprefentations, a and b, of any two Points, A and B, in any Line, be found; their Seats, d and D, are afcertained by drawing C a and C b, from the Center, to the Interfection (F I) of a Plane perpendicular to the Picture, in which the Original Line is fituated, cuting it in d and D.

By which means, the Reprefentation of any other Point, in that Line, may be found; making D L to D d, as the Diftance of the Point from B, is to A B, and draw C L, cuting a V in c, the Point fought. (See Prob. 8.)

SCHOL. After fome one or other of thefe Methods, all Lines inclined to the Picture are drawn on the Picture, and divided perfpectively, as the Original is divided; either by the real meafures, being applied, or by the known ratio of the Originals. Various Expedients for performing the fame thing will be found neceffary, becaufe the fame Method cannot always be applied, in all Cafes; for which reafon, I have given all the Methods, I would devife, which are really ufeful.

Thefe Methods of proportioning inclined Lines are general and applicable in all Cafes whatever; and, on infpection of the Figures, they will be found the very fame as for Lines perpendicular to the Picture. For, the Center of the Picture being the Vanifhing Point of all fuch Lines, its Diftance is laid down, on either Side as is moft convenient, on the Vanifhing Line, from the Center; and the Lines, which vanifh in the Center, are proportioned by the fame means. (See Prob. 15, Fig. 53.)

When

Plate XI. Fig. 55. When the Vaniſhing Point, V (being remote) cannot be had, on the Picture, the tranſpoſed place of the Eye, *E*, may be thus found.

The inclination of the Original Line, to the Interſection, being known, ſubtract it from 180 Degrees, or two Right Angles; take half the ſum of the remainder, and make the Angle V E *E* equal to it.

Or, through E, draw H E J, the parallel of the Eye, parallel to the Vaniſhing Line; make H E k equal to the Angle of Inclination of the Original Line; biſect the remainder, k E J, by the Line E *E*, cuting the Vaniſhing Line, at *E*, the tranſpoſed place of the Eye, for the Vaniſhing Point V.

§ 9. 1. El. For, the Triangle EV*E* is Iſoſceles (EV being equal V*E*) conſequently, the Angle VE*E*=V*E*E§ equal *E*EJ, by conſtruction, and by 4. 1. of Elements.

Hence it is evident, that Ib is the perſpective Repreſentation of IB.

For, ſuppoſe BIF, to be the Original of the Triangle biF; one, Side (IF) being in the Picture, therefore, it has, no Vaniſhing Point†. The Triangle BIF is Iſoſceles, and ſimilar to EV*E*.

† Cor. to Theo. 1. Now, EV is parallel to the Side IB, and E*E* to BF; wherefore, V and *E* are the Vaniſhing Points of thoſe two Sides‖, and IV, F*E* their indefinite Repreſentations; conſequently, Ib and IF repreſent equal Lines. IF, being in the Picture, retains it full dimenſions, and is its own Repreſentation on the Picture; and, Fb repreſents the other Side, FB; therefore, Ib repreſents IB; for, b repreſents B‡.

‖ Def. 22.

‡ C. 7. T. 12.

PROBLEM XVIII.

How to find the perſpective Repreſentation of a Triangle, having the Original given, in any Plane, whoſe Interſection and Vaniſhing Line is alſo given, and the Place of the Eye.

Fig. 56. A B C is the Original Triangle, D G is the Interſection, and L H the Vaniſhing Line; E is the Eye.

Produce the three Sides, A B, A C, and B C, to the Interſection, cuting it in D, F, and G, the interſecting Points of the Sides, reſpectively.

Draw E H, E I, and E L, reſpectively parallel to A B, A C, and B C, cuting the Vaniſhing Line in the Points H, I, and L, their Vaniſhing Points.

Draw the indefinite Repreſentations, D H, F I, and G L, cuting each other in the Points a, b, and c, and giving the repreſentation of the Triangle by their Interſections.

This is manifeſt, ſeeing that the Sides of the Triangle, are in the Lines AD, AF, and BG; conſequently, their Repreſentations muſt be in the indefinite Repreſentations of thoſe Lines, and conſequently, between their Interſections (by Theorem 8. and 12.)

N. B. The Angles LEI, IEH, and HEK, at the Eye, are equal, reſpectively, to ACB, BAC, and ABC; and they are alſo equal to two Right Angles†; and ſo are the three Angles of the Triangle‡.

† Cor. 3. 1. 1. El. Hence, if any one Side of the Triangle, as ab, be given, on the Picture, the Vaniſhing Points, I and L, of the other two Sides, are determined by Problem 4th; and, by drawing Ia and Lb, interſecting at c, the Triangle is completed.

‡ 10. 1. El.

See this illuſtrated, by moveable Planes, in Fig. 37.

Raiſe up the Picture AIKB perpendicular to the Ground Plane, and let the Horizontal Plane be placed parallel to the Ground Plane, i. e. perpendicular to the Picture; or, if the Picture be inclined to the Ground Plane, the Horizontal Plane making an equal Angle with the Picture, as that makes with the Ground Plane, will be parallel to the Ground Plane. In which Caſe, the Lines EN, EO, and EL are the Radials of the three Sides of the Triangle XYZ, on the other Side of the Picture; being reſpectively parallel to them§. Conſequently, the Points N, O, and L, are their Vaniſhing Points‖; B, P, and S are their Interſecting Points, and BN, PO, and SL are their indefinite Repreſentations, producing the perſpective Repreſentation, xyz, by their Interſections.

§ Def. 14.

‖ Def. 22.

Let the Picture be turned down, and imagine the Geometrical Plane turned over on its Interſection, AB, to the other Side; in which Caſe, the Triangle is inverted. Turn down the Horizontal Plane into the Picture; the Radials EN, EO, and EL, are ſtill parallel to the three Sides of the Triangle, and produce the ſame Vaniſhing Points, N, O, and L; alſo, the Viſual Rays EX, &c. being drawn on the Picture, paſs through their reſpective repreſentations, x, y, z; as it is obvious they would paſs through the Picture, in thoſe Points, to the Original Points X, Y, and Z, on the other ſide.

N. B. The Angle NEO, which the Radials EN and EO make, at the Eye, is equal to the Angle Y, of the Triangle; for, they are refpectively parallel to them. EO and EL, the Radials of YZ and XZ, make the Angle OEL equal to the Angle Z; and if NE or LE was produced beyond the Eye (E) they would make an Angle equal to X; which is confonant to Cor. 1ft, Theorem 6th.

PROBLEM XIX.

How to find the Reprefentation of a Square or other Rectangle, given in the Geometrical Plane; one Side being parallel to the Picture.

If the 15th and 16th Problems are well underftood, there will be little occafion to explain the Method of proceeding in this; for, if any Rectangle have one Side parallel to the Picture, the other two adjoining Sides muft, neceffarily, be perpendicular to the Interfection of the Plane it is in, if not to the Picture.

Let ABFD be a Square; one Side, AB, lying clofe to the Picture, AD and BF are therefore perpendicular to it. C is the Center of the Picture, or of the Vanifhing Line, *EE*, of the Plane of the Square, AB, is its Interfection. PlateXII. Fig. 57.

Now, AD and BF are perpendicular to the Interfection, therefore they vanifh in the Center of the Vanifhing Line. (See Problem 15.)

Draw AC and BC, their indefinite Reprefentations; make C*E*, on either Side, equal to the Diftance of the Picture, or Vanifhing Line, and draw B*E* (or A*E*) diagonal Ways, cuting AC in d, the Reprefentation of D, (or BC in f.)

Draw df parallel to AB. AdfB is the reprefentation of the Square A*D*FB.

For, B*E* is the indefinite Reprefentation of the Diagonal, BD, *E* being its Vanifhing Point; for E*E* is parallel to BD, (C*E* being equal to CE, and AD to AB) and DF being parallel to the Picture, its Reprefentation, df, is confequently parallel to the Interfection. (Prob. 16.)

2. If another fquare be required, draw f*E*, cuting AC in g, and draw gh parallel to df. By which Expedient any length of AC may be obtained.

If the length AG had been required, equal twice AD; A*G* on the Interfection being made equal twice AB, and *GE* drawn, gives the fame Point g; as before.

Or, if C*E* be bifected, at K, BK is the indefinite reprefentation of the Diagonal BG; for EK is parallel to BG; the Angle CEK is equal GBH, and, K is the Vanifhing Point of that Diagonal; draw BK, cuting AC in g, as before.

3. If the Square be at fome Diftance from the Picture, as *ABFD*, produce the two perpendicular Sides to the Interfection, cuting it in a and b; alfo, produce the Diagonal *FA* to I, its interfecting Point; or, make aI equal a*A*, and proceed as in the former Cafe. The Figure explains the reft.

COR. Hence, a Pavement of Squares, having their Sides parallel to the Picture, may be delineated, with great facility.

Let AB be the Ground Line, and ECD the Horizontal Line. No. 1.

Take the geometrical meafure of a Square, and apply it, on the Interfection, AB, as often as it is required, from A to B; as a, b, &c.

From each Divifion, draw Right Lines to the Center, AC, aC, &c. and draw a Diagonal, from A or B, to the Eye, at E or D, cuting each indefinite Reprefentation, in the Points *a*, *b*, *c*, &c. through which, draw the Lines FG, HI, &c. parallel to the Interfection, or Vanifhing Line, and the feveral Figures X, Y, Z, &c. are the Reprefentations of Squares on the Picture; which may be repeated, by drawing other Diagonals to any length or width.

Plate XII. Now, if one of theſe Squares (as Y or Z) be conſidered ſingly, it has ſcarce the appearance of a Square, having no other, contiguous to it, to biaſs the judgment; but, by the affinity of the whole, the Eye (being accuſtomed to ſee Objects as they appear, in all ſituations) is not offended, and readily gives the aſſent; although it is certainly capable of determining, that the ſeveral Repreſentations of Squares, are not Squares, but have the appearance, only, of Squares, in certain Poſitions and Situations.

PROBLEM XX.

To find the Repreſentation of a Square, or other Rectangle, whoſe Sides are all equally inclined to the Picture.

Firſt, by the Original Figure being geometrically drawn, in its determined Poſition to the Picture; the Interſection of the Plane it is in being given.

Fig. 58. Let A B C D be a Square, the Sides of which are equally inclined to the Picture. C is the Center of the Vaniſhing Line *E E*, or of the Picture; *E*, *E*, are the tranſpoſed places of the Eye, to the Vaniſhing Line, *viz.* C *E* equal C E.

As one Angle of the Square (A) touches the Interſection, it is, conſequently, the Interſecting Point of the two Sides A B and A D.

Let the other two Sides be produced to the Interſection, at F and G.

Draw A*E*, both ways; alſo, draw F*E* and G*E*, diagonal ways, cuting each other. A b c d is the Repreſentation of A B C D.

For the Point A being in the Picture, is its own Repreſentation, A*E*, A*E* are indefinite Repreſentations of AB and AD; and F*E* and G*E*, of FC and CG (Theo. 12) conſequently, they cut each other in the repreſentations of the ſeveral Angles, B, C, and D. (Cor. 7. Th. 12.)

If the Rectangle H I K L be at ſome Diſtance from the Picture, the Sides being produced to their interſecting Points, *a*, *b*, *c*, *d*, and the indefinite Repreſentations, *aE*, *bE*, &c. being drawn (as in the Figure) give the Repreſentation h i k l of that Rectangle; its Sides being parallel to the Sides of the Square.

E V, parallel to the Diagonal, I L, produces its Vaniſhing Point.

Note. In this Caſe, it may be obſerved, that there is no neceſſity for the Eye, i. e. the Diſtance of the Picture or Vaniſhing Line, being placed above it, but placed equally on either Side of its Center, C, in the Vaniſhing Line, as C*E*. Squares, or other Rectangles (wherever they are ſituated in the Original Plane) having the ſame Poſition to the Picture, have their Sides parallel, and conſequently, they have the ſame Vaniſhing Points.

N. B. The Diagonals of the Square are, in this Caſe, the one (B D) parallel, and the other (A C) perpendicular to the Picture; conſequently, their Repreſentations are either parallel, as b d, or vaniſh in its Center, as A c.

Let it be, here alſo, particularly noticed, that *E*, being the place of the Eye, in the Vaniſhing Line (commonly called the Point of Diſtance) is the Vaniſhing Point of the Diagonal of a Square, whoſe Sides are parallel and perpendicular to the Picture. Conſequently, if the Diagonals are parallel and perpendicular to the Picture, they are the Vaniſhing Points of its Sides.

By which Points, all Lines perpendicular to the Picture are proportioned, perſpectively; as it may be obſerved in the preceding Problems.

2nd. How to find the Repreſentation of a Square, in this Poſition, not having the Figure drawn out geometrically, only its meaſure and place known.

Let A be the Interſecting Point of an Angle of the Square, ſituate on the left ſide of the Station Line; at the Diſtance A J.

Make AF and AG each equal to the Diagonal, and proceed as before.

Or, by the meaſure of its Sides.

Make

Make EF equal to the Diagonal of a Square, whose Side is CE; i. e. make EF equal to the Distance of the Vanishing Point E, from the Eye, equal EE.
Make Af equal to AB and draw fF, cuting the indefinite Representation AE in b, the Representation of the Angle B.
Draw bd parallel to FG, the Intersection, cuting AE in d; and, lastly, draw bE and dE, diagonal ways, cuting each other, which compleats the Figure.

COR. Hence, a Pavement of Squares, diagonal-ways, may be delineated. Fig. 58. No. 1.

Let AB be the Ground Line and ED the Horizontal vanishing Line.

Make Aa, ab, &c. equal to the Diagonal of the Square; make CE, and CD each equal to the Distance of the Picture (C being the Center.)
Draw AE, aE, &c. and AD, aD, &c. cuting each other in the representations of Squares, placed diagonal-ways.
Through d, where AD and BE interfect, draw ef parallel to AB; and where it cuts the several Lines, drawn from a, b, c, &c. *viz.* in e, *a*, *b*, &c. draw *a*D, *b*D, and E*d*, &c. contrary ways; by which means they may be continued at pleasure.

PROBLEM XXI.

To find the Representation of any Rectangle, obliquely situated to the Picture.

First, by having the Original drawn in the Geometrical Plane, in its true place and position, in respect of the Picture and of the Eye. Fig. 59.

ABCD is the Rectangle to be delineated; FI is the Intersection of the Plane it is in; KL is the Vanishing Line, and *C* its Center. The Distance is known.

Make *C*E, equal to the Distance, and perpendicular to KL.
Draw EK and EL, parallel to the Sides of the Rectangle, AB and BC, respectively, producing their Vanishing Points, K and L.
The Original being at some Distance from the Picture, produce every Side, DA, CB, &c. to the Intersection, cuting it in F, G, H, and I, their Intersecting Points.
Draw the indefinite Representations FL, GL, HK, and IK; which, by their mutual Intersections, give the Representation, abcd, of ABCD.

METHOD 2nd. By the Seats of every Angle, on the Picture, and their Distances from their Seats, respectively.
If the Original be in a Plane perpendicular to the Picture, the Seats of all its Angles are in the Intersection of the Plane they are in.

Let *a*, *b*, *c*, and *d*, be the Seats of the Angles A, B, C, and D, respectively.

Draw *a*C, *b*C, &c. to the Center; make *ab* equal to the Distance of the Point A, from its Seat, *be* equal to the Distance of B, *cd* of C, and *df* of the Angle D.
Make *CE* and *CF* each equal to the Distance of the Picture, and draw *bE*, *eF*, *dF*, and *fE*, cuting *a*C, *b*C, &c. in the Points a, b, c, and d, respectively; which are the representations of the several Angles A, B, C, and D. (Pr. 14. Meth. 1.)
Join the Points a and b, b and c, &c. as in the Figure; and the Quadrilateral, abcd, is the representation of the Rectangle, ABCD, situated as in the Figure.

Note. This Method, is used by all the old writers on Perspective; particularly in the Jesuits. In which process it may be observed, that there is no occasion for the Vanishing Points, K and L; for if they made use of Vanishing Points, at all, they were found by producing the Sides ba, bc, &c. to the Vanishing Line; i. e. to the Horizontal Line, for they knew no other Vanishing Line. Which Vanishing Points, so produced, they called accidental Points.

Plate XII. Fig. 60. METHOD 3rd. By the known Proportion of the Original; its Pofition, Situation, and Diftance being determined.

The Original Figure, ABCD, is drawn out in the Geometrical Plane, to fhew how the meafures, &c. are applied, but it is, otherwife, of no ufe in the operation; as the careful and accurate obferver will perceive.

IN, the Interfection; KL, the Vanifhing Line; *C*, its Center; and *C*E its Diftance, are given, the fame as before.

Through E, draw HI, parallel to the Vanifhing Line; draw EK and EL, making the Angles, HEK, and IEL, equal to the known Inclination of the Sides of the Rectangle (AB and BC) to the Interfection, refpectively; i. e. make HEK equal to ABF, and IEL equal to CBG; FG being parallel to the Interfection, the Angles ABF and CBG are equal to their inclination to it†.

Make *DB* equal to JB; i.e. to the diftance of the neareft Angle (B) to the Station Line, and draw *BC*; alfo, make *Bd* equal to its diftance from the Interfection (equal B*B*) and, *CE* being made equal to *C*E, draw *dE*, cuting *BC* in b, the Reprefentation of the Angle B. (Prob. 14th.)

Draw the indefinite Reprefentations, bK and bL, from the Point b.

It remains, now, to cut off (from b) ba and bc, in the perfpective Reprefentations of AB and BC, the Originals.

Make KE^1 equal to KE, and LE^2 equal to LE; which Points are ufed for proportioning all Lines which vanifh in K and L, refpectively. (Prob. 17.)

Draw E^1b, cuting the Interfection in *b*; make *ab* equal AB, and draw *a*E^1, cuting bK in a, the Reprefentation of A.

Alfo draw E^2b cuting the Interfection in b^2 and make b^2 *c* equal BC (the other Side, whofe vanifhing Point is L) and draw *c*E^2 cuting bL in c, the Reprefentation of C, another Angle of the Figure.

From the Points a and c, draw aL and cK, interfecting at d, which compleats the Figure; for, the Originals of thofe Sides, being parallel to the Originals of ab and bc, they have, confequently, the fame vanifhing Points, refpectively.

† 4. 1. El.

SCHOL. This Method, which is perfectly confonant to the new Principles by Brook Taylor, may appear to fome Perfons more difficult than either of the other, particularly the firft, which is performed, alfo, entirely on the fame Principles; the difference is very obvious, notwithftanding the effect is the fame, as it may be feen, by comparing the Figures; or by comparing both, with the 17th Problem, which contains all the various Methods of proportioning inclined Lines, in general.

In the firft, there is a neceffity for having the Original Figure drawn geometrically, either on the Picture or fomewhere apart; for which there is not, frequently, room to fpare on the Picture; and we are liable to errors, in transfering the interfecting Points; but, without the Original being placed in the very Pofition, we cannot draw the Radials, or Parallels from the Eye, producing the Vanifhing Points; as by that Method.

Whereas, by the laft Method, the Angles, being known, are made equal to the Originals; in which Cafe, the Radials would be parallel to the original Lines being in their true places.

Or, if there be not room on the Picture, or interfere with the Objects, the Vanifhing Points and their Diftances, from the Center and from the Eye, are all determinable, by Problem 12th; the Vanifhing Points, K and L, and their Diftances KE^1 and LE^2 being afcertained by it.

There is, likewife, no need for the Original Figure on the Picture, but being any where drawn out, geometrically, or its meafures being known, and the Pofition determined, they are applied to the Picture with the greateft facility, for which, a little Practice will render it quite familiar; regarding always, carefully, to make ufe of the proper Points for particular Lines, and to diftinguifh between the Vanifhing Points and thofe which are ufed, for Vifual Rays, i.e. for cuting indefinite Lines in the Ratio required.

PROBLEM XXII.

To find the Reprefentation of a regular Pentagon, one Side being parallel to the Picture.

Pl. XIII. Fig. 61. Let ABCDF be a regular Pentagon, having one Side (AB) parallel to the Interfection (IK) confequently parallel to the Picture.

LM is the Vanifhing Line, and E the Eye.

Produce the inclined Sides, F D, C D, A F, and B C, to the Interfection, cuting it in G, H, I, and K.

Draw E L, E M, &c. refpectively parallel to them, producing the Vanifhing Points, L, M, N, and O, of each Side.

Draw the indefinite Reprefentations GL, HM, IN, and KO; producing, by their Interfections, the Figure, fjcd, of the Original F J C D, (No. 1.) and *IfdcK* of *IFDCK* (No. 2) fo that, the reprefentation of one Side (AB) is wanting, in each; which, on account of its parallelifm to the Picture, has neither Interfecting nor Vanifhing Point†; and muft be found by Problem 16; or, by drawing a Vifual Ray, E A, cuting the indefinite Reprefentation, IN, of the Side A F, in a; or, make NP equal NE, and I*a* equal I A, and draw *a*P. (Prob. 17.) Or, by producing either Diagonal, AD or BD; being parallel, refpectively, to the Sides BC, and AF, O and N are their Vanifhing Points. † Cor, to Theo. 1.

Having, by any of thefe Methods, found a, the Reprefentation of A, draw a b, parallel to the Interfection, cuting KO in b; which compleats the Figure.

Secondly. How to determine all the Vanifhing Points in this Pofition, without the Original Figure on the Picture.

Through E, draw RS parallel to the Vanifhing Line; and make the Angles REL, LEO, &c. each of 36 Degrees; i. e. divide the Ark of the Semicircle, R 2 3 S, into five equal Parts, and produce the Lines E 1, E 2, &c. to the Vanifhing Line, cuting it in L, O, &c. the Vanifhing Points of the Sides and Diagonals of a Pentagon, having one Side parallel to the Picture.

SCHOL. The reafon of this is obvious; for three of thofe Angles, *viz.* R E N, is equal to the Angle of a Pentagon; and fince one Side is parallel to the Picture, R E is parallel to it; and confequently, E N will be parallel to A F, another Side; the Angle R E N being equal B A F.

Alfo, the Angle which any two Sides, not contiguous (as A F and C B) make with each other, *viz.* FJC, is equal to one of thofe Angles; wherefore, E N being parallel to one of thofe Sides, E O is confequently parallel to the other. (By 6. 1. El.) Alfo to the Diagonals AD and BD.

This is fully explained, after Problem 11th, in the 4th Book of Elements.

To find the Reprefentation when every Side is inclined to the Picture (the Original being in its true Place) has nothing particular; for, if every Side be produced to its interfecting Point, and their Vanifhing Points are found by drawing parallels to every Side, from the Eye, the indefinite Reprefentations being drawn, produce the Reprefentation of the Original; as in the laft Problem, or Prob. 18.

If the Vanifhing Point of any Side be found, all the reft are determined as before; by producing the Radial of that Side and defcribing a Semicircle; and dividing it into five equal Parts, as in this Figure.

PROBLEM XXIII.

How to find the Reprefentation of a Pentagon, having only one Side given, in its true Place and Pofition, in the Geometrical Plane.

Let A B be the given Side.

It is required to find the Reprefentation of a regular Pentagon, whofe Side is equal to A B; and inclined to the Picture in the Angle B D G, or to the Interfection G D. KM is the Vanifhing Line, C is its Center, and C E its Diftance. Fig. 62.

Through E, draw EH parallel to the Vanifhing Line, and make the Angle HEN equal to BDG, the given Angle, producing the Vanifhing Point I.

Produce B A to D; draw the Indefinite Reprefentation DI; and find the finite part a b, the reprefentation of AB (by Prob. 17) i. e. draw the Vifual Rays A E and BE; or, make IP equal IE; and, DF, DG, equal to DA, and DB, refpectively; draw FP and GP, cuting DI in a and b.

Produce I E, and defcribe a Semicircle N*ac*O; divide the Ark into five equal Parts, at *a*, *b*, *c*, and *d*; draw E*a*, E*b*, &c. to the Vanifhing Line, producing the Vanifhing Points, K, L, &c. of the remaining Sides of the Poligon.

Having obtained a b, the Reprefentation of A B, as above; draw a M and b L the indefinite Reprefentations of two Sides, from thofe Points; for, I a M reprefents an Angle (I E M) equal to the Angle of a Pentagon; and I b L reprefents an Angle equal to I E L; which is equal to an external Angle of a Pentagon.

 Draw

Pl. XIII. Fig. 62. Draw aK, cuting bL in c; bc is another Side of the Pentagon; for, ac reprefents a Diagonal (which, in a Pentagon, is parallel to the oppofite Side) whofe Vanifhing Point is K.

Draw l f, through c, cuting aM in f; c f reprefents a Diagonal parallel to the Side a b; and confequently, a f reprefents another Side.

Laftly; draw fK, and bM cuting it in d, and draw cd, which compleats the Figure, abcdf, required.

The Vanifhing Point Q (which is out of the Picture) of the Side cd, and its parallel Diagonal b f, has not been wanted in this procefs.

N*c* and O *b* are Diagonals, EN, E*c*, or EO and E*b*, being fuppofed Sides of a Pentagon; and EQ, EK are Parallels to them, from the Eye, producing their Vanifhing Points. (See Prob. 10.)

Note. If a b or bc, or any Side in the Reprefentation had been given, the whole Figure may be completed, as above, without the Interfection.

PROBLEM XXIV.

To find the Reprefentation of a regular Hexagon, fituated any how to the Picture.

There is no Figure whatever, except a Square, eafier to defcribe than a Hexagon.

To have the Original Figure drawn in the Geometrical Plane, it is evident, is the fame as has already been explained, in the foregoing; I fhall, therefore, beg leave to pafs over that defcription.

Fig. 63. Let A B be a Side, given, of a regular Hexagon; the Original of which, is parallel to the Picture. IK is the Vanifhing Line.

Make IEK an equilateral Triangle, whofe Perpendicular is *C*E; as follows.

Through E draw HL parallel to the Vanifhing Line, and, on E, defcribe a Semicircle; which, divide into three equal Arks, at *a* and *b*.

Draw E*a* and E*b*, and produce them to the Vanifhing Line, cuting it in I and K; which, are all the Vanifhing Points required for this Pofition.

Draw AI and BK; make Ab and Ba each equal to A B, in A B produced, both ways; and draw aI and bK.

Draw BI and AK, or A*C* and B*C*, cuting them in F and G, and join FG, which is parallel to AB, and compleats the Figure. ADFGCB.

SCHOL. A regular Hexagon having an even number of Sides, the oppofite ones are confequently parallel; and the Diagonal, which paffes through the Center, is alfo parallel to them; and becaufe AB is parallel to the Vanifhing Line, its Original is parallel to the Picture; and confequently, the oppofite Side F G is alfo parallel to A B; and, becaufe they are equal, A F and B G are parallel, and confequently their Originals were perpendicular to the Interfection; therefore they vanifh in the Center of the Vanifhing Line. (Cor. to Theorem 11.)

The two Sides, A D and CG, and the Diagonal, B F, reprefent parallel Lines; confequently they have the fame Vanifhing Point, I; alfo BC, AG, and DF; all which may be feen in the Hexagon AB*CGFD*, which may be fuppofed the Original Figure, one Side AB, being in the Picture.

By drawing the Diagonals A*G*, B*F*, and *CD*, the Hexagon is divided into fix equilateral Triangles, AOB, BO*C*, &c. and IEK has its three Sides refpectively parallel to them; confequently, fince one Side in each Triangle is parallel to the Picture, and has no Vanifhing Point (Theo. 1.) EI and EK are parallel to the other two Sides of each Triangle (which are equally inclined) and confequently, to the Sides *AD*, *CG*, and B*C*, *DF*, of the Hexagon; I and K are, therefore, their Vanifhing Points.

COR. Hence, a Pavement of regular Hexagons, or equilateral Triangles, may be eafily delineated.

No. 2. Let AB be the Interfection of the Picture, or the Ground Line of the Pavement; and let Aa be the determined meafure of one Side of a Hexagon.

Make ab, bc, &c. equal to A a, as often as there is occafion.

If C be the Center of the Picture (its Diftance being known) find the Vanifhing Points, D and E, as I and K above.

Draw

Draw AD, aD, bD, &c. and aE, bE, &c. and, where they interfect, draw Lines parallel to the Interfection, as fg, hi, &c. which divide the whole into the reprefentations of equilateral Triangles; fix of which, around the fame Point o (as *acdef*b) form a Hexagon; and thus, as many may be defcribed as are required; for which, the infpection of the Figure is fufficient.

Cafe fecond. When the given Side is inclined to the Picture, in any Angle.

Let AB be the Side given, DF is the Interfection, and GL the Vanifhing Line of the Plane it is in; E is the Eye. Fig. 64.

Find ab, the reprefentation of AB, by any of the former Problems, its Vanifhing Point is G; draw GE, and produce it; on E defcribe a Semicircle, *abcd*.

Make *ab* and *bc* each equal *a*E, and draw E*b*, E*c*, cuting the Vanifhing Line, in H and I the Vanifhing Points of the other Sides.

Draw aI and bH, the indefinite Reprefentations of two Sides.

For, baf reprefents the Angle GEI, which is the Angle of a Hexagon; equal *a*E*b*+*b*E*c*, twice 60 Degrees; and abc reprefents the Angle HE*d*, equal *a*E*c*.

Make af and bc reprefent Lines equal AB (by Prob. 10, Cafe 3rd) i. e. bifect the Angle GEH, by the Line EK; or, E*d* being equal E*b*, having drawn *bd*; draw EK parallel to *db*; i. e. perpendicular to EI.

For, EI is the Radial of af; confequently parallel to its Original; and, EK is the Radial of a Diagonal of two contiguous Sides, which is perpendicular to the Original of af; parallel to *ab* (for *abcd* is half a regular Hexagon in the Pofition of the Original) confequently, EK is parallel to a Diagonal whofe Vanifhing Point is K.

Draw aK cuting bH in c, and draw Gc till it cuts aI in f; for, the Original of the Diagonal cf is parallel to the Original of a b. As *ad* is parallel to *bc*.

Draw eI; aH, and fH; and GD, cuting fH in e, which compleats the Figure.

Note. If the Vanifhing Point H had coincided with C, the Center of the Picture, G and I would be equally diftant from C; and the Original of the Hexagon would be regularly pofited; having two Sides and a Diagonal perpendicular to the Picture; or to the Interfection of the Plane it is in.

By which means, a Pavement of regular Hexagons may be drawn in that Pofition.

Let AB be the Ground Line of the Pavement, C the Center of the Picture, and DF the Horizontal vanifhing Line. No. 2.

On AB, take as many equal divifions Aa, ab, &c. as you pleafe, each equal to half the width of the Hexagon required.

Make CE equal to the Diftance of the Picture, and make the Angles CED, CEF, each of 60 Degrees, i.e. having defcribed the Ark *abc*, on the Center E, make *ba* and *bc* each equal E*b*, and draw E*a* and E*c*, to D and F; C, D, and F, are the Vanifhing Points of all the Sides.

From all the Divifions A, a, b, &c. draw Lines to the Center, C; and from every other Divifion a, c, e, draw aD and aF, &c. to both the other Vanifhing Points, D and F; and, where they cut AC and BC, *viz.* in f, g, &c. draw fF, gD, &c. as in the Figure; by which means they may be continued at pleafure. The reft is obvious, on infpection.

PROBLEM XXV.

To find the Reprefentation of an Octagon and of a Circle infcribed, having only one Side given.

An Octagon, having four of its Sides in a Square, is very readily delineated in Perfpective, feeing that, if one Side be parallel to the Picture, two are neceffarily perpendicular to the Interfection, and four are inclined to it, equally, *viz.* in an Angle of 45 Degrees, half a Right Angle; confequently, the Vanifhing Points are the Center, and Diftance, of the Vanifhing Line of the Plane it is in.

Pl. XIV. Fig. 65. Let AB be the given Side of an Octagon, parallel to the Picture; *EF* is the Vanishing Line of the Plane it is in, and C its Center.

Make C*E* and C*F* each equal to the Diftance; alfo make *F*J equal *F*E.

Draw A*E* and B*F*, which are two Sides, indefinite.

For, fince the given Side, AB, is parallel to the Picture, the adjoining Sides are inclined in half a Right Angle to the Interfection; therefore, *E* and *F* are their Vanifhing Points. (Pr. 19 & 20.)

Produce AB, both ways; make BD equal AB, and draw DJ, cuting B*F* in G; and draw FG parallel to AB, cuting A*E* in F.

Draw CF and CG, and produce them, till they cut AB, produced, in H and I; draw I*E* and H*F*, cuting HC and IC, in K and L, and draw KL.

Draw AC and BC, cuting KL in M and N; and draw *E*N and *F*M, and produce them, cutiug IC and HC in O and P, which compleats the Figure AFPMNOGB, of an Octagon.

Or, without drawing AC and BC, make KM equal NL, where DJ cuts KL, and draw *E*N and *F*M, as before.

Or, from a and b, where the Diagonals HL and IK cut AF and BG, draw aC and BC, cuting the Diagonals, again, in d and c; and through them, draw *E*c and *F*d.

A Curve defcribed, carefully, by hand, touching every Side of the Octagon, paffing through the Points a, b, c, and d, will be an Ellipfis; for it is the reprefentation of a Circle, in Perfpective.

Or, for greater exactnefs, through S, where the Diagonals HL and IK interfect, draw ef parallel to AB; and CS to g, bifecting AB and KL, in g and h; by which means there are eight Points obtained in the Circumference.

Note. The Figure below HD (which may be confidered as the Interfection of the Plane of the Original) is the Original, geometrically drawn. The Italics, correfponding with the Roman Letters in the Reprefentation, fhew the Original of each Point, or Line; S reprefents *S*, the Center.

To draw an Octagon inclined to the Picture has nothing fingular or particular in it; as every Line, which is inclined to the Picture, is projected, on the Picture, by the fame general Rule (Prob. 17) which, having been fo often exemplified, in the preceding Problems, it would be quite fuperfluous to repeat.

Of all the various Methods for finding the Reprefentation of a Circle, in Perfpective, there is none preferable to this; which I fhall exemplify more at large in the 8th Section. For, if any regular Poligon whatever, be drawn in Perfpective, a Circle may be defcribed through all the Angles, or touching every Side. Six Points are not fufficient for defcribing the Curve with accuracy; befides, an Octagon is readier to defcribe than a Hexagon, feeing that, the Vanifhing Points, required, are only the Center of the Vanifhing Line and its Diftance. The Octagon being infcribed in a Square, renders it, of all others, the moft expeditious for finding the Reprefentation of a Circle.

PROBLEM XXVI.

The Interfection of a vertical Plane being given, and its Inclination to the Picture, to find the Reprefentation of any Figure in the Plane.

Fig. 66. Let AB be the given Interfection, and C the Center of the Picture.

Through C, draw SD perpendicular to the Interfection, and CE perpendicular to SD, i. e. parallel to AB, and equal to the Diftance of the Picture.

Make CES equal to the Complement of the Angle of Inclination, cuting CS in S; and, through S, draw MN parallel to AB; MN is the Vanifhing Line, S is its Center, and ES is its Diftance. (See Prob. 3rd.)

If it be required to find the Reprefentation of a Square, whofe Side is parallel to the Picture, and equal to AB; at the diftance A*b* from the Interfection.

Draw AS and BS; make SE^1 & SE^2 equal SE, and draw *b*E^1, cuting AS in F.

Draw FE^2, cuting BS in H; and draw FG and HI, parallel to AB.

Then, FGHI reprefents a Square, in that Plane, at the diftance A*b* from the Interfection of the Plane of the Original.

If another Square be required, draw another Diagonal H*E*' or I*E*'.

Or, if a greater length than a Square be required, make SO to S*E* (equal SE) as one Side of the Rectangle, required, is to the other, and draw HO, cutting AS in K, and draw KL. For *E*O, being drawn, is parallel to the Original of HK.

See this illustrated by moveable Planes. (Fig. 37, No. 1.)

Turn up the Plane AFDC, (on the other Side of the Picture) at right angles with the Ground Plane, RS is its interfection with the Picture, and AB, with the Ground Plane.

AB and FD, being perpendicular to the Picture, vanish in its Center, C; and, if the Vertical Plane, V, be turned up, perpendicular to the Picture, *E*Q will be parallel to the Diagonals, AE and BD, and produce their Vanishing Point; and, Q, on the other Side of the Center, C (in the Vertical Line) equally distant from C, is the Vanishing Point of the other Diagonals, FB and EC.

The rest is obvious; if the Rectangles AFEB and BEDC were Squares, their Diagonals would vanish in E; CE being equal to the Distance of the Picture.

Second. Let ab be the given Side of a regular Heptagon (in the Plane AIHB) parallel to the Horizon; its Vanishing Point is S; it is required to describe the Heptagon. S*E* is perpendicular to the Vanishing Line MN.

Having made S*E* equal SE, make the Angles S*E*M, S*E*N each equal to the external Angle of a Heptagon, and bisect them, by the line *E*O and *E*P; also, make the Angles M*E*Q, N*E*R, equal M*E*O, &c. i. e. having described a semicircle, on *E*, divide the Semi-circumference into seven equal parts, and make ab, bc, and cd, each equal one seventh part.

*E*Q and *E*R, being produced to the Vanishing Line, MN, will generate two Vanishing Points, equally distant from S; which, with M, O, S, P, and N are the Vanishing Points of the Sides and Diagonals; none of them being, in this Case, parallel to the Picture.

Draw aM; and Nc, through b, indefinite; draw Pc, through a, cuting Nc in c, and draw cS, cuting aM at d. bc and ad are Sides of the Heptagon, whose Vanishing Points are M and N.

Draw cQ, and Rd indefinite (by Prob. 13.) and, through a, draw Ne, cuting cQ at e; and draw eS, cuting the opposite Side, at f; and, lastly, draw eO and Pg, through f, cuting eO at g, which compleats the Figure afecb, the representation of a regular Heptagon, whose Sides are all inclined to the Picture.

PROBLEM XXVII.

To find the Representation of any irregular Figure (entirely on the Principles of Brook Taylor) from the known dimensions of the Figure, and the Angles which every Side makes with the adjoining Side or Diagonal; the Place, Position, Situation, and Distance of the Figure, in respect of the Picture and of the Station Line, being given; in a Plane which is inclined to the Picture, in a given Angle, the Center and Distance of the Picture being given, and the Intersection of the Plane of the Figure.

Note. The Original Figure (A*BDFG*) being geometrically drawn, in the Original Plane, is of no other use (in the following Operation) than to determine its Situation and Position; otherwise it is not necessary. The Place of the nearest Angle (A) and the Inclination of a contiguous Side (A*B*, or A*G*) in respect of the Intersection, being all that is wanted; the rest are determinable, in respect of themselves; as it will be exemplified. Fig. 67.

C is the Center of the Picture, *HI* is the Intersection of the Plane of the Original Figure, and X is the Angle of its Inclination to the Picture.

Pl. XIV. Fig. 67. A is the place of the neareſt Angle of the Figure, and J A *B* is the Angle of the inclination of the Side A *B*, to the Interſection; A B is the Diſtance of the Angle A from the Interſection, and A e is its Diſtance from the Station Line.

Through C, draw EC perpendicular to the Interſection, which is the Vertical Line of the Original Plane of the Figure. (Def. D.)

Draw CE' perpendicular to EC, and equal to the Diſtance of the Picture.

Make the Angle CE' *C* equal to Z (the Complement of X) cuting E C in *C*; and, through *C*, draw DG, the Vaniſhing Line of the Plane of the Figure; *C* is its Center, and *C*E' its Diſtance. (Def. 19 and 20.)

*C*E, on the Vertical Line, being made equal to *C*E'; through E, draw ER parallel to the Vaniſhing Line; the Parallel of the Eye of that Plane.

Draw ED; making the Angle RED, equal JA*B* (i. e. to *BKH*, the Inclination of the Side A*B* to the Interſection) cuting the Vaniſhing Line in D; the Vaniſhing Point of A*B* †, and make the Angle DEG equal *BAG*, cuting the Vaniſhing Line in G, the Vaniſhing Point of A*G* ‡.

† Prob. 2.
‡ Prob. 4.

Alſo, make the Angle DEH, equal to A*BH*, the Supplement of A*BD*; and H is the Vaniſhing Point of *BD*; by the ſame.

Make the Angle GEI equal A*GI*, the Supplement of A*GF*; cuting the Vaniſhing Line in I, the Vaniſhing Point of *FG*.

Laſtly, make the Angle IEF equal to *DFG*; EF is the Radial of *FD*; which, if produced, would cut the Vaniſhing Line, in the Vaniſhing Point of *FD*.

Thus, are the Vaniſhing Points D, G, H, and I, produced, by the known Inclination of one Side to the Interſection, and the Angles which each Side makes with the adjoining Side, (by Prob. 4.)

The place of the Angle A being given in the Geometrical Plane (which is inclined to the Picture, in the Angle E'*C*C; and to the Horizon, in the Angle *C*E'C) find its repreſentation on the Picture (Prob. 14.)

Draw AB, perpendicular to *HI*, and draw B*C*. Make B*L* equal to BA, and *CE* equal to *C*E', and draw *LE*, cuting B*C* in a, the repreſentation of the original Point A.

Or, draw B*A* parallel to *C*E', and make B*A* equal BA; produce AB, and draw *AS* parallel to *HI*, or perpendicular to *BS*; *S* is the Seat of the point A, on the Picture; and *SA* its diſtance from its Seat.

§ Prob. 6. Draw S*C*, and *A*E' cuting it in a, the Repreſentation of A§, as before.

Draw aD and aG the indefinite Repreſentations of A*B* and A*G*, from the Point a; which, if produced, would paſs through their Interſecting Points, *K* & *L*.

Make D*E'* equal DE; and, through a, draw *E'a* to the Interſection, cuting it in *a*.

Make *a b* equal A*B*, and draw *b E'*, cuting a D in b; ab repreſents a Line equal to A*B* (Prob. 7, or 17.) Draw bH.

Produce DE, and make EK and EL reſpectively equal to A*B* and *BD*; or in the ratio of A*B* to *BD*; and draw EM parallel to KL, cuting the Vaniſhing Line, in M, the Vaniſhing Point of the Diagonal A*D*.

Draw aM cuting bH in d. bd repreſents a Line in the ratio to that which ab repreſents, as EK to EL, i. e. as A*B* to *BD*. (Prob. 10*.)

|| Def. 22. For, EK is parallel to A*B*, and EL to *BD*; wherefore KL, conſequently EM, is parallel to the Diagonal A*D*; therefore, M is its vaniſhing Point ||; and abd repreſents the Triangle A*BD*. (Pr. 18.)

Now, F, the Vaniſhing Point of *FD* (being much inclined) is not in the Picture.

Draw d*f* at pleaſure, cuting the Vaniſhing Line, and EF, in *e* and *f*; and draw *gi* parallel to d*f* (at any diſtance, at diſcretion) and make *hi* to *gh*, as *e*d is to *ef*.

* From the Poſition of A*B* and *BD*, the meaſure of A*B*, or its Ratio to *BD*, is taken beyond the Eye, in the Radial of A*B*, produced; as in the third Caſe, of that Problem.

e. g.

6

e. g. Draw the Diagonal *fh*; and draw dk parallel to the Vanifhing Line, cuting *fh*, produced, in *k*; draw *ki*, parallel to FE, cuting *gi* in *i*; *hi* is a fourth Proportional to *fe*, *e*d, and *gh*; i. e. *hi* : *gh* :: d*e* : *ef*. Draw d*i*; which will tend to the Vanifhing Point F; (by Prob. 13, No. 2.)

Produce GE, and make EN equal *AG*; and, on EI, take EO equal *GF* (or, make EN and EO in the ratio of *AG* to *GF*) and draw EP, parallel to NO, cuting the Vanifhing Line in P; the Vanifhing Point of the Diagonal *AF*.

Draw aP, cuting d*i* in f; and, laftly, draw If, and produce it till it cuts aG, in g, which compleats the Figure.

N. B. If the Sides of the Original Figure be produced to their interfecting Points, *H*, *I*, *K*, and *L*, the truth of the procefs may be perceived, by its affinity and agreement with the former methods, *viz.* when the Original Figure is drawn in the Geometrical Plane, its Vanifhing Line, Center, and Diftance being given; which, in this Problem, are given only by Pofition and Diftance; the Original Figure being ufelefs in the Operation, otherwife than to fhew the meaning of every ftep; and that, if the Original Figure be fuppofed in its true Place, and Pofition, each Line, from the Eye, producing a Vanifhing Point, would be parallel to the correfponding Side, or Diagonal of the Figure.

In this Problem, I have fumed up moft of the Rules given in the two laft Sections; which are called elementary, becaufe, by them, the whole of practical Perfpective is performed.

It will, I know, appear, to young practitioners in Perfpective, fomewhat difficult; yet, if they do but fee the Principles on which it is performed, and will be at the trouble to go through the Procefs a fecond time, and compare each Operation with the Problem it refers to, that apparent difficulty will vanifh; and they will find it of great advantage to them, in applying all the elementary Problems to real ufe, in delineating Plane Figures.

This Procefs, difficult and operofe as it may appear, at firft, is infinitely preferable to any other Method of performing it; nay, it is fcarce poffible to perform it at all, by any Rules given by the old writers on Perfpective, or indeed by any; and, confidering that I have given the whole procedure, from the begining to the end, it is not a long one. I have, not only, referred to the foregoing Problems for each particular ftep in the Operation, and (as is cuftomary) left the Reader to apply it; but, I have gone on with him, focially, hand in hand; and, like a trufty Guide, pointed out every ftep he fhould take.

I have, alfo, drawn every Line in the whole Operation, which remains in the Plate; yet there is not the leaft confufion, the ufe of each being obvious, on infpection; but, being taken regularly, in the courfe of the Work, every apparent intricacy is unravelled.

At the fame time, let it be obferved, that there is no need for half the Lines to remain at once; e. g. after having found the Vanifhing Line, DG, its Center and Diftance, and the Vanifhing Points of all the Sides of a Figure, all the operative Lines may be rubed out before we begin to draw the Figure; alfo, finding the Seat of A, is not abfolutely neceffary, but is only another Method to find the reprefentation of the Angle A; but they muft all remain in the Diagram; which circumftance frightens many, with apparent intricacy; which, in reality, is but apparent.

In order to compare the difference, I will fhew, how it may be performed otherwife. But, as a Figure of fo many Sides would be fomewhat confufed, I have made choice of a Triangle, in which, the Procefs is more diftinct, and will be better underftood.

Let ABD be the given Figure, in the Geometrical Plane; of which, GI is the interfection, and JI*M* is its inclination to the Horizon; let NI be confidered as a vertical Section of the Picture; NIM is the angle of Inclination of the Plane of the Figure with the Picture. Fig. 68.

Produce NI; draw AK, BL, and DM, parallel to GI, cuting IM in K, L, and M.

Make I*K*, I*L*, and I*M* refpectively, equal to IK, IL, and IM; and draw *M*N, *L*O, and *K*P perpendicular to the Picture; NI being perpendicular to JI.

Draw AF, BG, and DH, perpendicular to GI, and produce them, till they cut *K*P, *L*O, and *M*N, produced, in *A*, *B*, and *D*, the Seats, on the Picture, of the Angles of the Triangle. *ABD* is its orthographic Projection.

Having thus found the Seats of the three Points A, B, and D, the Reprefentation of each may be found by Prob. 6. as a, the Reprefentation of A, in the preceding Figure; by the fecond method.

C being

Pl. XIV. Fig. 68. C being the Center of the Picture, draw C E at pleafure, and equal to the Diftance of the Picture; and, having drawn *A*C, *B*C, and *D*C, draw *A* k, *B* l, (or *B l*) and *D* m (or *D m*) parallel to C E (or C *E*) and equal to the Diftance of each Point from its Seat, refpectively; *viz.* *A* k equal P*K*, *B* l (or *B l*) equal O*L*, and *D* m (or *D m*) equal N *M*.

Draw k E, cuting *A*C in a; l E (or *l E*) cuting *B* C in b, and m E (or *m E*) cuting *D* C in d.

The three Points a, b, and d, are the reprefentations of the Angles A, B, and D; which, being joined by Right Lines, compleat the Figure.

In this Procefs, it is obvious, there is neither Vanifhing Line nor Vanifhing Point required, the whole being performed by the 6th Problem. If the Interfection, GI, be parallel to the Horizon, ECE may be confidered as the Horizontal Line. But, if the Interfection of the Plane of the Figure, be given, it is immaterial what Pofition it has to the Horizon; the Center of the Picture and Diftance, the Place and Pofition of the Object, and the Inclination of the Plane it is in, being alfo given.

Although this method is perfectly confonant to the Principles of Brook Taylor, yet it is by no means fo mafterly or correct, that is, it is more liable to error, than the other, by means of Vanifhing Points; which, in complex Objects, having various Lines in various Planes, is preferable to all other. In fhort, it is truly Perfpective, which cannot be faid of the old Authors; having few or no elementary Principles, to fupport or demonftrate the Rules they prefcribed, the Operation is, merely, a mechanical Procefs.

As the old Authors had no Idea of any other Vanifhing Line than the Horizontal, nor of placing the Eye any where but in the Point of Diftance, therein; their method of finding the reprefentation of each Angle of the Figure, in a Plane which is inclined to the Horizon, is by means of their Seats on the Ground Plane, and their heights above it; as J of *M*, &c.

As the Procefs would, in this Place, be confufed, by mixing it with the other, I fhall defer it to a more proper Time and Place, in an Appendix to this Work; in which the difference will be clearly and fairly ftated, and exemplified.

PROBLEM XXVIII.

To find the Reprefentation of any Line, or Plane Figure, by means of the Directing Plane. (See the 14th Theorem.)

Fig. 69. Let A B be a Line perpendicular to the Picture, or to the Interfection, IK, of the Plane it is in. Let E be the Eye, in its true Place, as in the foregoing Problems.

At the Diftance E P, equal to the Prime Director *, draw O N, the Directing Line, parallel to the Interfection, I K †.

† Theo. 2.

The Space between O N and I K reprefents all that Space which lies between the Directing Line and the Interfection of the Original Plane with the Picture.

In which Conftruction, the whole of that Space, together with the Directing Plane, NEO, is fuppofed to be turned up, on the Interfection I K, till it falls into the Picture; the Original Figures being turned along with it, into the fame Pofition as ufual.

Produce BA, cuting the Interfection in I, its Interfecting Point; and the Directing Line in D, its Directing Point. Draw E D, which is the Director of AB‡; to which, draw IC, parallel.

‡ Def. 12.

For, C being the Center of the Picture, and I the Interfecting Point of a Line perpendicular to the Picture, IC is its indefinite Reprefentation; and EC, the Direct Radial, is parallel to A B, and equal to ID, a part of AB, produced; confequently, IC is parallel to E D. (15. 1. El.)

As E C (Fig. 37) is equal to *B* R, a part of AC, intercepted between the Interfecting and Directing Lines; alfo, E O is equal to G P, and E L to H *S*; and being parallel, refpectively, the Lines R C and E *B*, E G and O P, &c. are alfo equal and parallel.

* In Theory, the Prime Director is a Right Line joining the Eye, or drawn through the Eye, and the Station Point of the Original Plane. In common Cafes, EP is equal to the height of the Eye above the Ground Plane, always equal to the diftance between the Interfection and Vanifhing Line.

If

If the Sides of the Triangle, FGH, be produced, till they cut the Directing Line, in the Points M*, N, and O, their Directing Points, EM, EN, and EO, are their Directors; to which, their feveral Reprefentations are parallel; J, K, and L, are the interfecting Points of thofe Lines.

Draw Lh parallel to EO, Kh parallel to EN, and Jf parallel to EM; which, by their Interfections, produce the Figure fgh, the Reprefentation of FGH.

Through C, parallel to IK, draw RS, the Vanifhing Line of the Plane of the Triangle; and, having produced the Indefinite Reprefentations to their Vanifhing Points, R, S, Q, the Lines ER, ES, and EQ, are the Radials of the Sides of the Triangle, and confequently parallel to them, refpectively.

The agreement between thefe two Methods of producing the Reprefentation, fgh, is fo very obvious, as not to need any further explanation.

See this further illuftrated, in Fig. 37. Turn up the Picture and Directing Plane.

The Sides of the Triangle ZY and ZX, on the other Side of the Picture, being produced, cut the Interfection (AB) in P and *S*; and, the Directing Line (HG) in G and H; EG and EH, in the Directing Plane, are refpectively parallel to the Reprefentations of thofe Lines, PO and *S*L, the Directing Plane being parallel to the Picture. For, they are the Interfections of Radial Planes with the Picture and Directing Plane; which are parallel, by Def. 4.

The Planes W and X being turned over, till *E* coincides with E; the Section with the Picture, RC, is the indefinite Reprefentation of AC (Theo. 8 and 12) and E*B*, its Section with the Directing Plane, is the Director of the Line AC (Def. 12) which is parallel to RC (by Theo. 14) being the fections of parallel Planes, by another Plane.

SECTION VI.

Of the PRACTICE of PERSPECTIVE, applied to PLANE SOLIDS.

THE laft Section comprehends the whole Art of practical Perfpective, in refpect of Plane Figures; illuftrated with variety of Cafes and Examples; in which, I have ftudied more to render it eafy and familiar, than to embellifh the Subject with what is no way ufeful. Every Figure, which I have given, are fuch as are common, and applicable to Buildings and other Objects; the Pentagon and Heptagon are not fo ufeful in themfelves, as for the fake of managing inclined Lines, in all Pofitions; fo that the Leffons deduced from them are excellent. By varying their Pofitions and Situations, and diverfifying the Cafes as much as is confiftent with utility, I am perfuaded that no Perfon, who has a tolerable Capacity, will be at any lofs to know how to defcribe any Plane Figure, and in any Pofition whatever.

Although fome Perfons, of keener Talents, may imagine, that the preceding Section contained Inftructions fufficient for every occafion (it certainly contains the Elements of the whole) yet, I prefume, if no more had been given, it would be found of but little ufe to many; when, by applying thofe valuable Leffons to familiar Subjects, what, before, might appear fomewhat myfterious will, hereafter, be applied with eafe, as occafion may require.

* Suppofe FG produced till it cuts ON, produced, and EM a Line tending to that Point; then EM is the Director of FG.

Plate XV. It may, with equal propriety be alledged, that there is no need for ſhewing how plane Solids are formed; ſeeing they are only compoſed of plane Figures, in various Poſitions, all which have, already, been copiouſly treated of. Yet, a few Leſſons, and familiar Examples, will not be found ſuperfluous or unneceſſary. I ſhall not dwell on any thing not really uſeful, but haſten to apply the whole to proper Objects; for which, all the foregoing is but preparatory.

In this Section, I ſhall always conſider the Object as ſituated on a horizontal Plane, and the Picture as vertical; moſt of the foregoing Problems, being general, are applicable in all Poſitions whatever.

In the following part of this Work, I would adviſe the young Student, as he proceeds, to draw every Object, or a ſimilar one, at his own diſcretion. Or, if that be not neceſſary, in ſome Caſes, let him carefully notice what things are given, in the Propoſition, and what is required to be done; and then proceed, ſtep by ſtep; not ſuppoſing any thing as done, but what he has paſſed over; by which means, he will ſee, clearly, how every Line is produced, by which the Objects are formed; but to do it, himſelf, and varying the Poſition, a little, is better.

This Precaution would have been neceſſary in the third or fourth Section, but more particularly ſo here.

PROBLEM XXIX.

How to deſcribe a Cube, perſpectively, or any other right angled Parallelopiped*, having one Side, given, parallel to the Picture; at any Diſtance beyond the Picture, its Situation and Dimenſions being known.

No. 70. ECE is the Horizontal Line, and PM the Interſection or Ground Line.

C being the Center of the Picture, make CF, on either Side, or on both Sides, equal to its Diſtance; and let this be underſtood in future Examples, that CE (E being the Place of the Eye) either on the Horizontal or Vertical Line (the Picture being vertical) always denotes the Diſtance of the Picture; ſometimes C*E*.

Firſt; let the Picture be ſuppoſed applied cloſe to a Face of the Cube; in which Caſe, being parallel to the Picture, it coincides with the Picture.

This is generally the Caſe, when there is but one Object; as it anſwers no purpoſe to ſuppoſe it at any Diſtance, from the Picture, as in Figures 59, 60, &c. or, when there are various Objects to be delineated, it is, moſt commonly, ſuppoſed to be applied cloſe to the neareſt, as in this Example (ſee Fig. 38.) ſo that, the Diſtance of the Picture is the Diſtance of a Plane direct before the Eye, and touching the Object, or the neareſt Object, if there are more than one.

Let AB, in the Ground Line, be the meaſure of a Cube, to be repreſented; having one Face in the Picture.

On AB, deſcribe a Square, ABDF, which, being in the Picture, has the full dimenſions of the Side given; wherefore, ABDF is the repreſentation of a Face of the Cube required. (Cor. 5. Theo. 9.)

Draw DC and FC; and DE or FE, diagonal-ways, cuting FC in G, or DC in H; draw GH, parallel to FD, which compleats the Figure AGHB, the repreſentation of a Cube, in that Poſition, and Situation; having but two Faces ſeen.

Note. Each Side (AF, ED, and DB) of the Square AFDB, repreſenting the front Face of the Cube, is, in this Caſe, an Interſection of the Picture, as well as AB; *viz.* of the adjoining Face.

* See the Difinition of a Cube, &c. in the General Introduction, Section 1, Page 41.

If

If another, of equal dimenſions, be required, ſituated on one ſide of the Station Line, at the Diſtance LJ (as $ABEH$) and diſtant from the Picture La, equal LA.
Make LM equal AB; and, on LM, deſcribe the Square LMNO, as before.
Draw OC, NC, &c. and draw aE, cuting LC in a, and MC in e.
Draw ab parallel to LM; and ad, bc, to MN; alſo, draw cd parallel to ab. i. e. having drawn ab parallel to LM, deſcribe a Square on ab; and, at e, deſcribe the Square efgh, parallel to abcd, which compleats the Figure.

Or, having drawn LC, MC, and aE, compleat the Plan afeb. (Prob. 19.)
Deſcribe a Square on ab, and draw cC and dC; draw fg parallel to ad, and gh parallel to cd, which compleats the Figure.

DEM. abcd being parallel to the Picture (by Hypotheſis) is ſimilar to the Original, LMNO†; and its Sides have that proportion to the Originals, as the Diſtance of the Picture (CE) to the Diſtance of the Plane abcd; viz. as EC+LA; i. e. ab (or ad) : LM : : EC : EC+LA. (Theo. 9.) † Cor. 5. Theo. 9.
But, LMNO is a Square; therefore, abcd is a Square.
And, ſince it is parallel to the Picture, the Originals of the Sides af, dg, &c. are perpendicular to the Picture; conſequently they vaniſh in its Center, C ‡. ‡ Cor. to Theo. 4.
Alſo, afgd repreſents a Square in a vertical Plane, perpendicular to the Picture; for LO is the Interſection of that Plane §, and ECE its Vaniſhing Line ‖; wherefore, CE, being made equal to CE, E is the Vaniſhing Point of Lines (in that Plane) inclined to the Interſection in half a Right Angle; conſequently, if one Side (ad) be parallel to the Interſection (LO) and OP being made equal La, PE paſſes through a Diagonal of that Side †, therefore, afgd repreſents a Square; and Ob being equal La, bE is a Diagonal of the Top, dggh, which alſo repreſents a Square, parallel to afeb, and ON is the Interſection of that Plane, with the Picture. ‡ Theo. 10. ‖ Th. 2 & 10. † See N.B. Prob. 20.
Therefore, agc repreſents a Cube, having three Faces ſeen, each of which, is the repreſentation of a Square. Q. E. D. for the oppoſite Faces are alſo Squares.
PQ is the Interſection of a Diagonal Plane cdfe, whoſe Vaniſhing Line would paſs through E.

The Cube AGHB being ſituated on the Station Line (which, on the Picture, coincides with the Vertical Line) has but two Faces ſeen; the Front AFDB, and Top, FGHD, the Eye being above it.
The other Faces, AFGK and BDHI, ſeeing they apparently incline towards the Vertical Line, are conſequently loſt to ſight in this Poſition; as is obvious, ſuppoſing the Solid tranſparent.

EXAMPLE I.

To draw the Repreſentation of a high Wall, the End being parallel to the Picture, and at ſome Diſtance from it.

This Object, being a right angled Parallelopiped, is delineated as the Cube. Fig. 71.

Let PJ be equal to the Diſtance of the Wall from the Station Line, on the Left.

Make PQ equal to its thickneſs, and draw PC and QC; make Pd equal to the diſtance of the end of the Wall, from the Picture; draw dE cuting PC in R, and draw RS parallel to PQ.
Now, ſince P is the interſecting Point of RF, the common Interſection of the Plane of the Wall (which is ſuppoſed vertical) with the Ground; PT, perpendicular to the Ground Line, or parallel to the Vertical, is the Interſection of that Plane†. † Cor. 1. Theo. 2.
And, ſince the Wall is perpendicular to the Horizon, conſequently, its Angles or Corners, RU and SV, are parallel to PT or QX the Interſections of the Planes thoſe Lines are in (for they are parallel to the Picture, being vertical) therefore, draw RU and SV parallel to PT, i. e. to the Vertical Line.
Make PT equal to the known height, and draw TC, cuting RU in U, and draw VU parallel to RS; then RUVS, repreſents the End of the Wall, which is parallel to the Picture (on the Ground Line).
Laſtly. Make PM (on the Ground Line) equal to the Diſtance of the farther end of the Wall from the Picture, or dM equal to its known length.

Draw

4

Plate XV. Fig. 70. Draw E M, cuting P C, the indefinite Reprefentation of its common Section with the Ground Plane, in F; and draw F Y, parallel to R U, which compleats the Wall.

Or, if C *F* (on the Vertical Line) be made, to C E or *E*, as the height of the Wall to its Length, *F* will be the Vanifhing Point of the Diagonal, RY, which cuts T C, in the Point Y, for its length; draw F Y parallel to R U.

Let it be obferved, here, that if there be not room enough, on the Interfection, to fet off the whole length of the Wall, equal *d*M, take half the Diftance, P J (equal Pf, in the Plan below) make C e equal half C E (the Diftance of the Picture) and draw J e, projecting the fame Point F.

If the half meafure be too much, take a third, or fourth, or any equal portion, of P M (equal P m) the fame part being taken of C E will anfwer the fame purpofe (fee Prob. 17.)

E X A M P L E II.

How to reprefent feveral Parallelopipeds, as Blocks of Stone, &c. for the Bafement of a Building, ranged in a Right Line, perpendicular to the Picture.

Fig. 71. W, X, Y, and Z, are the Plans, or Seats of the Blocks on the Ground Plane.

Let ABD, the Front of the firft, be clofe to the Picture, i. e. in the fame Plane; confequently, in its geometrical Proportion, of height and width.

Draw AC, BC, and DC. Make A *a*, on the Interfection, equal to the length of the firft, *ab* equal to the fpace between them, and *bc* equal to the fecond.

Draw *a* E, *b*E, and *c* E, cuting A C in F, G, and H; draw F I, G K, and H L perpendicular, cuting B C; and I M, K N, and L O, parallel to BD, cuting DC, in M, N, and O, and compleat them, as in the Figure; having regard to the firft hiding part of the next, and that of the third, &c.

Now, in order to continue them further, to any length, this Expedient may be ufed, when the meafures exceed the limits of the Picture, and, confequently, cannot be applied to the Interfection, or Ground Line.

Draw H g, from the fartheft corner of the fecond Parallelopiped parallel to the Interfection, and draw *a* C and *b* C, cuting it in e and g.

If it be required to continue them of the fame dimenfions, and fpace, make H f equal e g; or, A *d* being made equal to *a b*, draw *d* C, cuting g H, at f, and draw f E and g E, cuting A C in a and b. Compleat the Parallelopiped a b c d as in the Figure.

After the fame manner they may be repeated as often as you pleafe by drawing b *g* parallel to g H, cuting *d* C and *b* C in *f* and *g*, and then proceed as before.

This Expedient is, in many cafes, better than applying the half-meafures, or other portion, on the Ground Line; as they may be continued infinitely, within the compafs of A *b* (one Block and Space) by drawing G h (inftead of g H) which, being cut by *a* C, at i, needs not to be tranfpofed, as e to f.

It muft be obvious, that G h (or g H) is cut in the fame proportion to the fecond Block (at the hither end, or the farther) as A *b* is to the firft; and confequently, i E, h E, &c. will cut A C in the fame Points, as if the full meafures were applied on the Ground Line; and fo of *g* b, to the next.

E X A M-

EXAMPLE III.

To draw several long Parallelopipeds, parallel amongst themselves, and perpendicular to the Picture; representing large Joists, supporting a rough Floor (over Head).

Let W, X, Y, and Z be the ends of the Joists, of equal dimensions, and equally spaced; and, since they are supposed to be parallel to the Picture, therefore, they have their geometrical proportion on the Picture. (Cor. 5, Th. 9.) Fig. 72.

Let them be drawn accordingly, by a Scale of equal Parts.

Then, because the Joists are perpendicular to the Picture, in respect of their length; (i. e. they are horizontal, and at right angles with the Picture) draw aC and bC, &c. from every Angle, necessary, as in the Figure; and determine their length, by Prob. 15, i. e. make AB, or *A B*, equal to the required length, and draw BE, cuting AC in D; or *A*C in *D*.

Through D, draw FG parallel to the Vanishing Line (CE) which determines the under Sides, Fe, &c. and the perpendicular Lines, de, &c. compleat them.

The divisions of the Boards are determined by Prob. 8th; i. e. take any measure, *Aa*, and repeat it, on *AB*, as often as there is occasion, if the Boards are supposed equal; otherwise *A a*, *a b*, &c. must be made in the same ratio as the Boards, to each other; and at the last division, *e*, draw *e D*, cuting the Vanishing Line in H; and draw *a*H, *b*H, &c. cuting *AD* in 1, 2, 3, &c. through which, draw Right Lines parallel to *AB*, i. e. to the Vanishing Line.

Or, divide *AB* into the same number of Parts (as *A*1, 12) and draw Lines to E, which will produce the same, as is obvious; seeing that it does not depend on the dimensions of the Parts, but on their number and ratio to each other. (See Prob. 8.)

Note. *A*1, 12, on *AB*, being each equal to a fifth Part of *AB*, is the geometrical width of the Boards, in proportion to the ends of the Joists, and to *Ac*, the space between them.

PROBLEM XXX.

To draw the Representation of a Cube, or other right angled Parallelopiped, any how inclined to the Picture.

First. Let the Square ABCD be the Plan of a Cube, in the Geometrical Plane, equally inclined to the Picture, and to the Intersection, at the Angle A. Fig. 73.

Lie down the Distance of the Picture, CE¹, from the Center, as usual; E and E are the Vanishing Points of all the Lines of its Sides. (Prob. 20.)

Draw AE, both ways, A being the intersecting Point of two Sides, AB and AD.

Find the perspective Plan *ABCD* (by Prob. 10) or of the two Sides, AB, AD, only.

Draw AF perpendicular to the Ground Line, and equal to AB, a Side of the Cube, and draw FE, both ways.

Draw *B*G and *D*H, parallel to AF; and draw GE and HE diagonal-ways, cuting each other, at I, which compleats the Cube, AGH.

Otherwise, without drawing the perspective Plan, by means of the Vanishing Lines of its Faces.

Draw AF perpendicular to the Ground Line. Then, AF is the Intersection of two Faces of the Cube. Make AF equal to a Side, AB.

Draw AE and FE, both ways; and through E, on either side, draw the Vanishing Line of the contiguous Face of the Cube. (Prob. 3.) E*E*, or EF.

Make E*E* or E*F*, on either Side, equal EE¹, and draw A*E* or A*F*, cuting the indefinite Representation FE, in G or H, and draw *B*G, or *D*H, perpendicular.

Draw *B D* parallel to the Intersection, and *D* H parallel to AF; and lastly, draw GE and HE, intersecting at I, as before.

Plate XV. Fig. 73. $ABGHD$ is the Reprefentation of a Cube, in the pofition required; fituated on a horizontal Plane, and its vertical Faces equally inclined to the Picture.

DEM. For, ABCD the given Plan is a Square, wherefore $ABCD$ reprefents a Square (Prob. 20) and FGIH alfo reprefents a Square, in a Plane parallel to $ABCD$. (Th. 3. and Cor. 1.)

And, they are between the fame Parallels, AF, BG, &c. therefore equal - - - (18. 1. El.)

But, AF is the Interfection of the vertical Planes, $ABGF$ and $ADHF$; and EE is the Vanifhing Line of $ABGF$ (Theo. 2 and 10) E is its Center (Def. 19) and EE^1 its Diftance (Cor. 2. 7.)

Wherefore, E is the Vanifhing Point of Lines inclined to the Interfection of that Plane, in half a Right Angle or 45 Degrees; confequently, it is the Vanifhing Point of the Diagonal of a Square, in that Plane, whofe Sides AF and BG are parallel to the Picture (Prob. 19;) and, AB and FG, perpendicular to them, vanifh in E, the Center of the Vanifhing Line, EE. - - - (Cor. to Th. 11.)

Therefore, AFGB reprefents a Square, at right Angles with $ABCD$ and AFHD, which (after the fame manner) may alfo be proved to be the reprefentation of a Square; and their oppofites, $DHIC$ and $CIGB$, being between the fame parallel Planes, are alfo reprefentations of Squares; confequently, $ABGHD$ reprefents a Cube. (See Defin. Page 41.) Q. E. D.

a c f g reprefents another Cube of the fame dimenfions, and in the fame pofition to the Picture; whofe fituation in refpect of the former, is as the Plan, W, to BD; which, on account of the fhort Diftance (CE) has a diftorted appearance, being remote from the Station Line, though wholly within the Radius of the Diftance.

By means of the Interfection (AF) of the Face a d f g, or af of the Face a b c d (AF, or af, being made equal to the Side of the Cube) the proportion of the parallel Lines (a d and f g, or b c) in thofe Planes are determined. For, AF, a d, and f g; alfo af, a d, and b c are in the fame Plane, and are all parallel to the Picture; confequently, they are parallel between themfelves, and to their Originals†.

† Theo. 9. Cor. 1.

EXAMPLE IV.

How to draw Parallelopipeds ranged in a Right Line, and inclined to the Picture obliquely.

Fig. 74. Let X and Z be the Plans of right angled Parallelopipeds; their inclination to the Picture is the Angle AIA, which they make with the Ground Line, CK.

C being the Center of the Picture, and CE its Diftance, find the Vanifhing Points, G and H, of the Sides and Ends of the Parallelograms, X and Z, (Prob. 2, and 4.) and lie down their Diftances on the Horizontal Line (Pr. 12.) at E^1 and E^2.

Transfer all the meafures, IA, IB, &c. of the Objects, and the fpaces between them, to the Interfection, at A, B, and C; and, having drawn the indefinite Reprefentation, IG, of the whole length, draw AE^1, BE^1, &c. cuting it in a, b, and c.

Becaufe the Angle I touches the Picture, IF, perpendicular to the Ground Line, is the Interfection of the Plane of the Fronts, and alfo of the End, IK.

Make ID equal to the height of the Objects, and draw DG, DH, and IH.

Make IK equal IK, and draw KE^2 cuting IH; Ik reprefents IK. (Pr. 17.)

Draw a d, b e, and c f, parallel to ID; and, where they cut DG, in d, e, and f, draw dH, eH, and fH; and from i, where k i cuts DH, draw iG, cuting them in j, h, and g, which compleats the upper faces.

h l, parallel to ID, &c. cuting kG, compleats the End b e h l.

If another Object be required, in the fame Line, feeing that the meafures on the Ground Line, would exceed the limits of that Picture, take GE^2 half GE^1, and draw E^2 c till it cuts the Interfection, at a.

Make aB equal to half the fpace between the Objects, and Bc equal half the width of the next Object, and draw BE^2, and cE^2, cuting IG in b and c.

If it be required higher than the reft, make IF equal to its known height, and draw FG. Draw perpendiculars from b and c, cuting FG in g and h.

Draw

Draw bH cuting k G at d, and draw gH ; and di parallel to bg, which compleats that Object.

Or, if c n be drawn parallel to the Ground Line, and BG cuting it in n ; divide c n, at m, as AC (equal AC) is divided in B ; and, from m and n, draw to E^1 cuting IG in the fame Points, b and c, as before.

By either of thefe Expedients they may be continued to any length.

Note. G, the Vanifhing Point of the front Line IC, and the parallels to it (being much inclined) may be fuppofed to be beyond the limits of the Picture ; confequently, Ic, D f, &c. are tending to a Vanifhing Point which is not on the Picture, and muft, therefore, be drawn by means of the Expedients, in the 13th Problem; which are very convenient when but few Lines are wanting.

But, a better Expedient than them all is (when the Point is not very remote) to fix a Lath to the Drawing Board, or ftraining Frame of the Picture, and continue the Vanifhing Line as far as is requifite, on the Lath ; fix a Pin in the Vanifhing Point, and, with a long Ruler, draw the Lines I G, DG, &c. for no other Expedient, whatever, can be fo true, i. e. it cannot be performed fo accurately as to have the Vanifhing Point itfelf. For large Work (as in Scenery) a fine, fmooth Cord, fixed in the Vanifhing Point, is a good Expedient.

Prifms of all kinds may be reprefented after the manner of the foregoing, which are right angled Prifms, having found the Plans of their Bafes, on the Ground Plane or other Plane, whether they be triangular, quadrangular, or multangular ; of which one or two Examples will be fufficient.

EXAMPLE V. Pl. XVI.

To reprefent a hexagonal Prifm, perpendicular to the Ground Plane.

The Situation, Diftance, and Pofition being determined, or AB being a Side, given or found, in Perfpective, whofe Radial is E V, find the Vanifhing Points, I and K, of the other Sides, and compleat the Plan, ABCDEF, of its Bafe (by Pr. 24.) Fig. 75.

Draw A a, B b, &c. perpendicular to the Ground Plane, i. e. to its Vanifhing Line (IK) or Interfection (GD.)

Produce any Side, as AB, to its interfecting Point, (G) and draw G H, perpendicular to the Ground Line.

G H is the Interfection of the Plane of the Face A a b B, or g h of B b c C (Pr. 3) for it is vertical ; and G, or g, is the Interfecting Point of one Line in the Plane §. § Th. 2. C. 1.

Make G H equal to the known height of the Prifm, and draw H V † cuting A a and B b in a and b ; the Original of a b being parallel to the Original of AB. † Prob. 13.

Then, becaufe A a and B b are the common fections of the contiguous Faces with A a b B, draw a I and b K, cuting the other Perpendiculars F f and C c, in f and c ; which compleat as many Faces as can be feen.

The other Faces, which are all Parallelograms (See Def. Page 41) F e, E d, and d C are fuppofed to be feen through, the Object being imagined tranfparent ; which, in fome Cafes, is a neceffary Expedient ; by which means, the connection of the feveral Parts are more accurately determined.

EXAMPLE VI.

To reprefent a pentangular Prifm, laid along on the Ground Plane ; the Interfection and the Seat of the Object, on the Ground Plane, being given.

AB is the given Side of the Pentagon, and BD the Seat of one Face of the Prifm. Fig. 76.

C being the Center of the Picture, and CE its Diftance, find the Vanifhing Points, I and K, of AB and AD (Prob. 2) by making the Angles J E I and O E K equal,

Pl. XVI. Fig. 76. equal, refpectively, to B*A*B and *DA*D; or, having determined one, find the other, by Prob. 4. making the Angle IEK equal BAD, a Right Angle.

Make a b to reprefent AB (Prob. 17.) and, on a b, conftruct a Pentagon, in a vertical Plane, on the fame Principles as in Prob. 23.

I being the Vanifhing Point of a b; FG, paffing through I, is the Vanifhing Line of the Plane of the end of the Prifm (being perpendicular to the Ground Plane) I is its Center, and E I its Diftance. (Prob. 3.)

As there is not room, on the Picture, beyond the Vanifhing Line, FG, to find the Vanifhing Points of the Sides of the Pentagon, take I*E* on this Side equal to I E, through which draw MN (the parallel of the Eye, of that Plane) parallel to FG, and, on *E* defcribe a Semicircle; which, divide into five equal parts.

Make *a b*, *b c*, and *a d*, *d e*, each equal to a fifth part, and draw *E b*, *E c*, &c. which, produce to the Vanifhing Line, producing the Vanifhing Points, F, G, & H, by which, the Pentagon a b c d e is completed, as in the Figure (by Prob. 23.)

Draw aK indefinite, of the Side A D; and make a f to reprefent a length equal to AD (Prob. 17) alfo, draw e K and dK.

Draw H g, through f, and g F, cuting dK in h, which compleats the Figure.

For, a f g e and e g h d reprefent Parallelograms, which are Faces of the Prifm a f h d b; and a b c d e reprefents an End which is a regular Pentagon; to which the other Faces are perpendicular, I E K being a Right Angle (which I a K reprefents, Prob. 4) equal B A D.

The Expedient of turning over the Vanifhing Plane, F*E*H, to find the Vanifhing Points of the Side of the Pentagon, is a very ufeful and neceffary Expedient; becaufe it often happens, that there is not room for turning it over on the other Side of the Vanifhing Line, if it be remote from the Center. And, notwithftanding it interferes with the Object in this Diagram, yet being done firft, and the Vanifhing Points found (as in the Figure) all the operative Lines (for finding them) are rubed out before we begin to draw the Object; or rather, they are never drawn, for, the Vanifhing Points are determined (as F.) by applying a ftreight Ruler to *E* and *b*, &c.

In Fig. 37, the horizontal Vanifhing Plane, N*IK*L, or the vertical Plane V, being turned over on their Interfections with the Picture (which are the Vanifhing Lines they produce) it is obvious that the Angles, formed by the Radials, EN, EO, &c. are the fame on both Sides, and confequently, they produce the fame Vanifhing Points, N, M, &c. whether the Vanifhing Plane be turned up or down, to the Right or to the Left.

EXAMPLE VII.

To reprefent a quadrangular or hexagonal Pyramid; or any other.

Fig. 77. Let W and Z be the Plans of the Bafes of two Pyramids, whofe Altitudes are known, the one a fquare Bafe, the other a Hexagon; two Sides, of which, are parallel to two Sides of the Square.

Find the perfpective Plan ABCD, of the Bafe, W (by Prob. 21.)

If the Original Figure be drawn in the Ground Plane, the fhorteft way is by producing its Sides to the Interfection, AK; and having found the Vanifhing Points, M and N, of the Sides, draw their indefinite Reprefentations (Method 1ft.)

Draw the Diagonals, AC and BD, interfecting at H, the Center of the Bafe.

Draw HI perpendicular; and where the Diagonal AC cuts the Interfection, at A, draw AF, alfo perpendicular to the Ground Line.

Make AF equal to the height of the Pyramid, and draw FG, cuting the Perpendicular HI at I, and draw AI, BI, and DI, which compleat the Figure.

If (as in this Example) on account of the Diagonal, AC, having very little inclination to the Ground Line, the Altitude cannot be determined with accuracy; and the other Diagonal, BD, is fo much inclined as not to cut the Interfection within the Picture; it may be thus determined.

From

From either Vanifhing Point, M or N, draw a Right Line through the Center of the Bafe, till it cuts the Ground Line, at a ; draw a f parallel and equal to A F, and draw f N, cuting H I in the fame Point, I, for the Vertex of the Pyramid, as before. Or, as in the following.

Second. Having found the perfpective Plan ABCDFG, of the hexagonal Bafe, Z (Prob. 24) M and O being two Vanifhing Points ; the other is out of the Picture. From any Point, P, in the Vanifhing Line, draw PH, cuting the Ground Line, in K.

Draw K L perpendicular to the Ground Line, and HI parallel to K L.

Make K L equal to the known Altitude of that Pyramid, and draw L P, cuting HI in I ; and draw A I, B I, &c. as before, which compleat that Object.

Prifms and Pyramids are very ufeful Subjects for practical Leffons. By the firft, we learn to form Pavillions, Temples, Cupolas, Alcoves, &c. which are, generally, of fome prifmatic Form ; by the other we may form a Spire, Obelifk, or Pyramid, of any Figure.

As thofe Objects admit of infinite variety (as many as there may be various Poligons) it would be to little purpofe to give more Examples of them, feeing that, if a Poligon of any number of Sides be drawn in Perfpective, for the Bafe (by the various Problems of the laft Section) the reft is the fame as in the Examples given.

It may, I prefume, have been obferved, how inconvenient it would be to have the Ground Plans of all the Objects, to be delineated, geometrically drawn, in the Geometrical Plane ; befides, the impoffibility of having room for them, when the Objects extend to any confiderable Diftance ; as the Plans, W, X, Y, and Z, of the Parallelopipeds in the fecond Example (Plate XV.) evince.

It may alfo be obferved, that they are of no other ufe than to know how the Objects are fituated, in refpect both of the Picture and of each other ; but fince it is obvious, from what has been done, that they are not abfolutely neceffary to be drawn, feeing that, if their meafures are applied to the Interfection of the Plane they are in, it will anfwer the purpofe ; therefore I fhall, in future Examples, where they are not abfolutely neceffary, do without them. Yet it muft not be underftood, that we can do entirely without, which is impoffible ; but, in many Cafes, their Meafures and Diftances, only, are requifite. In others, the Situation and Pofition of one Object to another, being irregular, require a true and correct Plan to be firft drawn (but not on the Picture) from which all the Vanifhing Points may be afcertained ; as every Line, in Perfpective, is determined from its geometrical Proportion, and the pofition of the Object to the Picture ; confequently, they muft be known or imagined, otherwife we have no foundation to build on.

EXAMPLE VIII.

To reprefent Steps with Kirbs at the ends ; *as at the entrance into a Houfe, &c.*

Firft, when they are parallel to the Picture ; and the Picture clofe to them.

Fig. 78 is a Section of the Steps, in their geometrical proportion, with the Inclination of the Kirbs to the Horizon, and to the Picture.

Take AB, on the Ground Line, equal to the width, or more properly, to the length of the Steps ; and, A D and BJ to the thicknefs of the Kirbs. Fig. 79.

Make AF equal to their height, and defcribe the Rectangles A F G D, and BHIJ, geometrically ; which are fuppofed to be in the Picture.

C is the Center of the Picture, and CE the Diftance. Draw A C and BC.

Make B*a* equal to the Diftance of the firft Step from the Front of the Kirb ; and *a b*, *b c*, and *c d*, each equal to the breadth of a Step, and draw *a* E, *b* E, &c. cuting B C in a, b, c, &c. the perfpective breadths of the Steps, on the Ground.

Make B*e*, *ef*, &c. each equal to the height of a Step, and draw *e* C, *f* C, &c. cuting the Perpendiculars, from a, b, c, &c. at l, m, &c. and transfer them, by the parallel Lines a f, b g, &c. on the Ground, to the other end, cuting A C at f, g, h, &c. from which Points, draw the Perpendiculars fr, gs, &c. and, from l, m, &c. draw Lines parallel to AB, cuting them at r, s, &c. draw r C, sC, &c. cuting them again, at *s*, *t*, &c. and the parallel Lines *lr*, *ms*, &c. at the internal Angles, being drawn, compleat the Steps.

 The

Pl. XVI. Fig. 79. The upper Step having a greater breadth than the other, make *d* D equal to its breadth, and draw DE cuting BC at e; the Perpendiculars d o and e p cut *b* C in o and p; and o u, p v, being drawn parallel to the Ground Line, cut the corresponding Perpendiculars i u and k v, which determine it; *viz.* o p v u.

If the Kirbs are square, they are right angled Parallelopipeds, and are delineated, as described in Problem the 29th. AFGD and BHIJ are the Fronts, AFvk and BHpe are the Sides, which inclose the Steps; and, FvwG and HpqI are their upper Faces.

If they incline with the Steps (as is common) make the Rectangles AK and BL equal to the Ends; suppose, equal to the height of a Step.

Draw C*C* perpendicular to the Horizontal Line; and, CE being equal to the distance of the Picture, make the Angle CE*C* equal to the inclination of the Steps, cuting C*C* at *C*, which is the vanishing Point of their Inclination.

Draw K*C*, L*C*, &c. cuting GC, IC, &c. in x y, &c. which compleat the inclined Faces of the Kirbs, as in the Figure; by drawing x u and o y.

In this Example, the front of each Step, being parallel to the Picture, are similar to each other; and have that Proportion to each other, respectively, as their several Distances, i. e. the second Step, is, in length and height, to the first, as CE is to CE + *a b*; and, the second is to the first, as CE is to CE + *a c*, &c. † † Theo. 9.

VL, passing through *C* parallel to the Horizontal Line, is the Vanishing Line of a Plane inclined to the Horizon, in the Angle *C*EC, equal E*C*L, and *C* is its Center (see Fig. 15, No. 3, P. 72.)

For C*C* is perpendicular to VL; and E*C* producing the point *C*, is the Radial of such Lines in the inclined Plane as are perpendicular to its Intersection KL; therefore, *C* is their Vanishing Point. (Cor. Th. 11.)

Case 2nd. EXAMPLE IX.

When the Steps are inclined to the Picture; the Inclination being determined, also their Situation, in respect of the Station Line.

In this Example I shall suppose, that there is not room on the Picture for ascertaining the Vanishing Points and their Distances, as usual; in order to apply the 12th Problem.

Fig. 80. C being the Center of the Picture, and KCL the Horizontal Line, draw CE perpendicular, and take CE any equal part (suppose one third part) of the determined Distance of the Picture.

Make the Angle CED equal to the Inclination known; and CD will be one third part of the distance of the Vanishing Point L, of one Side; which being determined, all the rest are determinable, arithmetically (by Prob. 12.)

Or, make DEF a Right Angle; CF will be a third part of CK; and FE will be a third part of its Distance, by the same, &c.

Having thus obtained the Vanishing Points, K and L, and their Distances E^1 and E^2 being laid down (CE^1 being equal to three times the difference between FC and FE; and CE^2 to three times the difference between CD and DE) then, proceed as usual; A*E* being the Intersection of the inward Angle of the Kirb.

Draw AK, the indefinite Representation of the Front, and AL of the End.

Let AJ be the length of the Steps (eq. AB, Fig 79) draw JE^1 cuting AK at *A*.

Make A*a* equal to the distance of the first Step; *ab*, and *bc*, &c. each equal to the breadth of a Step, and draw *a*E^2, *b*E^2, &c. cutting it in a, b, &c. and, seeing there is not room on the Intersection A*c*, to apply the measure of the top Step, which, suppose equal to A*c*, draw d*f* parallel to A*b*, and *c* L cuting it in *f*; and draw *f* E^2 cuting AL at B.

Draw the Perpendiculars aj, bf, &c. indefinite.

On A*E*, take A*B*, *B C*, &c. each equal to the height of a Step, and draw *B*L, *C*L, &c. cuting the Perpendiculars from a, b, &c. as in the Figure.

Draw

Draw aK, and *AL* cuting it in k; draw kl perpendicular, and jK cuting it in l; draw lL, and eK cuting it in m, and fK cuting the Perpendicular, mn at n, and proceed, after the fame manner, throughout.

Or, having drawn all the Lines eK, fK, &c. indefinite, draw lL; and mn perpendicular to the Ground Line; then nL, and op, &c. by proceeding after that manner the Steps are completed.

The Kirb being equal to BJ (as in the former Figure) draw BE^1, cuting AK at *D*; *AD* reprefents the thicknefs of the Kirb, at that end.

The Original of the Point, G, being fuppofed on this fide of the Picture, and being in the Ground Plane, the Reprefentation (G) is confequently below the Ground Line, AB. (See N.B. Prob. 8. Fig. 45. The meafure being applied on this fide, i.e. below AB, as AH, fuppofes AB proportioned perfpectively; but, being at the other Extreme, as aB, it is projected; and is, confequently, larger than the Original.)

Make A*d* equal to BJ, and draw E^1d, cuting KA produced, in G; AG is the projective reprefentation of the end of the Kirb, the Original, of which, is equal to A *d*, equal BJ.

Draw the Perpendiculars GH, *AF*, and *DG*, indefinite; and, A*E* being made equal to the height of the Kirb, draw *E*K, cuting them in *F*, and *G*; and, being produced, it cuts GH at H.

*E*L cuts a Perpendicular from B, at I; and, if *F*L, *G*L, and HL be drawn, KI, cuting them, compleats the upper Faces, *F*J and *E*I*b*H. Draw *gb* perpendicular, cuting GL, at *g*, which compleats the End.

If the Kirbs are inclined with the Steps, as in the former Cafe, draw LV, perpendicular to KL; which is the Vanifhing Line of the Planes A*E*IB, *AF*J, &c.

Make LE^2 equal to its Diftance, and draw E^2V, making the Angle LE^2V equal to the inclination of the Steps, cuting LV in V, the Vanifhing Point of the inclined Sides*. KV is the Vanifhing Line of the Inclination. (Theo. 10. Cor. 1.)

A*B* being made equal to the height of the Front, draw K*B*, cuting *DG*, *AF*, and GH, in *K*, *H*, *B*, and *L*; from which Points, draw *K*V, *H*V, &c. cuting the horizontal Lines *G*L, HL, &c. in s, r, t, and u; through which, draw Ku, or join the Points rs, tu, only.

Note. The Picture might, with equal propriety, have been fuppofed on this fide of the Kirb, entirely; but, as it frequently happens, in Practice (the Picture being fixed inadvertently) that fome parts of the Object would project on this fide; I thought it neceffary to give an Example how to proceed in that Cafe.

SCHOL. In this Example, it is obvious that, the Plane A*E*IB, in which, the Steps are perfpectively proportioned, being farther removed from the Vanifhing Line, VH, is fitter for the purpofe than GHL; and, if the Original of GH was in the Picture, the meafures on A*E* would be lefs than the full meafure of the Steps, feeing it would be beyond the Picture.

A*Bob* is the Section made by the Picture, with the inclined Kirb, and fhews how much is fuppofed to project on this fide. A*b* is the fection of the Bottom, confequently, parallel to the Horizontal Line; *ob* is the fection of the End, which is vertical, therefore parallel to VL its Vanifhing Line; and *Bo* is the Section with the inclined part, confequently, parallel to the Vanifhing Line, KV, of the inclination of the Steps. (Theorem 2nd.)

* This will be clearly demonftrated and made manifeft, in the 12th and laft Section of this Book.

N.B. The Vanifhing Line (VW) of the Roof of the Object in the Apparatus, is determined in this manner (as KV) and is in a fimilar Pofition, on the direct Picture, MNOP.

V, the Vanifhing Point of HG, anfwers to K, in this Figure; Y anfwers to L, and W to V.

SECTION

Plate XVII.

SECTION VII.

Of the application of Perſpective, to MOULDINGS, &c.

THE foregoing Section contains many uſeful Leſſons in plane Solids, which may be conſidered as parts of Buildings, &c. In this, I intend to ſhew how the more decorative parts are formed, ſuch as Pedeſtals for Columns, Cornices and Entablatures, &c. of the various Orders; without which embelliſhments, a Building ſeems naked and deſtitute of Ornament.

Theſe Leſſons, of Steps, are very neceſſary to the art of delineating Mouldings; which, when we know how to manage well, particularly ſuch Steps as return at the Ends, or on four Sides, forming mitre Angles, the greateſt difficulty is ſurmounted. For Mouldings, breaking round a Pedeſtal, or the internal and external Angles of a Cornice, &c. properly conſidered, are but ſo many Steps, one above or below another, of different dimenſions; formed by the Fillets, between the cylindrical parts, which are, properly, the Mouldings; and are effected only by Light and Shade. The Fillets, between them, are narrow Planes, cuting the curved Surfaces in parallel Lines; which being deſcribed, by the Rules given, and the mitre Angles of the Mouldings drawn, the buſineſs of the linear part is done, and nothing remains but to give the appearance of ſolidity, convexity, and concavity, by a proper diſpoſition of Light and Shade.

EXAMPLE XI.

How to delineate ſquare Steps, returning on every Side.

Fig. 81. Firſt, when they are parallel to the Picture. AB is their length, ABCD is half the geometrical Plan of the firſt Step; FGHI of the ſecond, and KLMN of the third; and let OP be the meaſure of a right angled Block, on the upper Step.

C being the Center, draw AC and BC; and AE, which is a Diagonal, repreſents AQ, in which are the Seats of G, L, and O, the Corners of the Steps.

Compleat the Square A*DC*B (Prob. 19) and draw the other Diagonal B*D*; which, it is evident, tends to a Point on the left hand of the Center, equal to CE, the Diſtance of the Picture.

Produce FG, KL, &c. to the Ground Line, cuting it in *g*, *l*, &c. i. e. make A*g*, *g l*, B*h*, &c. equal to the breadth of the Steps, and draw *g*C, *l*C, &c. cuting the Diagonals at *G*, *L*, *H*, *M*, &c. the perſpective Seats, on the Ground Plane, of the ſeveral corners of the Steps.

Draw AF perpendicular to AB, and make Aa, a*b*, and *bc*, equal to the height of the Steps, as in the former Examples; and, having drawn the Perpendiculars from *G*, *L*, &c. draw aE, *bE*, and cE, cuting them at c and e, g and i.

Draw ab, cd, ef, &c. parallel to AB, cuting Perpendiculars from B, *H*, &c. at b, d, f, &c. which give the repreſentations of the Fronts, correſponding with GH and LM. AabB, being in the Picture, has its full dimenſions, in height and length; the other two are in proportion to their Diſtances.

Having obtained the Fronts (by means of an imaginary Plane paſſing through the Diagonal A*C*, which A*cFC* repreſents) finiſh the ſquare of the Top, iklm†.

Then, draw fC and bC, till they cut hk and df, which repreſent the return of the Steps on that Side, ſo far as they can be ſeen; on the other Side they are ſeen to their full extent, the Point of view being on that Side.

† Prob. 29.

Draw aC and eC, &c. cuting Perpendiculars from *D*, *I*, &c. at *f*, j, and m; and, having compleated the Face g m i, which is somewhat seen, from j and *f*, draw Lines parallel to the Ground Line, cuting the adjacent Step, which terminate their appearance, and compleat the Steps.

To represent the Block on the upper Step; draw the Perpendiculars *O*r, and P q indefinite; make *cd* (on the vertical Intersection, AF) equal to its height, and draw *d*E, cuting *O*r, in r; *c*E cuts it at o.

Draw op and rq, parallel to the Steps, which cuting a Perpendicular (*P*q) compleat the Front, orqp. OC and rC, cuting *N*s, finish the End, orsn, as in the Figure.

I presume that the Reader will, ere this, have discovered, how inconvenient it is to have the perspective Plans, of complex Objects, in the very place where the Object itself is drawn; which, I am sensible, must render it, to young practitioners, perplexing and intricate, to distinguish one part from another. That difficulty shall be obviated in the next Example; but I must observe, that the connection of the several parts, and the agreement between the Plan and Elevation, would not have been so distinctly seen, without the Plan in its true place; hereafter we shall place it either above or below, as is most convenient.

Let it be observed, notwithstanding, that this apparent intricacy will vanish at a second process; and let it be noticed, that, as the Side *BC*, of the ground Plan, passes through the Angles, d and h, of the Steps, it is not so necessarily; as it must be obvious, if the Center of View (C) was moved, on either hand, it would not. The Line Bp, being considered as a diagonal Line, passing through the Angles of the Steps, d and h, cannot be in the Ground Plane, as *BC*, a Diagonal of the Square, but making an Angle of elevation with the Diagonal B D; and consequently, its Vanishing Point cannot be at C, in the Horizontal Line, but above it, in the Vanishing Line of a Diagonal Plane, passing through B Q.

Above the Vanishing Line, let *AB* be drawn parallel to it, which let be equal to AB; the Point *A* perpendicularly over A, and *B* over B. Fig. 81. No. 2.

Then, if *A*C and *B*C be drawn, and the Diagonal *A*E, cuting *B*C in *C*; and, *CD* being drawn parallel to *AB*, compleats the Plan of a Square, *ABCD*, corresponding with A B *C D* below.

And, if the measure of the Steps be set off, from *A* and *B*, at *a*, *b*, *c*, and *d*, *e*, *f*; *a*C, *b*C, &c. being drawn, cut the Diagonals *AC* and *BD*, in *g*, *h*, &c. which correspond with *G*, *H*, &c. below; as may be seen by the doted Lines *gG*, *hH*, *mM*, *pP*, &c.

Wherefore, if there be room above, and not below, the Plan may be formed above; by means of which, the Object may be completed; drawing the Perpendiculars *g*c, *l*g, &c. cuting aE, *b*E, and *c*E; and *h*d, *m*h, &c. cuting the parallel Lines ef, &c. which terminate the other ends of the Steps.

EXAMPLE XII.

Is an Expedient, by means of which, Mouldings, &c. may be delineated, without having the perspective Plan in the Work.

Let ABCD be a perspective Plan of a Square, whose measure is AI, on the Intersection (described by Prob. 20 and 21) F and G are the Vanishing Points of the Sides of the Square, and H of the Diagonal; consequently, FG is the Vanishing Line of the Plane of the Figure; S is its Center. Fig. 82.

The Distance of the Vanishing Line, F G, is a mean Proportional between S F and S G (Prob. 12) the Point E, which is the Distance of the Vanishing Point F, from the Eye, is determined by the same; and, the Vanishing Point, H, of the Diagonal, by bisecting the Angle, which the Radials of AB, and AD, make at the Eye; or, by making F H to HG, as the Radial of AB is to the other (3. 6. El.)

See Prob. 10, Case 3rd, Fig. 49. EH, bisecting the Angle DEG, made between the Radials of A C and CB, gives the Vanishing Point H; by which, the Lines AC and CB are cut, perspectively, representing equal Lines. As, in this Example, by drawing AH (H being obtained, as above) A B being made to represent a measure equal to AI (Prob. 17) and BG drawn; BG is cut, in C, representing an equal Line as A B represents, i. e. equal to AI (Prob. 10.) Then, draw AG; and FC, till it cuts AG, at D; ABCD represents a Square, whose Side is equal AI.

It is required to draw, within the Square ABCD, the representations of several less Squares, of certain dimensions in proportion to the other, about the same Diagonals (AC and BD) and consequently, having the same Center; i. e. to draw the representations of Borders or Margins within it.

Plate XVII.

Make A*a* and *ab* refpectively equal to the breadths of the Margins.

Draw *a*E and *b*E, cuting AB in *c* and *d*; then, draw *c*G and *d*G, cuting the Diagonals in e and h, i and m; and draw eF, iF, &c. cuting the Diagonals again, at f, k, &c. and join fg and kl; or, draw fG and kG.

efgh and iklm are the reprefentations of Squares, having the fame Center, o; and are about the fame common Diameters, AC and BD.

Now, becaufe the Interfection (AI) of the Plane thofe Squares are in, is fo near its Vanifhing Line (FG) the Sides of the Squares, AB, ef, &c. cut the Diagonal BD very oblique; for which reafon, the Points of fection, f and k, cannot be afcertained with accuracy; nor can the true breadths of the Margins be determined. Therefore, either below or above the Vanifhing Line, draw *A*B, or a*B* parallel to AI; in which, take *A* or a, in a perpendicular Line from A, and make *A*B, or a*B* equal AI; and make *A*1, or a1, and 12, refpectively equal to A*a* and *ab*.

Draw *A*F and *A*G, or aF and aG; and BE, 1E, 2E, cuting *A*F in *B*, p, and q.

Compleat the Square *ABCD*, or abcd, by the fame Vanifhing Points, F, G, and H, and draw the Diagonal *BD*, or bd. Draw pG, and qG, cuting the Diagonals, in *E*, *H*, *i* and *m*; from which Points draw *E*F, *i*F, &c. and finifh the interior Squares, *EFGH*, and *iklm*, as in the Figure.

If Perpendiculars are drawn from the Points *E*, *F*, or *e*, *f*, *i*, *k*, &c. they will cut the Diagonals AC and BD in the Points, e, f, i, k, &c. as it is obvious from infpection of the Figure.

For, if AI be the Interfection of the Plane of ABCD, with the Picture, *AB*, a*B* may be confidered as the Interfections of other Planes, parallel to the former, and therefore, they have the fame Vanifhing Line, FG†; and confequently, *AB*, AB, and ab; alfo *AD*, AD, and ad, and all other Lines, whofe Originals are parallel to their Originals, have the fame Vanifhing Points, F and G‡; notwithftanding, the Lines are in different Planes; and, H is the Vanifhing Point of the Diagonals, *AC*, AC, and ac, feeing that, they reprefent parallel Lines, and in parallel Planes.

† Theo. 3.
‡ Cor. to Theo. 3.

Therefore, *AB*bad*D* reprefents a right angled Parallelopiped or Prifm, whofe Bafe, *ABCD*, and Top, abcd, are Squares; confequently, they are equal and parallel to each other§; and fimilarly pofited.

§ See Def. Page 41.

ABCD may be confidered as a Section of that Prifm, by a Plane parallel to its Bafe; and confequently, it is alfo a Square, equal, fimilar, and parallel to the other.

The fame may be faid of all the interior Squares; which, with the Perpendiculars *Ee*, *Ff*, &c. alfo form Prifms. The Border, around, reprefents the edge of a hollow Prifm.

In Example 5, the hexagonal Prifm has its Bafe and Top fimilar Figures, and fimilarly pofited; and being in parallel Planes, their Sides are parallel, and confequently, they have the fame Vanifhing Points.

Hence it is manifeft, that, either the extra Plan *ABCD*, or abcd, may be ufed, for the purpofe of drawing perpendicular Lines, in order to determine the extremes of Steps, &c. inftead of the real Plan ABCD; in which, on account of their being farther from their Vanifhing Line, the Interfections are determined with greater exactnefs, and without incommoding the Object.

EXAMPLE XIII.

How to reprefent fquare Steps, obliquely fituated to the Picture and having a Pedeftal fituated on them.

Fig. 83.

Let AB be the Ground Line, and DF the Horizontal vanifhing Line, C is its Center; A is the interfecting Point of the neareft Angle of the Ground Plan.

The Diftance of the Picture being known; and the Inclination of the Sides of the Object determined, find the Vanifhing Points, D and F, of the Sides (by Prob. 12) and G of the Diagonal (as O, Fig. 51; EO bifecting the Angle AEB) alfo, find E, making DE equal to the Diftance of the Vanifhing Point D.

Draw AD and AF, indefinite; make AB equal to the Side of the Square of the Steps, and draw BE, cuting AD in d.

Draw *AB* parallel to AB, at any diftance at difcretion.

Draw

Draw AA perpendicular to AB; and at the Point A, describe the representation of a Square $ABCD$ (by means of the Vanishing Points D and F; and O, of the Diagonal) whose Sides AD, &c. represent the measure AB; as Ad, Ab, above.
Make Aa, ab, each equal to the width of the Steps; and draw aE, bE, &c. cuting AD in g and h.
Draw the Diagonals AC and BD; and draw gF and hF, cuting them in F, G, &c. and compleat the interior Squares, as in the last Figure.
Draw AH, perpendicular to AB. Take Aa and ac equal to the height of the Steps, and draw aD and aF, terminating at b and d, where Perpendiculars from B and D cut them.
Draw aG and cG, cuting a Perpendicular from F, at e and f.
Draw eD and fD, eF and fF, cuting Perpendiculars from G and H, at g and h, which terminate that Step; bD and dF cuting the next Step, at g and h, compleat the first; and, by the same means, the next is compleated.

It remains, now, to delineate a Pedestal upon the upper Step.

Set off the geometrical projectures of the Mouldings, from b to c, on the Intersection; and transfer them to AD, by means of the Point E; and draw iF, cuting the Diagonals at L and A; and draw LD, cuting the Diagonal DB at J.
Then, on the Intersection AH, make cH equal to the height of the Pedestal; make ci equal to the Plinth, ik to the Base Moulding, kl to the Dado, lH to its Cornice or Sirbase, and Hj to the Plinth of the Base of the Column; which, notwithstanding it is a part of the Base, always goes with the Pedestal. Fig. 83.

At X and Z, on the right hand, the Profile of the Mouldings and Steps are described, geometrically; from which they are transfered by means of parallel Lines, to the vertical Intersection AH.

Draw cG, iG, kG, &c. cuting Perpendiculars from K and L, in m, n, o, p, and q; the representations of the corresponding Angles in the Profile.
Draw mD, nD, mF and nF, &c. cuting Perpendiculars from J and I, in r, s, t, m, n, o, &c. and having divided ik into the smaller divisions of the Mouldings (as at Z) draw to G, cuting Perpendiculars from the corresponding parts in the Ground Plan, at JIK, by which means, the whole is compleated; as in the Figure.

Now, when this Object is well considered, what are the several Parts which compose the Pedestal, but so many Steps of different heights and breadths. The Plinth, mnnm, is in the place of the next Step, about twice the height; and the Fillet, which follows it, is another of a much less height; the Dado, after that, is another still higher, equal kl; and the Cornice, vx, is but Steps reversed.
The Curve Lines st, no, and no, joining the Angles of the several Fillets, are all that have any linear curviture; although noon represents a cylindrical Surface, yet the appearance of it is effected by Light and Shade only; which, according as it is supposed to be situated, in respect of the Light, will appear either convex or concave; to which, the curve Lines, as its extremes, do not at all contribute.

And now I might take leave of the Reader, and of this Section, and endeavour (as others have done) to persuade him, that I have, in this Lesson, given him sufficient instructions for delineating all kinds of rectilinear Mouldings; but, I have not quite so good an opinion of his Capacity (if it be his first essay) as to suppose any such thing; my intent in this, is only to give him some little Idea of it, by comparing the Process with that of delineating Plane Steps. There remains yet somewhat more to be said on the Subject; therefore I shall not trespass on his time, but give some other Examples for his further Instruction.

EXAMPLE XIV.

How to represent Mouldings around the Basement of a Building, being a regular Pedestal, with an internal and external Angles; whose Planes are parallel and perpendicular to the Picture, and also one to another.

Let AB be the Ground Line, and AD the Intersection of the Picture with the vertical Plane of the Dado, bcg$e$; on which, set off all the measures of the heights of the Mouldings, and their distances from each other. Fig. 84.

Plate XVII. Aa is the height of the Plinth, ab of the Bafe moulding; bc of the Dado, and cD of the Sirbafe, or Cornice of the Pedeftal.

Let each Moulding be drawn, geometrical, cdD the Cornice, and ab*c* the Bafe; draw the Perpendicular *ac*, and draw the diagonal Lines b*c* and cd.

C being the Center of the Picture, and EC the Diftance, draw AC indefinite. Make AF the meafure of the firft break, and draw EF, cuting AC at G.

Draw GI parallel to AB; make AF equal to the projecture of the Pedeftal, and draw FC, cuting GI, at I.

Make IJ to GI as the front of the Pedeftal to its projecture; that is, as FB to AF; and draw JE, cuting FC, at K; or BE, cuting AC, at H, and draw HK parallel to AB, cuting FC.

From the feveral Angles, G, I, and K, thus obtained, draw Gr, Is, and Kt, perpendicular to the Ground Line; which conftitute the feveral Planes, from which the Mouldings are fuppofed to project.

Draw bC, cC, and DC, cuting Gr, at *e*, g, and e; draw *ef*, gm, and ek, parallel to the Ground Line, and where they cut Is, at *f*, m, and k, draw Lines to the Center, cuting Kt, at *g*, n, and o.

Now, Degc, ekmg, and konm, are the Seats of the Cornice on thofe Planes; cg*e*b, &c. is the Dado or plane part between; and Ab*e*G, G*ef*I, and I*fg*K are the Grounds of the whole Bafe, thereon.

Draw *a*C, cuting EF at *b*; draw *bh* parallel to AB, cuting EI produced, at *h*; and draw *h*C, indefinite.

The Angle *k* is determined, by drawing a Line from K to the Eye, on the other Side of the Center, equal CE; for K*k* is a Diagonal of a Square; whofe Vanifhing Points are, in this Cafe, the Points of Diftance. (Prob. 19.)

Or, if the Vanifhing Point of the Diagonal K*k* is not on the Picture, draw K*d* parallel to the Ground Line; on which defcribe the Square K*ikd* (by Pr. 19) which will give the Angle, *k*, required; i. e. draw *dE* cuting KC; and *ik* parallel to K*d*, cuting *h*C at *k*, the Angle fought.

From the feveral Angles, *b*, *h*, and *k*, draw the Perpendiculars *b*f, *h*l, and *k*p; and, through e and k, draw Ee, and Ek, cuting the two firft, at f and l.

Or, from *c* and d, draw Lines to the Center, cuting the firft at *d* and f; from which, draw *dB* and fl, cuting the fecond, at *B* and l; and from them draw Lines to the Center, cuting *k*p, at *l* and p; which give all the extreme Angles of the Mouldings.

Draw the diagonal Lines *ed*. *fB*, and fg, &c. and obferve how the parallel Lines in the geometrical Mouldings, cut their Diagonals; as, at h, the great projecture of the Facia goes beyond it.

Draw the lower Line of the Cimma or Ogee, at h; and, where it cuts the Diagonals, at i and j, draw the perpendicular lines of the Facia, which will terminate it, at the feveral Angles; by drawing hi, tending to the Center, and ij parallel to the Vanifhing Line, &c. The reft is obvious on infpection of the Figure.

The Fillet at z between the Ovolo and Cavetto, touches the diagonal Line, cd; by which, it is eafily defcribed, as above.

As the Fillets of the Bafe Moulding touch the diagonal Line b*c*, they are very readily returned at the internal Angle, *ed*, to the external, at *fB*, and from thence to the other, at *g l*.

It may perhaps be imagined, that b*c*, and cd are mitre Angles, at the Corner AD, which they are not. For, fuppofe ADvu to be a returning Plane, parallel to the Picture, and at right angles with ADeG. Then, becaufe AD was confidered as the Section of that Plane, ADvu is in the Picture; and confequently, Ab*ca* and cdD are geometrical fections of the Mouldings, by the Picture; and if they were returned on the Plane ADvu, they would project on this Side of the Picture.

Draw EA, meeting *ba*, produced, at *n*; and draw *n*x parallel to AD.

Produce

Produce fd to x; alfo produce *dc*, till it cuts the Perpendicular *n*x at *o*; and draw *op* and *nq* parallel to the Ground Line; a diagonal line, *o*b, is the mitre Angle of that Moulding.

It may be obferved, that, if AD paffed through E, the mitre Angles would be in that Line entirely; indicating the Eye to be in the diagonal Plane of that Angle; if AD was beyond E, the mitre Angle would be on the other hand of it, when it becomes greatly diftorted.

AB being the Interfection of the Ground Plane with the Picture, all the Mouldings on this fide project through the Picture, and they are confidered as projected to the Picture.

Dq is the height of the Plinth of the Bafe, which determines it, at r, s, t, on the Pedeftal.

EXAMPLE XV.

Is a general and univerfal Rule for delineating Mouldings, parallel to the Picture; whether they are above or below the Eye.

At either extreme of the Picture, if there be room (or it may be done on another drawing Board, and the Scale Line transfered, carefully, to the Picture) defcribe the Cornice, &c. geometrically; as at No. 1.

If the Student be not acquainted with Architecture, or has not a knowledge of the geometrical proportions of Mouldings, let him not attempt the delineation of them in Perfpective, before he has ftudied, and practifed himfelf a little in that kind of Projection; as he will but lofe his time; it being impoffible to fucceed in one without the other; feeing that, the projection of Mouldings, in Perfpective, depends wholly on the geometrical Profile.

Let AB be the height, and BD the projecture of a Cornice, intended to be delineated; to which let there be added an Architrave, FG; between which, the plane part, AF, is called the Frize; the whole together compofes an Entablature. Fig. 85.

Let the feveral Mouldings be geometrically defcribed, according to their heights and projectures; at a, b, c, &c. and, CD being drawn perpendicular, may be confidered as a vertical Section of the Picture, applied clofe to the greateft projecture of the Cornice, at D.

According to the height which the Cornice is fuppofed to be above the Eye, take the Point C, and draw CE perpendicular to CD, i. e. horizontal; and equal to the Diftance of the Picture.

Now, E being confidered as the Eye of a Spectator, viewing the Cornice, AD, &c. EA, ED, EF, &c. are Vifual Rays, in which direction the feveral parts are feen; and, CD is fuppofed to be a Plane, interpofed, cuting thofe Rays; confequently, the feveral Points, *A*, *a*, *b*, &c. projected by them, are the apparent places of the feveral edges, or parallel Lines of the Mouldings, on the Picture.

C is the Center of the Picture, and EC is its Diftance; by which, the perfpective reprefentation of the Mouldings muft be projected; provided that, the drawing be required to the Scale CD, of the Profile, ABD.

But, it may, from the fame Profile, be made either greater or lefs. e. g.

If the Rays ED, &c. be continued, beyond D to H, and HI be confidered as the Plane of the Picture, the projection of the Cornice is JH, which is greater than *A*D, and fo of the reft; EI is the Diftance of that Picture.

Or, if any other Plane be interpofed, as at Kd, or *CH*; the proportions of the feveral Parts on each, *viz.* *GF*, *FA*, and *A*D; gf, fa, ad, or *gf*, *fa*, *ad*, are, to each other, as their feveral Diftances, EC, to EK, or E*C*. (6. 6. El.

Hence it is evident, that, from parallel Sections of the Rays, EA, ED, &c. any where, the Mouldings will have the fame proportion to each other.

The feveral Sections, CD, Kd, and *CH*, are Scales of the perfpective Proportions for each refpective Diftance.

It is required to delineate the Entablature (No. 1.) by the Scale *CH*.

Plate XVII. Let E be confidered as the Center of the Picture or Point of View, and E *C* the Diftance of the Picture; the Terms being inverted.

To the fartheft Corner (B) of the given Cornice (i. e. to its height, or Seat of the extreme projecture, D, on the Perpendicular A B) draw EB, cuting *C H* at *H*.

Take any length, required, as *CI*, for the Ground Plane of the Cornice.

Draw *IK* parallel to *CH*, i. e. perpendicular to E*C*, the Horizontal vanifhing Line; and from *H*, where EB cut *CH*, draw *HB* parallel to the Horizon, cuting *IK* at *B*. Draw *dD* alfo parallel; and *CB*, cuting it, at *D*, gives the Angle *D*.

Draw E h and E i; and, from their Interfections, with *CH*, draw Lines parallel to *D d*, till they cut *B D*; and, from *h* and *i* draw Perpendiculars, cuting the Parallels from *c* and *b*, at *k*, which compleats the Corona. *c k b* reprefents the Planceer, b c, in the Profile. The reft is obvious, for the parallel Cornice.

The projecture of the Architrave, at *F*, is obtained, by drawing *CF*, after the fame manner as the Cornice, at *D*.

If the length of the returning Plane exceeds the limits of the Picture, take any equal part of the whole length; as a*B*, half, in *HB*, produced; make E*E* equal to half E*C*, and draw a*E*, cuting E*B*, at b.

To afcertain the mitre Angle, e, feeing that the Vanifhing Point of the other Diagonal is not in the Picture, draw b d parallel to a*B*; and *D*E, cuting it in c; make c d equal b c, and draw d*C*, cuting *D*E in e, the Point fought.

Or, draw b*L* parallel to a*B*, cuting the Diagonal, *DC*, at *L*; bifect *BL* perfpectively, at *J*; draw *J*b, and produce it, till it cuts *D*E, at e. Draw ef.

For, becaufe b*L* is parallel to the Picture, and, *C* is the Vanifhing Point of one Diagonal of a Square, in that pofition, *B*b reprefents a Line equal to b*L*; and *B*b*L* reprefenting a Right Angle, is bifected by *J*b, half the other Diagonal; which, being produced, cuts *DE* at e, the extreme Angle of the Cornice.

Draw E*h* and E*i*, cuting e b at 1 and 2; from which, draw Perpendiculars; and draw Lines to E, from the Corona, at *k*, and all the Fillets, cuting ef; obferving, that the extreme Angle of the Corona, goes beyond the Diagonal.

This Method of reprefenting Mouldings is deduced from the moft natural and fimple Ideas of it, and cannot, I think, fail of being intelligible to the meaneft capacity, who has any emulation to attain to the art of drawing in Perfpective.

As there are no Modilions, or other Enrichments, of that kind, in this Cornice, there is no apparent diftortion; if there were fuch, they would be diftorted towards the extreme *CH*, feeing that, the Mouldings extend to the full Diftance (E*C*) by which they are projected.

EXAMPLE XVI.

Plate XVIII.

To reprefent a Doric Entablature, with an internal and external Angles.

On account of the variety of Parts, and the many breakings of the Fillets, around the Triglyphs, in the Doric Frize, it is neceffary to have it defcribed, geometrically; fo that, the Parts may be clearly underftood. Indeed every Moulding, delineated in Perfpective, ought firft to be geometrically drawn, to the Scale of the Drawing; although the meafures may be applied from a Scale only.

I have fhewn, in the 12th Example, how a perfpective Plan may be drawn either above or below the Work; by which the projectures of the Mouldings, &c. may be determined at the Angles, as exemplified in the 13th. But, that procefs (though the moft to be depended on, for accuracy) is attended with extraordinary trouble, which may be leffened greatly, when we are tolerably acquainted with Mouldings. A Specimen of both will be fhewn, in the following Example.

Fig. 86. No. 1. Let A B be the height of the Entablature, according to the Scale of the Drawing. Let it be bifected at C; and A C, or C B, is a Module, or Diameter of the Order. A C divided into 60 Minutes, is the Scale; by which the whole is proportioned.

Q M, the projecture of the whole Cornice, is a Diameter; the reft is proportioned as by the Scale A B. D F is the Architrave, F G is the Frize, with its Triglyphs, and G M Q is the Cornice. H I is the projecture of a Mutule or Modilion, and K of the whole Planceer; which, being feen in Perfpective (the Eye being below it) has a fine effect, and adds greatly to its auguft appearance.

No. 2 is a geometrical Plan of the Cornice, with an internal Angle at S, and an external, at T; ſhewing the Planceer of the Mutules, and panneling between them; for, unleſs we know the true geometrical form, it is not poſſible to deſcribe any thing, perſpectively. Each return of the Cornice is the ſame, having two Mutules each way, except the laſt, on the Right, which is ſuppoſed unlimited; and, on the Left, it is limited by the bounds of the Picture; which, on account of its diſtance from the Center, would be diſtorted, if it was continued much farther. No. 2.

BAFG is a perſpective Plan of the whole; which is the beſt method of proceeding, if accuracy be required in the ſeveral parts, and we do not grudge the time ſpent in doing it. It is formed as follows. No. 3.

At any Diſtance from the Horizontal Line, *E*CE, either above or below it, draw *AB* parallel to it; one Side, in this Example, being parallel to the Picture.
Having determined on the Place of the Angle a, in the Deſign, take *A* perpendicular over it, and draw *A*C, indefinite; C being the Center of the Picture.
Take *AD* equal ST, in the geometrical Plan; and, having made C*E* in the Horizontal Line, equal to the Diſtance of the Picture (equal ES) draw *D E*, cuting *A*C in *F*; then, *AF* repreſents the length of *AD* (equal ST, No. 2) and *F* is the internal Angle of the extreme Moulding in the Cornice.
Draw *FG* parallel to *AD*, and *D*C cuting it, in *G*, the external Angle; for *FG* alſo repreſents an equal length as *AF*†, each equal to ST. † Prob. 19.
Make *AC* equal MQ (No. 1) the projecture of the whole Cornice, and draw CC and *AE* cuting it, at V. The Diagonal *F*X (i. e. *D E*) alſo cuts *C*C at X; from which, draw X Y parallel to *AB*.
The Angles *K*, *L*, and *M*, of the Corona (K, in the Profile) are determined by making *Ad* equal to MN, and drawing *d*C, cuting the Diagonals *A*V and *F*X; by means of which, they are carried around, to *L* and *M*, and, by the ſame means, the whole Plan is compleated, as in the Figure.

To deſcribe every Step, by which the whole perſpective Plan is formed, would be as tedious as it would be uſeleſs; ſeeing that, the various Leſſons, already given, are ſufficient for any right-lined Figure whatever. Fig. 86.
The Plans of the Mutules, Z, Z, are the repreſentations of Squares having one Side parallel to the Picture (found by Prob. 19) their places are determined as follows.

Having made *Ae* equal M O (in the Profile) draw *e*C cuting the Diagonals *A*V and *F*X at *S* and *W*, and draw *WM* parallel to *A B*.
Make *Aa* equal to J M, in the Profile (equal T 1 in the geometrical Plan) make *ab*, *bc*, &c. equal to the width of the Mutules and the ſpaces between them (12, 23, &c. in the Plan) and from each Point, *a*, *b*, &c. draw Lines to C, the Center, cuting a parallel Line from *S*, in *R*, *P*, &c.

For, the returning Side, *AF*, make A*f*, *fg*, &c. equal *A a*, *a b*, &c. and draw *f E*, *g E*, &c. cuting *A F*, at 1, 2, 3, from which, draw Lines parallel to *A B*, cuting *S*C at *T*, V, &c. and *H*C at *m*, *n*, &c.

For the other parallel Side, *F G*, make *D m*, *ml*, *lk*, alſo equal to *Aa*, *a b*, &c. from which draw Lines to the Center; cuting *WM*, at *X*, *Y*, *Z*.

For the apparent width of each Moulding, &c. in Front, draw MS perpendicular to the Horizontal Line; take SE equal to *E* C, on the Horizontal Line; and, from every Angle, G, H, K, &c. in the Profile, draw E G, E H, &c. cuting M S in the ſeveral Points, g, h, k, &c. as in the laſt Example.

Being thus prepared, we now proceed with the Repreſentation.

a is the determined Angle of the extreme Moulding; draw the Fillet a z of the upper Moulding (which is in the Picture) geometrical. Fig. 86.
From V, X, and Y (in the perſpective Plan) draw perpendicular Lines indefinite; which are the Angles of the Ground Planes* of the whole.
Then, from the ſeveral Angles *H*, *K*, &c. in the perſpective Plan, draw Lines perpendicular; and, from the correſponding parts on the vertical Section, M S, draw Lines parallel to the Horizon, cuting them. e. g.

* By Ground Planes, here, is not meant the real Ground or Floor, as defined, but the vertical Planes, from which the whole Cornice, &c. project; called ſo by Workmen.

From

Plate XVIII.

From *K* draw a Perpendicular, *K*c, and from k (where K E cut M S) draw horizontal Lines cuting it, at c, the reprefentation of the external Angle of the Corona.

Draw cC, cuting the Perpendicular from *L* at l, the internal Angle of the Corona; and from l, draw parallel Lines, cuting the Perpendicular from *M* at m, the other external Angle of the fame.

Then, from h (where HE cuts *C*M) draw horizontal Lines cuting the Perpendicular from *H* (in the Plan) at h, the reprefentation of the Angle H.

Draw h C cuting a Perpendicular from *I*, at j, and the horizontal Line j n gives the other external Angle, n.

For the Mutules, draw a Perpendicular from *S*, in the Plan (where they would meet if they were continued to the Diagonal *A* V) and from i in the Scale Line, M S, draw parallel Lines cuting it, at f; which muft be returned at all the Angles, as if it was a continued Facia.

From *O*, *P*, &c. draw perpendicular Lines cuting them, in the feveral Points, *o*, *p*, &c. which are the front Faces of the Mutules. Draw *r* C, *q* C, &c. cuting the horizontal Line h h; where they fall againft that Facia.

In the returning Side, draw Perpendiculars from *TV*, &c. till they cut fC, the returning Facia, and fo of all the reft; which are beft defcribed by the Figure; obferving that, if from the feveral parts, in the firft or neareft front of the perfpective Plan (*B A* V) perpendicular lines be drawn, and from the correfponding parts in the Section, or Scale Line, M S, parallel Lines cuting them; and, by carefully remarking the mitre Angles of each Member, on A V, the true perfpective proportions of them are carried round as many Breaks, in the Object, as are required; by means of the Vanifhing Point C (the Center) for one return, the other parallel; the Originals of them being, in this Example, parallel to the Picture, and the Object right-angled.

The Mouldings around the Mutules are determined, at their projectures, from the Plan, above (at *O*, *P*, &c.) and the width, in Front, from the Section of the Picture, MS (at m) by means of which, and the Vanifhing Point C, they are very eafily continued around the Mutules, from one Angle to the next, adjoining. Firft, drawing parallel Lines, from m, till they cut Perpendiculars from *O*, *P*, &c. at *e*, *f*, &c. and *f*C, *h*C, &c. cuting Perpendiculars from J, at *i* and *j*; through which they are drawn, parallel, to the Angle at *k*, and from *k* they are continued, after the fame manner, around the whole.

Every thing being correct, and very particular in the Figure, makes it unneceffary to give a further defcription of it. To defcribe every Line in the whole Procefs, would take feveral Pages, be extremely tedious and apparently prolix.

For if, as I have before obferved, the Student be not acquainted with Architecture, nor underftands the feveral Parts of the Order, it will be impoffible for him to fucceed in this Example; but if he is, it will be found fufficiently intelligible.

The Architrave, of this Order, it is needlefs to fay any thing about; feeing it is compofed of plane Facias, only; projecting, one over another, like Steps, of different heights and projectures, and is managed in the fame manner; which, from infpection of the Figure, may readily be defcribed.

The Frize and Triglyphs have nothing difficult in them, efpecially in this Pofition. In the parallel Faces they are geometrically proportioned and fpaced (Theo. 9. Cor. 5) and in the returning Side, their places are determined from the Plan above, or by the 10th Problem; dividing the upper Line of the Frize, st, in the fame Ratio, perfpectively, as the front Line, n s, is divided, at o, p, q. (Prob. 8.)

At No. 4, I have given two Triglyphs, more at large; in order, that the Parts may be more diftinctly made out; in which, after dividing the front one, as in the Profile, geometrically, for the flutings (which are right angled) the indented Angles may be obtained, as in the Plan ABD, above; by drawing a Line from each Angle *a* and *b*, to the two Diftance Points, if they are within reach, cuting at *c*.

Or,

Plate XII.

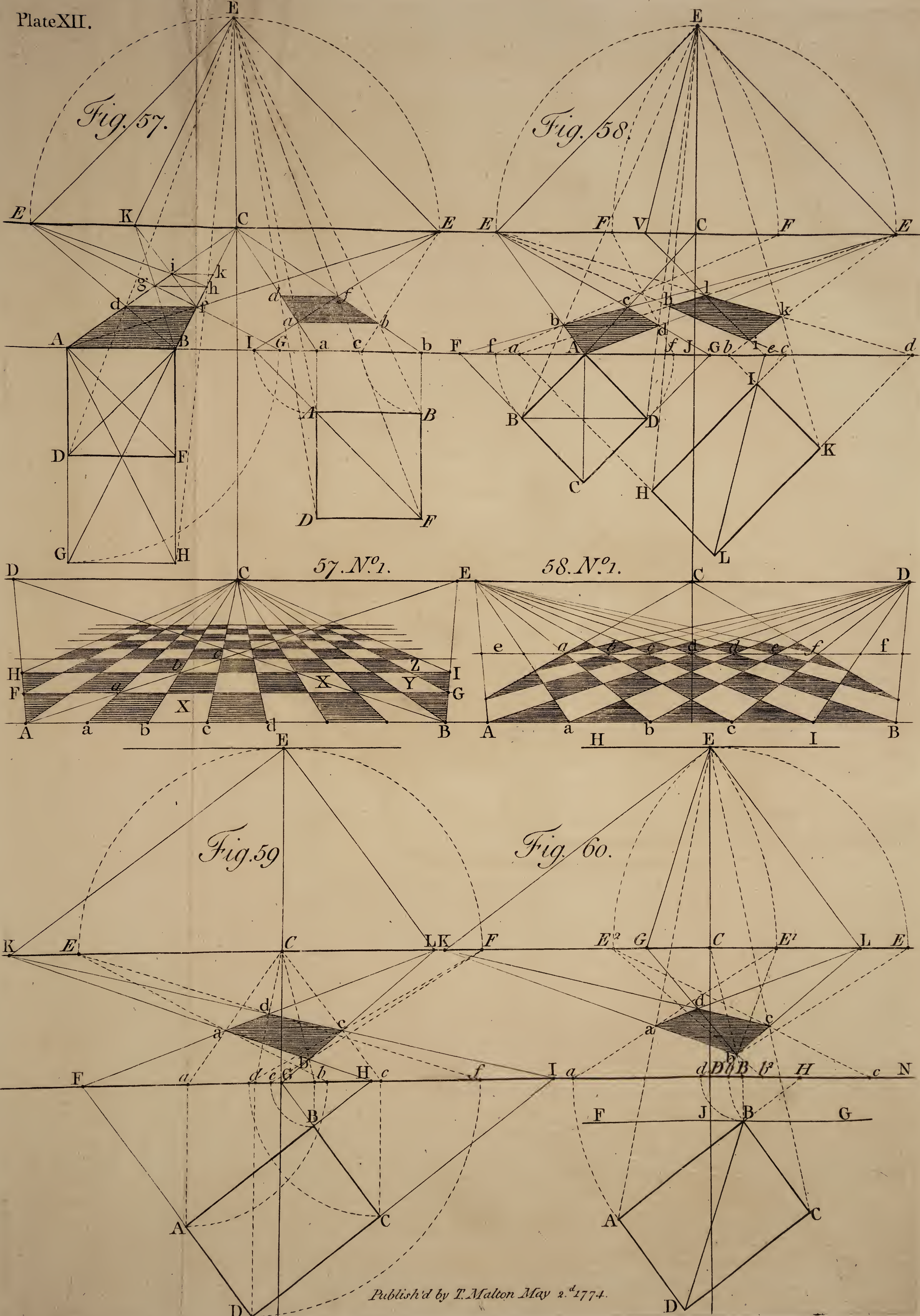


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Plate XIII.

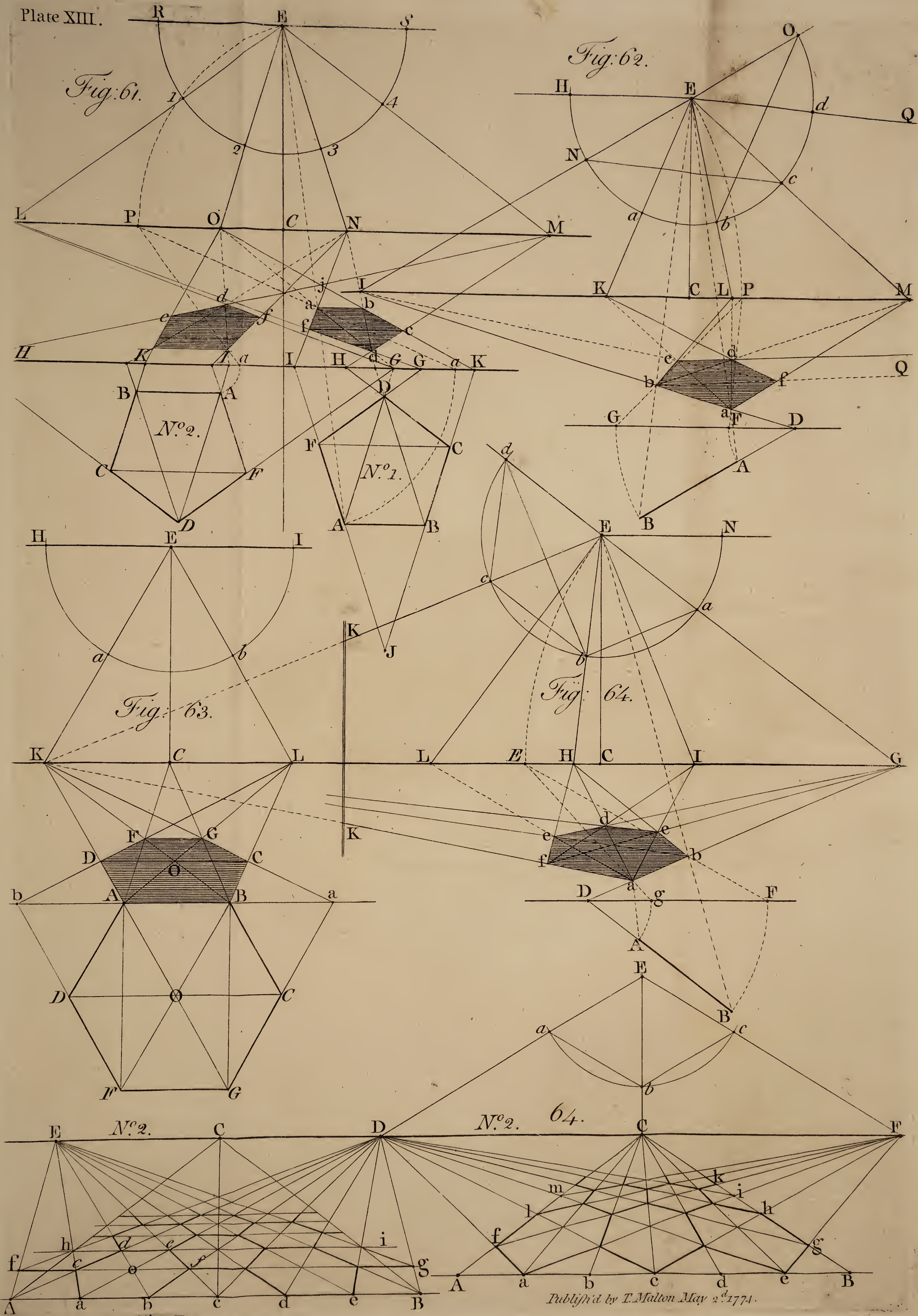

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Plate XIV.

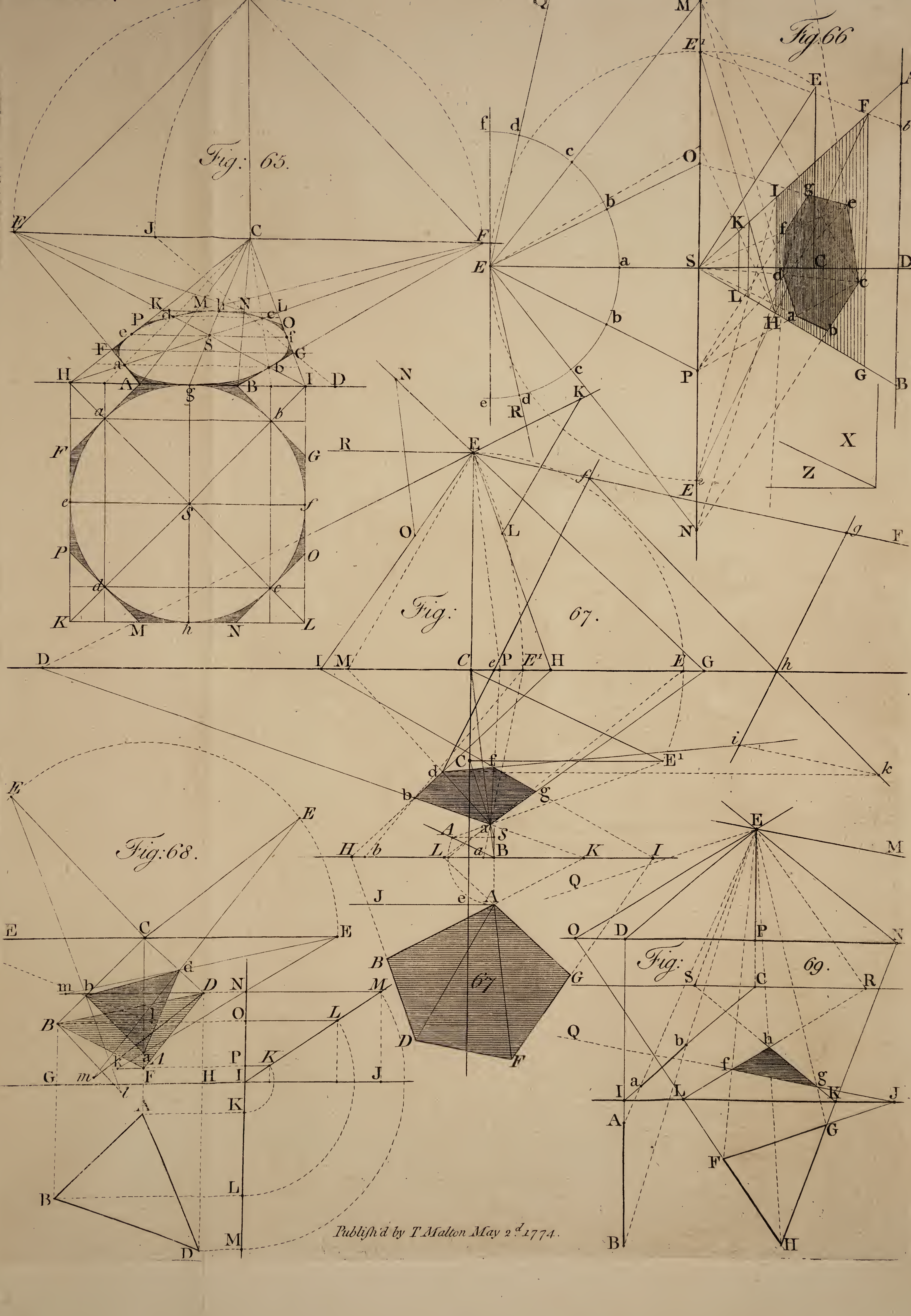

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Plate XV.

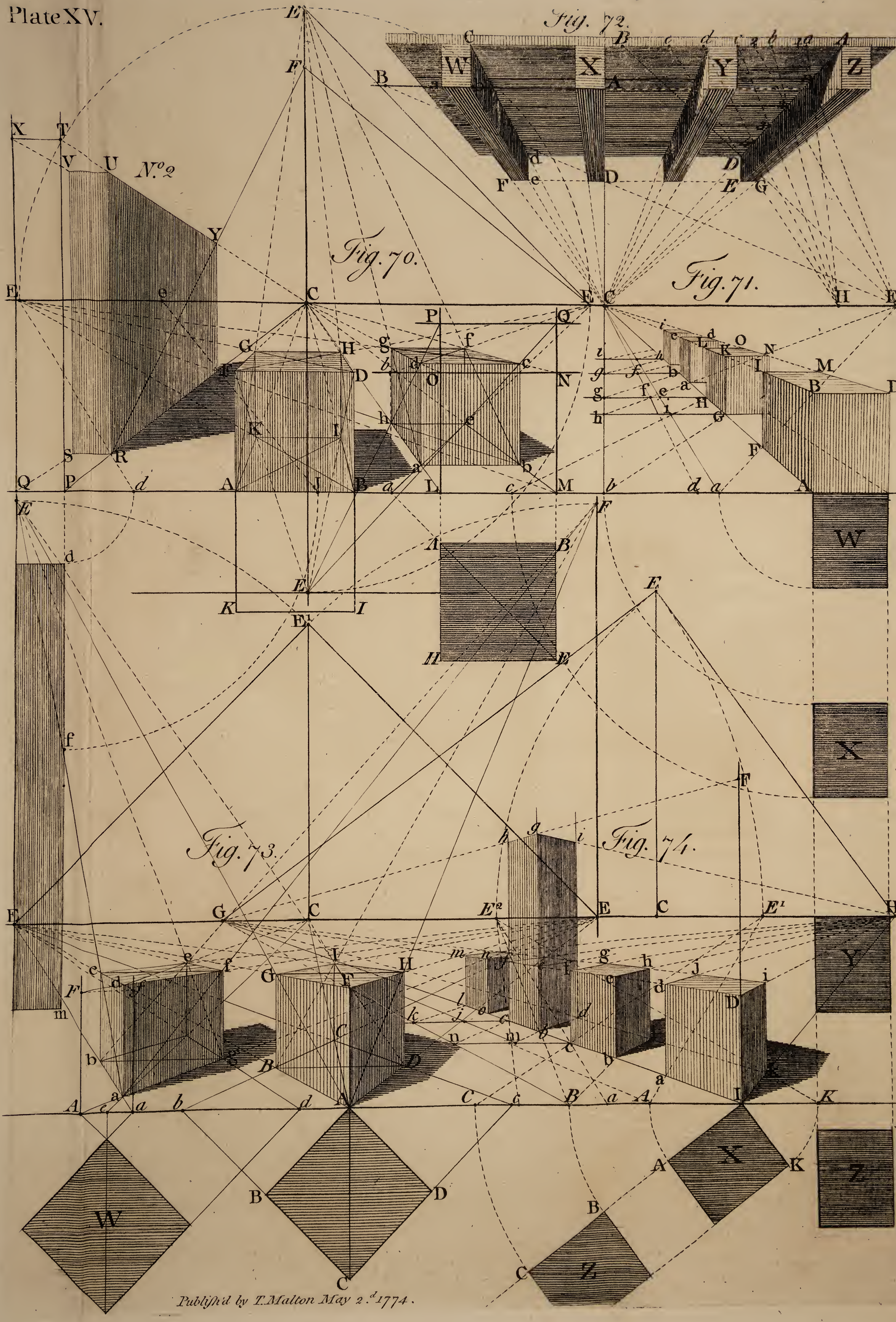

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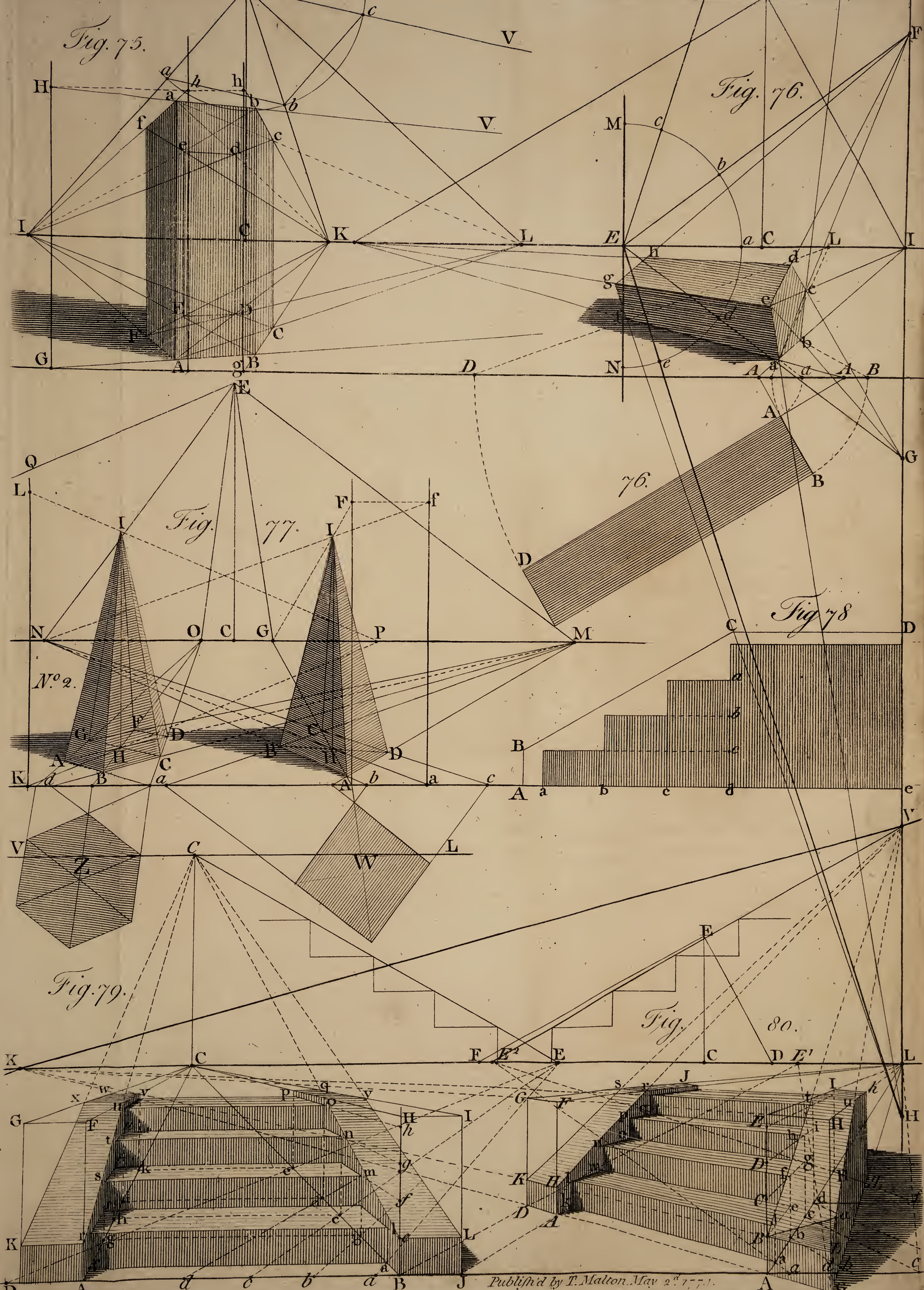
Plate XVI.



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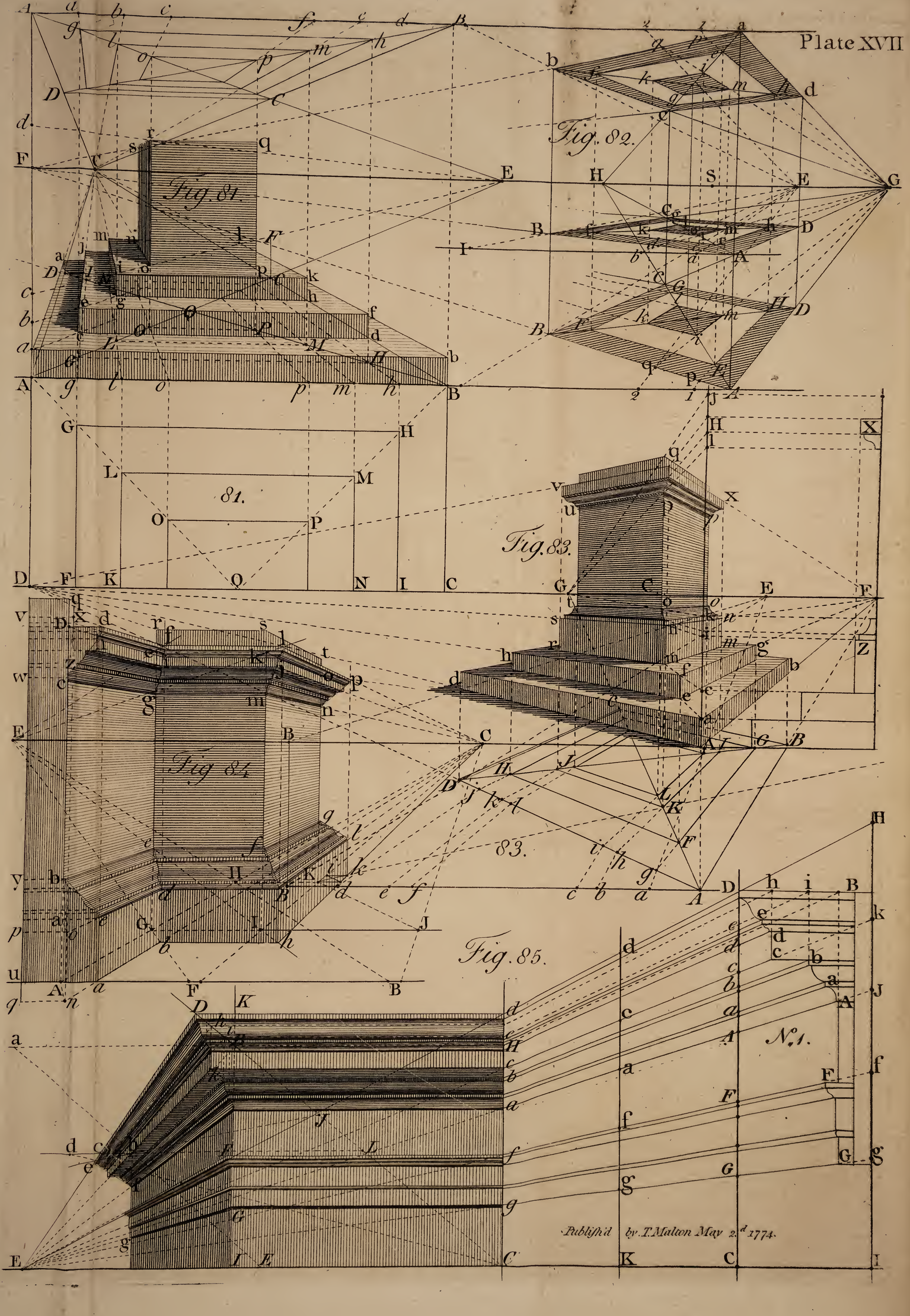

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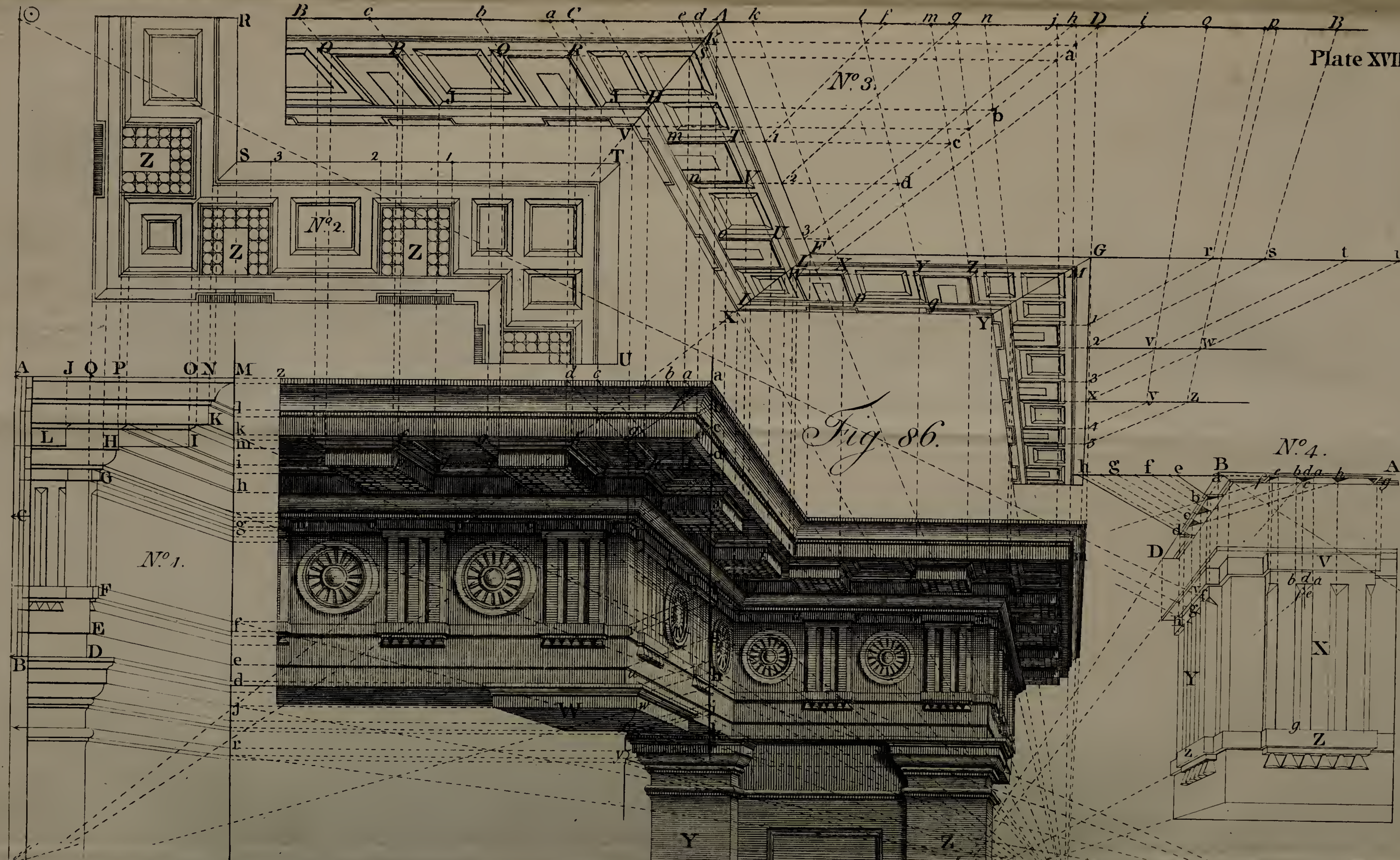
Plate XVIII.



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Or, if one Diftance (E) only; bifect the width of the Flute, at d, and draw d c, to the Center, cuting aE at c, as before; ef, &c. drawn, to E, give the reft.

For the Triglyph in the returning Side; having drawn the front Line, ad, to its vanifhing Point, c; from each angle, a, b, &c. in the front Triglyph, draw lines to E, cuting ad, at b, c, and d.

Or, if it be convenient to have the Diftance of the Eye on the other Side, it would be better; draw a h parallel to AB, and take the divifions, from a, at e f, g and h, equal to the geometrical meafures; and, from them, draw Lines to the other Diftance, as in the Figure (fuppofing it within reach) which give the fame Points, more accurately defined. (See Prob. 8.)

From the feveral divifions, on each Side, draw Lines perpendicular, as in the Figure; by which means, the Triglyphs, X and Y, are compleated. The Perpendiculars, from c and b, (in the Plan, above) give the indented Angles in the front Triglyph; in the other they are not feen.

This operation is fuppofed either at the top or bottom of the flutings.

The inclined Lines, at the Top, are determined, truly, by drawing a Perpendicular at the middle point, d, making de equal to half its width, and drawing a line to c, the Center, cuting the Perpendicular from c, at f; join af and bf.

It is unneceffary to be more particular, or their vanifhing Points may be eafily determined; by making the fame Angle at the Eye, as the inclined Lines make, either with the horizontal or vertical Line (Prob. 4.) and where it cuts a Line, paffing through E, perpendicular (the Vanifhing Line of an inclined Face) is the Vanifhing Point of af. Either being found, determines the other; and, the inclined Planes of Y being parallel to thofe of X, have the fame Vanifhing Lines, refpectively, paffing through the Eye in the Points of Diftance.

The Vanifhing Points of the inclined Lines, in the front Triglyph, vanifh below the Horizon, and thofe in the returning one (Y) which are feen (at f, g, h) vanifh at the fame Diftance above it, at ⊙.

Although this method will feldom be practifed, yet I affirm it to be the beft, being by much the readieft and the moft correct. Nor, is there any occafion to be at the trouble of forming the Plan, to get the indented Angle; for, the Flutes being proportioned at ab (as above) and the Vanifhing Points determined, the Lines, af and bf, being drawn to their refpective vanifhing Points, determine at the fame time, the internal Angle.

The Tenia (V and Z) both above and below, breaks regularly around; as it may be feen, and would be needlefs to explain; fimilar Subjects having fo frequently been repeated. Such minutias are beft defcribed by the Figure, only, being accurately drawn, and well defined.

The Abacus (W) which may be fuppofed over a Column, is a Square; and the entire Capitals, over the Pilafters, Y and Z, at the internal and external Angles, although they are detached Mouldings, are managed (from the Profile) after the fame manner as if they were continued; their perfpective Plans being delineated, as above; from which, the doted Lines, correfponding with the Profile Section, (MS) determine their feveral Angles.

Method 2nd. EXAMPLE XVII.

How to reprefent the fame thing without having the Profile drawn, or a perfpective Plan.

Let AB be the given height, which divide into two Modules, and into Minutes, as before (the reft being fuppofed not drawn,) The Scale (A M) of the projecture of the Mouldings, &c. at the top, is alfo neceffary.

From a, the determined Angle, draw ak perpendicular, and transfer all the meafures of the feveral Mouldings, from the Scale, to ak; at b, c, d, &c.

Make aa, ab, &c. equal to the feveral projectures MN, NO, &c. and proceed as in the Plan, above; drawing aE for the diagonal Line of the top; and, from the feveral divifions a, b, c, &c. draw Lines to the Center (C) cuting the Diagonal, at e, f, g, &c. from which, draw perpendicular Lines; and, from the meafures on a k, draw diagonal Lines, to E; which give the fame Angles c, h, &c. as before; from which they are carried around the feveral Faces.

Plate XIX. The perfpective Plan, in this, is fuppofed to be formed on the Top of the Cornice, the fame as above; and fince ak is the Interfection of a Plane, paffing thro' the Diagonal or mitre Angle of the Cornice (one Side, in this Cafe, being parallel to the Picture) *E*, the Diftance Point, is the vanifhing Point of that Diagonal, confequently, all horizontal Lines, in that Plane (being parallel) vanifh in *E*.

Wherefore, a*E*, b*E*, &c. are each, the indefinite Reprefentation of a Diagonal of a Square, in different horizontal Planes, whofe Interfecting Points are a, b, &c. and, the diftance of each Angle from the Picture, is a*a*, a*b*, &c. which being transfered, perfpectively, to the Diagonal a*g*, each Angle, in its proper place, will be perpendicularly oppofite to the Points *e*, *f*, *g*, &c. as in the Example.

And thus may the whole be compleated, when we are a little verfed in Mouldings, by transfering the meafures, perfpectively, from one Diagonal to another, which will be further illuftrated in the following Example.

EXAMPLE XVIII.

How to reprefent a Cornice when it is inclined to the Picture, on both Sides.

If the laft Example be tolerably well underftood, in the laft Procefs, this will be found eafy, being performed by the fame means. It muft be obferved, that the firft Method, refpecting the Profile, cannot be applied here, nor in any Cafe; but when the Mouldings are parallel to the Picture. The extra Plan may be ufed in all Pofitions; but I fhall in the following Examples, do without; as it is only fuppofing the Plan formed at the Top, from which the Perpendiculars are drawn, and the feveral Angles of the Facias and Fillets determined.

Fig. 87. Let AEB be a Profile of the Cornice, to be drawn, and *AC* the Side of one Plane, on which it is to be projected.

ML is the Horizontal Line, S is the Center of the Picture, and L the Vanifhing Point of *AC*; which is fuppofed the upper edge of the Plane from which the Cornice is to project. Let the Top be fuppofed a Square.

S being the Center of the Picture, and L the Vanifhing Point of one Side of a Right Angle (the Diftance of the Picture being known) find M the Vanifhing Point of the other Side; by Prob. 12. SM is a third Proportional to SL and the Diftance of the Picture (found by Prob. 32, Geo.)

Alfo, find N the Vanifhing Point of the Diagonal; by making MN to NL as one Side of the Triangle (whofe Perpendicular, at S, is the Diftance of the Picture) to the other. As N, or O (Fig. 51.) GN, or EO bifecting the Angle FGH, or AEB.

Draw *A*M and *C*M, and *A*N cuting *C*M at *K*; draw L*K*, till it cuts *A*M at *D*. *ACKD* is the Top, of a fquare Prifm, from which the Mouldings project.

From each Angle, *A*, *C*, and *D*, draw perpendicular Lines, which reprefent the Corners of the Prifm; the Profile being proportioned to the neareft, *AB*.

From *B* (*AB* being equal to AB) draw *B*L and *B*M, cuting the Perpendiculars, from *C* and *D*, at *G* and *H*. *ACGB* and *ADHB* are the Grounds or Seats of the Cornice on the two Planes, which are feen.

The Projecture of the Cornice, around the Prifm, is next to be determined.

Draw the Diagonal *DC*, and produce it both ways; alfo, produce N*A* indefinite, and draw *A*k parallel to the Horizon; on which, take the diagonal projectures of the Cornice, as on AK, in the Profile.

f EK being made equal and perpendicular to AE; confequently, AK, the Diagonal of the Square AJKE, of the projecture AE, is the mitre Angle of a Right Angle, for the Cornice AEB.

From the feveral Projectures, C, D, &c. draw CG and DF, parallel to AB, and produce them to the Diagonal, cuting it in H and I.

Make *A*h, hi, and ik refpectively equal to AH, HI and IK; and having made NO equal to the Diftance of the Vanifhing Point N, of the Diagonal, draw Oh, Oi, and Ok, till they cut N*A*, produced, at *h*, *i*, and *E*; from all which, draw

draw Lines to both Vaniſhing Points, L and M, cuting the other Diagonal, *CD*, produced both ways, at *F*, *I*, *k*, *l*, &c. which form the perſpective Plan of the principal Mouldings. *EB*, *FG*, and *HI*, being drawn, are diagonal Lines of each Angle, from the lower edge of the Fillet, at the top, to the bottom of the Cornice.

Let it be obſerved, that N, being the Vaniſhing Point of the Diagonal of a Square, i. e. of a Line biſecting the Right Angle, is the ſame whether the Top be a Square or other Rectangle; but, if it be not a Square, *CD*, produced, would not be the Mitre of the other Angles; becauſe it would not biſect them; neither would *E*N paſs through the oppoſite Angle, or Mitre *KL*; yet N would be the Vaniſhing Point of both, becauſe they would ſtill be parallel. The others are alſo parallel, and conſequently, they have the ſame Vaniſhing Point; which, being much inclined, is at a great diſtance.

Having ſhewn how the Mouldings are projected forward, on the Diagonal *AE*, and transfered to the other Diagonal; ſuppoſe No. 2 the ſame thing, prepared in the ſame manner; being diveſted of ſeveral preparatory Lines, which, altogether, render the Work confuſed, and are ſuppoſed to be rubed out. Fig. 87. No. 2.

On *AB*, take *a*, *b*, and *c*, equal to the ſeveral heights of the Mouldings (as at a, b, and c, in the Profile) and, from N, project them (as N *a*, &c.) till they cut Perpendiculars, from *i* and *h*, in *d*, *e*, *f*, and *g*; and, from them, draw Lines to both Vaniſhing Points, L and M, cuting Perpendiculars, from *k*, *l*, and *m*, *n*, &c. at o, p, q, &c. which compleat the Facias and ſmall Fillets; and there remains only, to join them by curved Lines, of the ſame kind as in the Profile.

If the Mouldings are very large, as many Points may be found, in the Curves, as are neceſſary to deſcribe them with accuracy; by Perpendiculars from *AE*, and meaſures transfered to *AB*, from correſponding parts in the Profile. But, they may be as well performed by a careful Hand; for after all attempts at exactneſs, in ſuch minutias, a judicious Perſon, in Mouldings, and Perſpective, would deſcribe them as perfectly, regarding the Poſition in which they are ſeen.

Let it be obſerved, that the Curves are always flatter, in mitre Angles, than in the Profile, and more ſo, the leſs the Angle; likewiſe, in the diagonal Section, *AEB*, they are flatter than in *CFG*; becauſe, the Plane of that Diagonal is nearer to a coincidence with the Eye; for if it be in the Plane (however ſituated) they are Right Lines.

The Mouldings, in the diagonal Plane *AEB*, ſeeing that, the full meaſures are applied, on *AB*, and projected forward to *E*d, are larger than the Originals; that is, than the given meaſure; as *E*abcd, than *Aabc*, on *AB*; equal Aab, &c. in the Profile. The Picture cuts the plane of the top in ef.

In which Caſe, if *AB* be ſuppoſed in the Picture, *AecB* on one Side, and *AfgiB* on the other, are ſections of the Mouldings by the Picture; and, the whole, efg*B*, of that Section, being on this ſide of the Picture, is projected, to the Picture. *AEeB*, *CF*p*G*, and *HI*c, are diagonal Sections through the mitre Angles; and, jklm is a Section perpendicular to the Plane *ACGB*.

If what I have advanced be well underſtood, all that follows, reſpecting right-lined Mouldings, will be eaſy and intelligible, almoſt from inſpection of the Figures.

EXAMPLE XIX.

How to repreſent an Entablature with Modilions in the Cornice, obliquely ſituated to the Picture, having internal and external Angles.

ABC is the geometrical Profile of the Cornice; F is the front of a Modilion, and FG is its Profile. CD is the Frize, and DE the Architrave; with two Facias. Fig. 88.

In this Example, I ſhall ſuppoſe the whole Cornice to be beyond the Picture, and touching it at the Angle *A*.

AFG is the Interſection of the plane of the top, HL is the horizontal Vaniſhing Line, C is its Center, and CE the Diſtance of the Picture. H and L are the Vaniſhing Points of the Sides; i. e. of the horizontal Lines of the Mouldings; M and N are the Diſtances of thoſe Vaniſhing Points, for proportioning Lines which vaniſh in them, reſpectively; and O is the Vaniſhing Point of the Diagonal, biſecting the Angle, HEL, made by the Radials of the Sides.

Draw the vertical Interſection, *AE*, of the diagonal Plane, paſſing through the mitre Angle; and, transfer all the meaſures of the heights of the Mouldings, from the Profile, to *c*, *d*, &c. equal Bc, cd, &c. alſo, their projectures, to *Aa*, *ab*, *b*B. equal A a, a b, and b B.

Plate XIX. Draw *A*L, indefinite; and from all the Points *a*, *b*, &c. draw Lines to N, cuting *A*L at 1, 2, 3; from which, draw Lines to H, cuting a Diagonal, A O, at *f*, *g*, &c. the perſpective Seats of the projectures; i. e. of the mitre Angles of the Corona, K, and the Facia at F, in the Profile.

From the ſeveral Angles, k, l, &c. obtained, as in the foregoing Examples, draw Lines to both Vaniſhing Points, H and L, indefinite; and proportion them to their reſpective lengths, by means of the Diſtances, M, and N, reſpectively, of thoſe Vaniſhing Points.

The length being determined, according to the number of Mod lions, contained, or otherwiſe; make *AD*, equal to the firſt break, and draw *D*N, cuting *A*L at f, the internal Angle; and draw fO, the Diagonal Line of that Angle, in the plane of the Top; indefinite.

Draw b L, cuting that Diagonal, at g; from which, draw a Perpendicular; and c L cuting it at e, and join e f; the Diagonal of the Projecture.

Draw H e, H f, and H g, indefinite; and, make f *F* to repreſent a Line in the proportion of the Original, of f *F* to *A* f; (by Prob. 10) or, draw M f, and produce it to the Interſection, at *B*; make *B F* to *A D* in that proportion, and draw *F* M, cuting H f produced, at *F*. (Prob. 17.)

Draw *F* O, cuting H g, produced, at h; from which, draw a Perpendicular, cuting H e, produced, at i; and draw *F* L, h L, and i L, indefinite.

Produce N *F* to the Interſection, and make *F G* equal to the length of the front Line of the Cornice; draw *G* N, cuting *F* L at m; and, *G* 6 being made equal to the projecture of the Cornice (A B, in the Profile) draw 6 N, cuting *F* L at n; and n H, cuting h L, at p; from which, draw a Perpendicular, cuting i L at q.

Fig. 88. Thus, are all the Angles at the top and bottom of the Cornice obtained; and, having ſet off the meaſure of the upper Fillet, at *A*, let it be continued around, by means of the Vaniſhing Points H and L; then draw diagonal Lines, *F* i, and m q, by which the Mouldings are all determined (as at A C in the Profile) having, before, obtained their places on the diagonal Line, *A* c, at the firſt Angle.

The perpendicular Lines of the Corona fall below it, from the Point where they are cut by the upper Line of the ſame, at k.

On the returning Side, on the left hand, the Cornice is ſuppoſed to fall againſt, or is cut off by a Plane, parallel to the front Planes, and conſequently has not a mitre Angle.

Draw *A* H, indefinite; make *A* a repreſent a length equal to what the whole projects from the Plane (by means of *A*f, Prob. 10; or, by its meaſure, Pr. 17.)

Draw a L and b H cuting at r; from which, draw a Perpendicular.

Draw c H cuting it at d, and draw a d, which will terminate all the Mouldings; as the diagonal Lines determine the mitre Angles; by drawing Lines from every Angle, on *A* c, to the Vaniſhing Point H.

The Architrave, *E*, *H*, *I*, *K*, being compoſed of plane Facias, with one Moulding, only, on each Face, needs no particular deſcription.

The Modilions are determined, in the ſame manner, as the Mutules, in Ex. 16.

Make *A*g, on the Interſection, A F, equal to A g, in the Profile, and draw g N, cuting *AD*, at o, and draw o H cuting the Diagonal *A*O, at *f*; and, from *f*, draw a Perpendicular, giving the mitre Angle, S, of the front Planes, of the Modilions. Let it be continued around the ſeveral Breaks (as the Corona) in Pencil-Lines.

Draw *f*L, cuting the next Diagonal, at *l*. *f l* is the Seat, on the top, of the front of the Modilions on that Side; which may be continued around, from one Diagonal to the other, by means of the Vaniſhing Points H and L.

Make *A b* equal to A b, in the Profile, and draw *b* N, cuting *A D* at 2; and draw 2 H, cuting *f l* at *h*. From N, project the Point *h* to the Interſection, at *i*; and make *i j*, *j i*, *j k*, &c. reſpectively equal to the width of the Modilions, and to the ſpace between them, alternately, as often as is requiſite; from all which, draw Lines to the Point N, cuting the Seat, *f l*, in their perſpective widths, at *m*, *n*, &c. from which, draw Perpendiculars, giving the ſeveral Fronts, x, x, as in the Figure.

I The

The Modilions yy and zz, in the returning Side and Front, are determined after the same manner, *viz.* those in the Side, y, y, and also v, v, by means of the Point M, and the front ones, z, z, by the Point N; their measures being set off at 1, 2, 3, &c. on the Intersection of the Top, as those at x.

Having thus obtained the Fronts, draw Lines to the respective Vanishing Points H and L, till they cut the Plane from which they project; and, having drawn Right Lines for the edge underneath, first, the curved Lines may be drawn by hand, as in the Figure, regarding the different appearance of each, as they recede.

If the Parts are large, and require to be accurately projected, the Moulding, around the Modilions, may be managed the same as in the last Example, by a perspective Plan, above the Cornice. But, a judicious Person will save that unnecessary trouble; for, having obtained their mitre Angles, as at s, t, u, &c. the front Moulding being drawn, as any other continued Moulding, they may be returned at the sides, and meet the inner Moulding, sufficiently correct without the trouble of planing them.

In this Example, there is all the variety that is requisite for such Cornices; but, as the Corinthian Modilion may, to some, appear more difficult than the Ionic, I shall give a specimen how it may be delineated, as briefly as possible.

In the following Lesson, I shall not describe, over again, how the Mouldings are to be drawn; and, I have (in order to have the Modilions larger) omited the upper Mouldings of the Cornice, and the Corona; having retained only the Planceer; which I shall consider as the Plane, on which the Modilions are seated, and AB the Intersecting Line of that Plane.

The Vanishing Points, H and L, are also the Vanishing Points of this Figure; the Center or Point of view is at S, and the Distance is ES. Fig. 88. No. 2.

Draw the vertical Intersection of the Diagonal, AF; and, by a geometrical Scale of the Proportions, set off the heights of the Modillions and Mouldings at *a*, *b*, &c. also, make ab, bc, &c. equal to the width of the Modilions and spaces between; and transfer them, by means of the Point P (the distance of the Vanishing Point H) to *AB* (the first doted Line) which is the Seat of the fronts of the Modillions; and finish the square Blocks *efgh*, &c. which enclose the Modillions, as in the former Example; by which means, they are truly proportioned, perspectively, as in the Figure, which would otherwise, be somewhat difficult.

In the side, of each Block, must be drawn, by hand, the Profile of the Modillion, perspectively, as it is represented at X (the geometrical Profile) according as they are contracted, or obliquely situated to the Eye.

In the Front, of each, is also described the end of the Scroll, and of the Leaf. In short, having first obtained the Blocks or Cases which contain them, the rest must be delineated by a judicious hand and Eye; for such Figures will baffle all Rules, nor is it possible to subject them to any, by which they may with greater certainty be described; except the Profile, in the side of each, as the Truss in the next Example (No. 2) which is a similar kind of Figure.

The Moulding breaking around the Modillions, is first described as other continued Mouldings, at the front, on which their mitre Angles are obtained, from the Intersection AB; the measures, on which, not being equal, on both sides, is owing to the front Lines of the Fillets, being in another Plane, standing forwarder than the front of the Modillions.

The Dentils have nothing particular in them; the proportion being known, of their width and spaces between; a Right Line drawn through either Corner, as fg, parallel to the Intersection, being divided in the Ratio, and Lines drawn to any Point, G, in the Vanishing Line, according as the measures are taken, greater or less, will give their measures perspectively (as by Prob. 8) but, their true measures can only be applied, at the Intersection of the Plane they are in, with the Picture (drawn through C) and projected, by means of Visual Rays, to the Eye, at P.

Plate XIX.

EXAMPLE XX.

How to represent a Door Head, with a Pediment, supported by Trusses or Consoles.

Fig. 89. Let *A* be the determined Angle of the Cornice of the Object, in the Picture.

At No. 1. is the true geometrical proportion of the whole, in Front.

† Prob. 3. ‡ Prob. 17. Through *A*, draw *A*B perpendicular, the vertical Section of a diagonal Plane, passing through the mitre Angle, whose Vanishing Point is G†. H and L are the Vanishing Points of the front and side Lines, which are horizontal; S is the Center of the Picture, and SE its Distance; I and K are the Distance Points of each, respectively, for proportioning Lines which vanish in them‡; EG bisects the Angle HEL; conf. G is the Vanishing Point of horizontal, diagonal Lines. (Pr. 21.).

These things, here premised or given, are supposed to be found and determined, from the known position and situation of the Object, as in the former Examples. (See Prob. 12.)

All the measures, from the Profile, are produced, or applied to the Intersection, *A*B; and, by means of the Diagonal *AG*, the mitre Angle of each Moulding is obtained, as in the foregoing Examples.

To determine the breakings of the Mouldings around the Trusses, draw a*D*, the Intersection of the Plane they are in, above the Cimma reversa; and, by means of the Point I, project the Point *a* to that Intersection, at a.

Make a b, b c, and c *D*, respectively, equal to the Breaks, and to the opening between them, geometrical (as at v u, No. 1.) and draw b I, c I, &c. cuting *a* L, at *b*, *c*, and *d*, their perspective Proportions.

Draw *c* H, indefinite; and by means of the Point K (the Distance Point of H) project the Point *c* to the Intersection, at d.

Make d e equal to the returning Side, and draw e K, cuting *c* H, at *e*; and, through *e*, draw L *e*, cuting *b* H; which determine all the mitre Angles.

The Mouldings are described, or delineated, as in the last Examples.

To determine the proportion of the Truss, or Console; of which X is the Profile, and Z the Front (No. 1.) Through D, draw a horizontal Line, D F, and produce H *f* (from H) to that Intersection, cuting it, at h.

Draw h o, perpendicular; and, from h, let off, geometrically, all the proportions of the Truss, at i, k, &c. in respect of its height (as in the Profile at X) from which, draw Lines to H, cuting *f n*, at *k*, *l*, *m*, &c. from which they are transfered to the other side, by the Vanishing Point L, at *v*, *u*, *x*; and, by the Point H, projected forward; which proportions the heights in the other Truss.

The geometrical projectures are perspectively proportioned, in the same manner as the returning Moulding, at *c e*, by projecting the Point *f* (from K) to the Intersection DF, at 1, and making 1, 2, 3, equal to the projectures in the Profile; from which, draw Lines to K, cuting h H at *g*, *h*, &c. and from them, draw perpendicular Lines; by which means, finding as many Points as are necessary, the true perspective form may be determined.

At No. 2, is shewn (more at large) how the Truss is described, having all the same Letters of reference; and, in proportion to the other, as 3 to 2.

If the width be not already determined, by the Mouldings, draw I *h*, till it cuts the Intersection, D F, at f. H, I, K, and L (No. 2) are supposed to be the Vanishing Points, in the Horizontal Line, answering to those below.

Make f h equal to the width of the Truss, and draw h I, cuting *b* L at *i*.

In the Profile (No. 1) take as many Points, in the Curve (1, 2, 3, 4) as are necessary; from which, draw lines parallel to the Horizon, cuting h o, at i, k, l, and m; and also perpendicular. Let those measures be transfered to h o (No. 2)

from which draw lines to the Vanishing Point H; and, having proportioned *fh*, perspectively, as f h (No. 2) by the Intersection DF (where their geometrical proportions are transfered, at 1, 2, 3) draw perpendicular lines from the divisions on *fh*, cuting the Lines from i, k, &c. respectively, at 1, 2, 3, and 4, through which, a Curve being described will represent the out-line of the Console.

From 1, 2, &c. thus obtained, draw Lines to L; by which means, the exterior Curve may be described, as in the Figure; and, also, transfered to the other Console, which must be described after the same manner.

To determine the Pediment. Having obtained the Angle *B*, its perspective length (Prob. 17) equal twice A *A*, geometrical; at I (the distance of the Eye from the Vanishing Point L) make the Angle, LIV, equal to the Angle of the Pediment, cuting VL (the Vanishing Line of the front Planes, Prob. 3) at V, and set off an equal Distance on the other side L; from which Points, Lines drawn through *A* and *B*, determine the middle, at *C*; all the Mouldings vanish in those Points.

The height and true pitch, or inclination, may be thus determined.

Make B*E* equal *A*B; i. e. make *AE* equal to twice the height of the Pediment. Draw *BF* also perpendicular, and draw *E* L, cuting it at *F*.

Draw *A*F and *BE*, interfecting at *C*; *ACB* is the true perspective outline.

Or, having bisected *AB*, perspectively at *G* (by Prob. 8) draw a Perpendicular; and, from B, draw BL, cuting it, at *C* as before; and draw A *C* and *BC*.

From all the Mouldings at B C, draw Lines to the Vanishing Point L, cuting *CG*; and the projectures of the Mouldings being also perspectively proportioned, on *C*H, at r, s, t; perpendicular Lines, from them, will cut others corresponding, from *CG*, drawn to H, in the true mitre Angles; and, from those Angles, draw Lines to both Vanishing Points (V) of the Pediment, which will give all the Mouldings, with accuracy, beyond any other method whatever.

The horizontal width of the Frize and Architrave, are proportioned to the Truss and Mouldings, on *fn*; by drawing Lines to L; the perpendicular widths of the Architrave (being parallel to the Picture) and the opening of the Door, by the Vanishing Points of the Diagonals of a Square (as at Y) in the Vanishing Line VL of that Plane; regard being had to the projecture of the Mouldings (See Pr. 26.)

The recess of the Door, pq, is determined (by Prob. 10) by its width, m p.

Produce the Radial LE; make EM equal to the width, and EN to the recess; or, make EM and EN, in the ratio of one to the other.

Join MN, and draw EO parallel to MN. O is the Vanishing Point of a Diagonal of the Soffit, or head of the Door Case; and mO being drawn, cuts pH at q, as required. The rest is obvious, from the Figure.

This Method, when it is applicable, is preferable to any other; for, having obtained the true proportion of one Line (mp) in any Plane, any other Line (pq) in that Plane is easily determined, by the 10th Problem. Indeed, it may always be applicable (provided, the Diagonal Line mq (i. e. MN) be not very oblique to the Picture; and consequently, its Vanishing Point (O) very remote; for, if the whole Distance (SE) cannot be used, half, or any other portion, may be taken, with equal propriety; and the Point, O, ascertained the very same (by Prob. 12.)

Otherwise; if no Line, in the same Plane, be found or determined, there is but one general Method; which was applied, to proportion the return of the Moulding *ce*, or the front Line at *b*, *c*, and *d*, *viz.* by the true measure applied to the Intersection of the Plane it is in (Prob. 17) which being frequently used, would be needless to repeat; particular regard being had to the true Intersection of either Plane, mpq, of the Soffit (which is horizontal) or of the vertical Plane, W, of the Door Jamb.

For, since pq is the common section of both Planes, it is consequently in both; and therefore, either Intersection will answer the same purpose.

PQ is the Intersection of the horizontal Plane mpq; its true Distance, from any other, being known, equal DQ; a Line drawn through Q, parallel to the Horizon, is its Intersection with the Picture; for, all parallel Planes have parallel Intersections (8. 7. El.) And, if Hp be produced, till it cuts PQ; a Line drawn through P, perpendicular, is the Intersection of the vertical Plane W.

For, pq cuts the Picture at B; therefore, P is its intersecting Point (Def. K) consequently, PR is the Intersection of the vertical Plane, W, that Line is in (Prob. 3), and, PQ of the horizontal Plane, mq.

EXAMPLE

Plate XIX.
Fig. 90.

EXAMPLE XXI.

How to delineate a Block-Cornice, and to break the fame, or any other, around a Bow Window; which is half a hexagonal Prifm.

H L is the Horizontal vanifhing Line, C the Center of the Picture, and C E is its Diftance; H is the vanifhing Point of the Front of the Bow, and L of the Side of the Building, at right angles with it.

Draw EH and EL making a Right Angle (HEL) and, with any Radius, on *E*, defcribe an Ark, *x*, *y*, *z*, cuting EH, at *x*.

Make *xy* and *yz* each equal to E*x*; and draw *Ey*, cuting the Horizontal Line, at M; and E*z*, which, would cut it, if produced; M, and N (the fuppofed Point where E*z* would cut HL, produced) are the vanifhing Points of the Sides and Diagonals of the Hexagon.

Bifect the Angle *v* E*x*; (i. e. HEL) at *u*, and draw E*u*, to P; which is the Vanifhing Point of the Diagonal of the Right Angle.

Let AD be the Interfection, of the neareft Angle of the Object, with the Picture; and A the determined height of that Corner; let A a be the height of a Plinth, above the Cornice; and a b the height of the Cornice, of which, X is the Profile.

The Dimenfions of the Bow window, and other parts of the Building, being known, it would almoft be fuperfluous to defcribe how it is to be determined in Perfpective; fufficient inftructions for that purpofe are contained in the 4th and 5th Examples. However, as the application of them to real Objects, may not, to fome, be familiar, I fhall, briefly, defcribe it.

At No. 1. is a Plan of the Window, to a Scale of half the meafures applied in delineating; and about one fixtieth part of the real Object.

a k is the pofition of the Picture, applied clofe to the Angle a, and cuting the Building at the Angle c; wherefore, great part of the Cornice is projected to the Picture, feeing it is on this fide, a k.

Draw A F the Interfection of the Plane of the Top, parallel to the Horizontal Line; make HD equal to E H (the Diftance of the Vanifhing Point H) and draw A H and A L indefinite.

Make AB equal to the fhort returning Plane (V) which is at right angles with W, the long fide of the Building (a h, continued) i. e. make AB equal twice a b in the Plan, and draw BD, cuting AH, at *B*.

Draw N*B* indefinite (Prob. 13) and having found the point P, the diftance of N (by Pr. 12) draw P*B* to the Interfection AF, cuting it at *G*.

Make *Ge*, on the Interfection AF, equal to twice b c (No. 1) and draw P*e*, cuting N*B* at *C**. Draw *C*H, indefinite; and draw *C*D cuting AF at *d*; make *dF* equal to *Ge* (i. e. to the Front) and draw FD, cuting *C*H at *D*.

From all the Angles, *B*, *C*, and *D*, draw Perpendiculars, which give all the Faces of the Prifm, that can be feen (Y, and Z) i. e. b c, and c d (No. 1.)

As there is not room, on the Interfection, to fet off the whole meafure of the Plane W (equal four times a h, No. 1) take AG a third part of it, and L*E* one third of EL, and draw G*E*, cuting AL at *H*; from which draw a Perpendicular.

Thus, having determined the Building, we now proceed to the Cornice.

From X, the Profile, transfer all the meafures, to AD, of the heights of the Mouldings, at a b. Draw the Diagonal Pa; and determine the mitre Angle of the Mouldings (as in Example 17th) at a*ab*b.

* Becaufe the point *C* (in *B C*) is beyond the Interfection, A F, it is confidered as projected to the Picture, its place being on this fide, in the Original; as a k (No. 1) the Picture, cuts the Angle c.

 Draw

Draw aH and bH, cuting *B*h, at c and h. Draw Nc and Nh, till they cut *C*g, at d and g; and draw dH and gH, cuting *D* f, at e and f. Also, from i, draw iL, representing a Facia, below the Cornice; and iH, cuting *B*h, and continue it around each Face of the Hexagon.

The mitre Angles of the Hexagon are thus obtained.

In every regular Poligon, Right Lines drawn through the Center, bisect the Angles of the Poligon; and in a Hexagon, they divide it into six equilateral Triangles, each Diagonal being parallel to two opposite Sides. (See No. 1.)

*x*E*y* and *y*E*z* are two such Triangles; consequently, E*x*, E*y*, and E*z*, are parallel to all the Sides and Diagonals of the Hexagon †, producing the Vanishing Points, H, M, and N, of the Diagonals or mitre Angles. † Cor. 1. Th. 6.

But, the Angle, at b (No. 1.) where the Hexagon joins with the Building is internal, and its Mitre is gb, produced; consequently parallel to fc; therefore, they have the same Vanishing Point (M)

a b is the Angle of the Building (which is a Right one) where the Mouldings are first projected; ch is the internal, and dg, ef, two external Angles of the Hexagon.

Produce Se, Sd, and Mc, indefinite; and draw H*a*, cuting Mc, produced, at *c*.

Draw N*c*, cuting Sd at *e*, and *e*H cuting Se, produced, at *f*; *a*, *c*, *e*, and *f*, are the extreme Angles of the Cornice; which being obtained, the rest is managed as in the former Examples; by transfering all the measures, of the heights of the Mouldings, from ab to ch, and from ch to dg, &c. also, the projectures of the Mouldings, from the Diagonal a*a* to c*c*, from c*c* to d*e*, and from d*e* to e*f*; by means of the vanishing Points H and N; from which Diagonals, perpendiculars being drawn, give the Corona, at *b*, which is carried around in the same manner; and also the small Moulding at b, h, g, f.

The Blocks, are obtained after the same manner as the Modilions, in Ex. 19th.

The internal Angle at q being obtained, as above, and the hither face of the first Block described (by means of the Vanishing Point L) through the outer Angle draw pq parallel to the Horizon, and set off geometrically, the number of Blocks and spaces between them, at 1, 2, 3, &c. from which, draw Lines to some point (P) in the Horizontal Line, cuting p*q*, at *g*, *h*, &c. the perspective proportions of the fronts of the Blocks, on the Face Z.

On the other Sides, they may be determined after the same manner, or otherways; as on the Face Y, by their Seat, *BC*, at the top, projected to it from the Intersection AF; how they are finished is best explained by inspection of the Figure.

On the Side W, they are determined from their true measures, by drawing mn, through n, parallel to AF; mn is the Intersection of the Plane they are in with the Picture (which pq is not) therefore, having (from the point Q) projected the first to mn, at r, set off the true geometrical measures, from r towards m, at r, s, &c. and, by means of the same point Q, project them to rL; i. e. draw rQ, sQ, &c. cuting rL, which give their true places, on that Side. Fig. 90.

When the full measures, on nm, exceed the bounds of the Picture, make use of the half measures and distance, &c. as in former Examples.

The Blocks, in the returning Side, V, are determined, after the same manner, from the true measures; by means of the point D, the Distance Point of the vanishing Point, H, of that Side.

The Windows, and the Mouldings around them, are determined as the Door, &c. in the foregoing Example; by setting off the true space and measure of the Moulding from I to k, and transfering them from one Plane to the other, as in the Figure.

G*e* and *d*F, on the Intersection AF, being the true geometrical measure of each Plane; divide them in the true measures of Piers, Mouldings, and Sash Squares, at *i*, *k*, *l*, and *m*; from which draw Lines to the Points D and O, respectively, cuting *BC* and *CD*, at *n*, *o*, *p*, *q*; and, from them, Perpendiculars being drawn, determine their perspective proportions on those Planes, at *r*, *s*, &c. and the proportions of the Sash Squares are projected to their true places, from *t* and *u*, by means of the Vanishing Points L and Q, respectively.

I cannot help remarking, here, that the apparent intricacy of reprefenting Mouldings, in Perfpective, with accuracy, has led fome Perfons to imagine that the whole Work is intricate, and the Diagrams complicated; imagining they muft neceffarily go through the whole, to be acquainted with it. No fuch thing is abfolutely neceffary. If a Perfon be well acquainted with the Principles of the Theory, and the Elements of Practice, contained in the 3d, 4th, and 5th Sections of this third Book, he will find no myftery in their application to any Object whatever; the different Examples, here given, fhew how they may be applied, with fuccefs, in various cafes, and under various circumftances, which might not readily occur, to a Perfon not thoroughly converfant in it. They muft not imagine, becaufe there are a multiplicity of Lines in a Plate, in which there are fuch variety of Parts in the Object, that the whole is incomprehenfible; let them firft make themfelves Mafters of the fimple Leffons and Examples, before they attempt a complex one, or it will be impoffible for them to fucceed in it. Can they imagine, that in fuch a Subject, as the 86th Figure, where every effential part is minutely defcribed, it can be done without Lines, by contemplating the Figure only? Or, more particularly, the three laft Figures, in which, every horizontal Line is inclined to the Picture; every one of which muft be perfpectively proportioned, to its place and pofition. Although the defcription is full and explicit (the preceding Leffons being underftood) yet, to give a minute inveftigation of the 89th Figure, would fill, at leaft, half a dozen Pages, and would appear the moft prolix and tedious defcription imaginable.

Notwithftanding the operative Lines in them neceffarily crofs each other, yet, if they are traced to their proper References, that intricacy (which fome object to) will no longer exift. It is obvious, that, to have avoided it, wholly, the Figures muft have been much fmaller, the Plates much larger, or their number greatly augmented, which would have enhanced the price, unneceffarily. If they imagine that others have managed thofe matters more fimply, let them compare, and fee what they can make of Pozzo, the only Author, amongft the Italians, who has attempted it; nor has he attempted the defcription, but the delineation only, and that, in the moft eafy pofitions; wherefore, his elegant Defigns, convey no better inftructions, than any other good Print, by infpection. Mr. Kirby, in his firft Effay, has run away from the Subject before he has well begun it; and, in his pompous Work, what has he done? nothing, to any purpofe. Mr. Highmore, in his 37th Plate, has delineated a Cornice, at No. 3. tolerably, parallel to the Picture; but No. 4, being inclined, is intolerable, and his defcription of it more fo. In fhort, a complicated Subject cannot be defcribed without Words, nor can the operation be performed without Lines; although, as I have obferved, in the Preface, not half the Lines, which appear in thefe Diagrams, are neceffary to be drawn at all, in the operation.

SECTION VIII.

OF CURVILINEAR OBJECTS.

THE Perfpective of curve-lined Objects, the Subject of this eighth Section, is the moft difficult, of all other; feeing that, Curve Lines cannot be projected as Right Lines, by means of interfecting and vanifhing Points, indefinite; neither can any portion be taken, or cut off, perfpectively, otherwife than by drawing a Chord Line from one Point to another in the Original, and finding its Vanifhing Point; or, by any means, finding the reprefentation of the extreme Points, from their correfponding Points in the Original.

There are various ways of projecting the reprefentation of a Circle, in Perfpective; all which, do no more than find the reprefentations of various Points in the Circumference. For, by the Theory of curvilinear Perfpective, it is fuppofed, that the defcription or delineation of a Circle, or Sphere, in Perfpective, is fome one or other of the Sections of a Cone; as in Fig. 28, Plate 7; it is obvious, that if Right Lines (EA, EB, &c.) from the Eye to the feveral Points A, B, C, &c. in the Curve (which is fuppofed to be a Circle) in the Plane Z; be cut by another Plane (X or Y) the Points a, b, c, &c. being joined, carefully, by a fteady hand will generate a Curve (adg) which, to the Eye at E, will (as it is obvious it muft) exactly coincide with the original Curve; feeing that, it is in the furface of the fame Cone, of which, the Eye is its Vertex.

Hence it is manifeft, that, the more Points there are found in the Reprefentation, the more exactly may the Curve be defcribed; but after all, it depends greatly on the Hand and Eye; infomuch that, without great nicety in both, the reprefentations

ſentations of curve-lined Objects will have a lame, and very diſagreeable appearance. It is, therefore, no wonder to ſee ſuch bad Repreſentations of round Objects, as are to be met with; but it is a matter of ſurprize, that any Perſon, who attempts it, ſhould have ſo little judgement as to turn the Curves the contrary way; or to make them flatter in thoſe parts, where, it is obvious that, they would have a greater curviture.

Without having the leaſt notion how to project Curves, perſpectively, is it poſſible for a Perſon, who has been uſed to ſketch at all, to place a cylindrical Object, of any kind, or a Vaſe, &c. before him, and not ſee, immediately, how the curves are to be deſcribed? or, what part of the Object appears more or leſs curved than others? is is not obvious to a common Eye? yet may we frequently ſee Vaſes repreſented, whoſe greateſt ſwell, at the top, is a Right Line, whilſt the Curves of the lower part take a contrary direction, to each other; indicating, that the Eye is between them; in which Caſe, the top would be the moſt curved. Would not ſuch attempts at Perſpective be better let alone? and content themſelves with a perfectly geometrical Repreſentation? in which they would all be Right Lines; ſave only, the external Figure of a vertical Section through the middle.

But, what can be ſaid for the performances of thoſe Artiſts, whoſe Works are an honour to their Country and to the Age they lived in, to ſee them, often, greatly deficient in thoſe particulars. I could wiſh to ſee the Works of the preſent Age more perfect; which, in other reſpects, ſeem to vie with the moſt celebrated amongſt the Antients; yet do not pay ſufficient attention to thoſe neceſſary Appendages, which are eſſentially requiſite, to a perfect Picture.

As a Circle is the firſt and principal of curve Lines, ſo it is the only one that can be reduced to any certain Rules, in delineating it perſpectively. And, of all the various ways to project the repreſentation of a Circle, in Perſpective, the beſt, and moſt practicable, is to ſuppoſe a regular Octagon to be inſcribed, or the Circumference divided into eight equal Parts; or, if very large, into ſixteen.

Irregular curved Objects are not Subjects for Perſpective; all attempts at a Spiral, or twiſted Column, &c. by Perſpective Rules, would be in vain. Various other Objects, as Rocks, Mountains, Rivers, Trees, &c. are not fit Subjects for Perſpective; and conſequently Landſcape Views cannot be taken or delineated by its Rules; becauſe it is impoſſible to have the true geometrical Figures and Proportions of ſuch Objects, as before mentioned. I ſhall however, in an Appendix, deſcribe the uſe and application of an Apparatus for taking Views, with eaſe and great exactneſs; the beſt calculated, for the purpoſe, of any I know or have ever heard of. For, notwithſtanding what many imagine and affirm, of the poſſibility of taking Landſcape Views with accuracy, by Sight only, I know it is impoſſible to be done; and cannot conceive it to be any way derogatory to the abilities of the moſt eminent Artiſt, to make uſe of any expedient; by means of which, he may be enabled to make a more correct Portrait, and Picture. I do not mean that he ſhould, rigidly, deſcribe every minutia of the Objects, as in Trees, &c. by it; but I muſt affirm, that he would take the apparent Magnitudes of the ſeveral Objects, their Figures, and their Bearings in reſpect of each other, with much greater accuracy than it is poſſible to do by Sight.

P R O B L E M I.

To find the Repreſentation of a Circle; the Original being given, in any Plane, whoſe Vaniſhing Line, its Center and Diſtance are given; according to Brook Taylor.

Firſt; by means of the Vaniſhing Line, and one Vaniſhing Point only.

Let AFG be a Circle, in the Geometrical Plane; of which *GK* is the Interſection, and V*E* the Vaniſhing Line; *C* is its Center, and *CE* its Diſtance. Fig. 91. No. 1

Plate XX. Fig. 91. Draw AB, at pleaſure, cuting the Interſection at a; and through C, D, G, &c. (Points aſſumed at pleaſure, in the Circumference) draw Lines parallel to AB, cuting the Interſection, at a, c, d, &c.

†Cor. Th. 3. Draw EV parallel to AB, producing the Vaniſhing Point, V, of thoſe Lines†.

Draw aV, cV, &c. and, to E, draw Viſual Rays from every Point, A, D, G, &c. cuting the indefinite Repreſentations of the Lines paſſing through thoſe Points, reſpectively, at *a*, *d*, *g*, &c.

Having thus obtained as many Points as are neceſſary, a Curve (*adifb*) deſcribed carefully through thoſe Points, will be an Ellipſis, and it is the true Repreſentation of the Circle AFG.

Or, having drawn the parallel Lines AB, &c. and their indefinite Repreſentations, as before; make V*E* equal to VE; alſo, make ab equal to aB, ge equal to gH, &c. and draw b*E*, e*E*, &c. which will give the ſame Points as before.

This needs no Demonſtration; ſeeing that, the Points *a*, *b*, *c*, *d*, &c. are projected the ſame, as in Prob. 6th, where it is fully demonſtrated. And, that the Curve is an Ellipſis, is demonſtrated in Th. 2nd. Sect. 5; of Curvilinear Perſpective; for it is evident, and manifeſt, that the Viſual Rays EA, ED, &c. from the Eye to every Point in the Circumference, would cut the Picture in the correſponding Points *a*, *c*, *d*, &c. therefore the Curve *adf*, deſcribed through thoſe Points, is the true ſection of the Cone of Rays, and conſequently it is an Ellipſis.

Secondly; by the Directing Line and one Director. No. 2.

Fig. 91. No. 2. Let No. 2. be a Circle, nearly in the ſame Poſition to the Eye (at E) as before; having the ſame Interſection, *G*KI, and the ſame diſtance of the Eye (SE.)

Draw CD parallel to the Interſection, and diſtant from the Eye equal to the diſtance of the Vaniſhing Line (VL) from the Interſection (KI.) CD is the Directing Line of the plane of the Circle. (Def. 10.)

Draw AD, at pleaſure, cuting the Directing Line at D; and, from ſeveral Points, B, F, &c. in the Circumference, draw Right Lines to D, cuting the Interſection at c, d, e, &c. the Interſecting Points of thoſe Lines.

Draw ED; and, from the Interſecting Points, K, c, &c. draw Right Lines parallel to ED, which are the indefinite Repreſentations of thoſe Lines, reſpectively.

Draw the Viſual Rays EA, EB, &c. as before, cuting thoſe Lines at *a*, *b*, *f*, &c. and deſcribe a Curve through them, which is the true repreſentation of the Circle AFH. Draw VL, the Vaniſhing Line of the plane of the Circle.

Dem. Becauſe the Lines AD, BD, &c. have the ſame Directing Point (D) their Repreſentations are parallel between themſelves (Cor. 1. Theo. 14.)

Conſequently, ſeeing that Ka, cb, &c. paſs through the Interſecting Points, K, c, d, &c. parallel to ED, they are the indefinite Repreſentations of thoſe Lines; and conſequently, the Viſual Rays EA, &c. will cut them, in the ſame Points as before, which is manifeſt; for, if Ea, Eb, &c. be drawn, parallel to AD, BD, &c. reſpectively, their Vaniſhing Points, a, b, &c. are produced; by which, the affinity between the different Methods of producing the ſame thing is accounted for.

PROBLEM II.

To deſcribe the Repreſentation of a Circle, having the Repreſentation of one Diameter given.

Fig. 92. Let AB be the given Diameter; let *ECE* be the Vaniſhing Line of the Plane it is in; *C* is its Center, and *CE* its Diſtance.

Produce AB to its Vaniſhing Point, V; and biſect AB, perſpectively (Prob. 8) e. g. Draw A*bc* parallel to the Vaniſhing Line, and take two equal Diviſions A*b*, *bc*, at pleaſure. Through B, draw *c*B, and produce it to the Vaniſhing Line at G, and draw *b*G cuting AB, at C, the Center of the Circle; through which, draw DF, parallel to the Vaniſhing Line.

Make VG equal to VE, and draw AG, cuting DF at D; and, make CF equal to CD; or, through B, draw GF. DF is a Diameter of the Circle.

Make *CE*, on both Sides, equal to *CE*; draw *CC* indefinite; and, through D or F, draw *ED* or *EF*, cuting *CC* produced, at H; through which, draw ab parallel to DF; and, through D and F, draw *CD*, *CF*, cuting it at a and b.

Draw the Diagonals a*E* and b*E*; which will pass through C, the Center of the Circle, and cut a*C*, and b*C*, at d and c; draw cd, cuting *C*H at I; IH is a Diameter, and abcd is the representation of a Square, circumscribing the Circle.

Make $E E^2$, and $E E^3$, each equal to *E*E; from both which Points, draw Lines through D and F, cuting the Diagonals at e, f, g, and h; which Points are also in the Circumference.

Through the Points, A, H, f, F, g, I, h, D, and e, if a Curve be described, it will be the true Representation of a Circle, whose Diameter is equal to ab.

Dem. For, suppose ab the Intersection of the Plane the Original Circle is in; and *ECE* its Vanishing Line. *E*, *E* are the Vanishing Points of the Diagonals of a Square †, and the two Diagonals, ac and bd, cut each other in its Center, C‡; which is, consequently, the Center of a Circle, inscribed. † Prob. 10. ‡ 16. 1. El.

And, because HI passes through C the Center of the Circle, HI represents a Diameter perpendicular to ab, seeing it vanishes in *C*; and DF, passing through C, is also a Diameter, parallel to the Picture.

Also, because CD is equal to CF, and $E E^2$ is equal to *E*E; E^2 and E^3 are the Distance Points of the Diagonals; and consequently, seeing CD is equal to CF, the diagonal Diameters, eg and fh, represent Lines equal to DF. (See Prob. 8th, and 10th, Case 3rd, for a further Demonstration.)

PROBLEM III.

How to describe the Representation of a Circle, in any Plane whose Vanishing Line, Center and Distance are given; and Intersection.

Let AB be the Intersection of a vertical Plane, and *ECE* its Vanishing Line; its Distance is *CE*. Let AB be the Diameter given. Fig. 93.

Bisect AB, at D; and draw CD perpendicular to AB.

Make DC equal to AD, and describe the Ark DEF, a fourth part of the Circumference. Draw AC, cuting the Circumference, at E; from which, draw a Perpendicular, Ee, to the Intersection.

Make Bc equal to Ae, and draw A*C*, B*C*, and D*C*; also c*C* and e*C*; and draw the Diagonals A*E* and B*E*, cuting A*C* and B*C*, at I and K; draw IK; and *FH*, through S, parallel to AB.

Through the Points, D, *F* and *H*, also through *a*, *e*, *b*, *c*, and *d*, where the Diagonals, and IK, are cut by c*C*, e*C*, and D*C*, if a Curve be described, accurately, it will be an Ellipsis, and the true representation of a Circle, viewed oblique.

If the Circle be large, and eight Points are not sufficient; bisect the Arks, DE and EF, at f and g; from which, draw Perpendiculars to AB; and make b and d, equally distant, as a and f, from D; or from A and B.

Draw a*C*, f*C*, d*C*, and b*C*. From *A* and *G*, &c. where the Diagonals are cut, draw *Af* and *Gg*, &c. parallel to AB, cuting f*C* and a*C* at *f* and *g*; and the others at *h*, *i*, *k*, *l*, *m*, and *n*, through which Points, the Representation will also pass, and may be described with more accuracy.

The affinity between the Original and the Representation, which the corresponding Characters particularize, is sufficient Demonstration. S represents the Center of the Circle, C.

This Method of projecting the Representation of a Circle is, of all others, the best and most practicable. It is nearly the same thing as Prob. 25th; for, the eight Points *a*, D, *e*, *F*, *b*, *c*, *d*, and *H* being joined by Right Lines, will be the Representation of an Octagon, which is circumscribed by the Circle; as, in the other, the Circle is inscribed, i. e. touches every Side, as this passes through its Angles

Plate XX. Fig. 93. Notwithftanding, the Methods, in Prob. 1ft, by Brook Taylor, are facile and fimple, yet I believe, they are fcarce ever ufed in Practice. If the Circle be large, and Diftance adequate thereto, they are utterly impracticable: becaufe there is a neceffity for having the whole Circle and Diftance, at once, in their true places. Whereas, by the laft, the Diftance is applied on either, or on both fides of the Center, as in all other Cafes whatever. Nor is AB neceffarily the Interfection of the Plane of the Original Circle. For, if the place of the Circle be determined, on the Picture (either its Center, or the neareft part of the Circumference, at D) a Line drawn through S or D, parallel to the Interfection, or Vanifhing Line, anfwers the fame purpofe. AB or *FH* being made equal to the known Diameter, in that place, and a quarter of a Circle defcribed, as DEF, of that proportion (which, it muft be obvious, is as fufficient as the whole) the reft is as already defcribed. Other Points, if requifite, may be taken, as f and g, and more, if neceffary.

In common Practice (in a Circle not very large) eight Points being fufficient, there is no real neceffity for defcribing an Ark, geometrically; for e (or c) the Point where a Perpendicular, from E, cuts the Diameter, is diftant from A or B fomewhat more than one feventh part of the whole Diameter, AB; fo that, in all common Cafes, it may be afcertained near enough.

The 2nd Problem is ufeful in many Cafes. Having obtained a Line, AB, by any means, on the Picture; which being known to be the Diameter of a Circle, the whole Circle may be projected by the means there defcribed; or by Prob. 10th, Cafe 3rd, with the greateft exactnefs. Having obtained the parallel Diameter, DF, the affinity with this laft Problem is difcernable.

EXAMPLE XXII.

How to reprefent a plain, circular Arcade, cafually inclined to the Picture.

Fig. 94. Let VM be the Vanifhing Line of the Horizon, C the Center of the Picture, CE its Diftance, and A the Interfecting Point of the hither Angle of the Steps.

Through A, draw A J parallel to the Horizon; which may be confidered as the Ground Line; alfo, draw AG perpendicular, the vertical Interfection of a Diagonal Plane; on which, fet off, from A, the feveral heights and proportions of the Object, at D, G, &c.

† Ex. 13. The near Angles of the Steps being firft obtained, as at A, a, b†; their length may be acquired by Prob. 7th. If they exceed the bounds of the Picture, take any equal portion of their length, as A J, one third part; alfo, V being the Vanifhing Point of that Side (fee Prob. 21, Meth. 3rd) and VE its Diftance, make V*D* one third of VE, and draw J*D*, cuting the indefinite Reprefentation AV, at N. AN reprefents a length equal to thrice A J. (Prob. 17.)

The Steps are finifhed as in the 13th Example.

Next, the proportion of the Plinth, ec f, is determined by the fame; and the others, gh, ik, and l, by Example 4; which, with the Piers, are fo many Parallelopipeds, of equal magnitude and equally fpaced. (See Example 4.)

Their meafures are fet off, at 1, 2, 3, on the Ground Line; and projected to their Seats, on the Ground, at 3, 4, 5, 6, &c. by means of the Point *E*, on one Side; the Diftance, VE, of the other Vanifhing Point (V) falls out of the Picture.

Their height, and the upper Border at H, are obtained, by their geometrical height, AD (as the Pedeftal in Example 13th.) Thofe above are perpendicularly over the Plinths below; the Band, or Caping on the Top, is determined from G; and, by drawing Lines to both Vanifhing Points, V and Y.

The laft is not in the Picture, and muft be drawn by Prob. 13, or the Point muft be afcertained; where, EY, produced, would cut the Vanifhing Line VM. *E* is the Diftance Point, of the Eye, for the Vanifhing Point (Y) of the Lines AY, &c.

To reprefent the Arches; which, in this Example, are inclined to the Picture (and confequently, their Reprefentations are Ellipfes) the laft Figure is adapted.

D, on the vertical Interfection AG, is the height of the fpring of the Arch.

Make DE and EF equal to the meafures on AB (Fig. 93) i. e. make DF equal to the height of the Arch (half the fpace between the Piers, equal CF) and DE equal De (equal LE) and, by means of the Vanifhing Point of the Diagonal (M) transfer them to the Angle of the Building, at H, I, K; or, if HK be the determinate height of the Arch, at that Angle, the reft was unneceffary.

From

From H, draw Lines to both Vaniſhing Points, V and Y, cuting the Piers, at e, f, m, n, &c. Biſect m n, perſpectively, at S ‡, the center of the Arch. ‡ Prob. 8.

Draw S p perpendicular, cuting a Line, from K to the Vaniſhing Point, Y, at p, the crown of the Arch; alſo, draw n o and m q perpendicular; m q o n is half a Square, circumſcribing the Arch.

Draw the Diagonals S o, and S q, cuting a Line from I, at r and s.

A Curve deſcribed through m, r, p, s, and n, is the front of the Arch, the Repreſentation of a Semicircle. If more Points are requiſite, ſee the laſt Problem.

The inner Curve is deſcribed as the outer; n*n* being the thickneſs of the Pier, obtained below; from C and d in the corner Pier.

From all the Points in the outer Curve, draw Lines to the Vaniſhing Point V; and, *m n*, the inner Diameter, anſwering to m n, being drawn, S V cuts it at *ſ* the Center; from which, the Diagonals *ſo*, *ſq*, being drawn, in the Rectangle *m q o n*, give the Points *r*, and *s*; and a Perpendicular from *ſ* gives *p*; then, a Curve drawn through *m*, *r*, *p*, *s*, and *n*, determines as much of the Soffit of the Arch as can be ſeen.

After the ſame manner, the Arches *a d ſ*, &c. are deſcribed, in the returning Side; which would be ſuperfluous to repeat over again; as the Lines themſelves ſhew their uſe and application the ſame.

If V E^2 be made equal to V E, on the Vaniſhing Line of the front Plane, E^2 is the Vaniſhing Point of one Diagonal (as *ce*) in each front Arch.

E X A M P L E XXIII.

To repreſent an upright Cylinder, of any given Dimenſions.

Let A B be its Diameter. The height of the Cylinder is ſuppoſed to be known. Let V be the Center, and VC the Diſtance of the Picture. Fig. 95.

Deſcribe a Square A B D g (Prob. 19) and inſcribe a Circle a c e g (by Pr. 2 or 3.) At the two Extremes of the Ellipſis, a and d, draw Perpendiculars, which repreſent Sides of the Cylinder. Their height muſt be determined by a Perpendicular either from A or b, or any Point in the Circumference.

Make A *A* and b*b*, each equal to the determined height of the Cylinder; the Point *b* is in the Circumference at the Top; and, if A C be drawn, it will cut Perpendiculars from a and e, at *a* and *e*, which are in the ſame Circumference. *b*V will cut a Perpendicular from f at *f*; and thus, as many Points as you pleaſe may be obtained, as *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h*, correſponding with a, b, &c. below; through which, the upper Curve may be deſcribed.

Or, having obtained any three Points, the Curve *a*, *b*, *c*, *d*, may be determined, by Prob. 11, with the greateſt accuracy.

The Plinth, A B D is parallel to the Picture; and the Abacus, *B* F G, is equally inclined on both ſides, each of which repreſents a Square (ſee Prob. 29 and 30) the Curves are ſtill the ſame; and if they are equally diſtant from the Vaniſhing Line (VC) they are equal and ſimilar Ellipſis.

It is obvious that, as the Diameter of the Cylinder depends on the Ellipſis, at either Baſe, it is very liable to error; as the Ellipſis is contracted or lengthened, it will be ſmaller or larger in Diameter. To obtain the true Diameters of ſeveral Columns, with accuracy, they muſt be drawn in their true geometrical proportions, as in the next Figure.

Let A B, D F, and G H be the Sections of three Columns by a horizontal Plane, in which the Eye may be ſuppoſed to be, at E; and, let *A H* be a Section of the Picture, i. e. ſuppoſe a Plane ſtanding upright on *A H* parallel to the Columns; E C perpendicular to *A H* gives its Center, and Diſtance. Fig. 96.

The

Plate XXI. The Vifual Rays EA, EB, &c. being drawn, are in the fame Plane; and the Points *A*, *B*, *D*, &c. where they cut the Picture, it is evident, are the places, where the apparent edges of the Columns will appear, and confequently, *AB* is the apparent width of the Column AB, *DF* of DF, and *GH* of GH; which, on account of the Rays, EG, EH, interfecting the Picture more oblique than thofe from DF, has a larger Reprefentation; notwithftanding it is really further from the Eye, and from the Center of the Picture, at C.

If the Eye be moved to *E*, the doted Lines fhew the difference in their Proportions, from the two Stations or Points of View; where, the difference between the Reprefentations of DF and GH are much larger than at E.

If the Picture be at a h, parallel to *AH* (the Eye being at E) the Proportions of the Reprefentations are in the fame Ratio, as their Diftances, E c to E C.

The true pofition of the Picture from that Station, is *a h*, whofe Center is at *c*, E*c* perpendicular to *a h*, bifects the Optic Angle, *a*E*h*; which Optic Angle, of the other Pictures, is iEh; c i being equal to c h.

And thus the Diameters may be obtained on any Picture, fituated to the Columns in any Angle, at pleafure as (*HAK*) the apparent Diameters on *AK*, are where the Vifual Rays cut it, at *A*, *i*, *k*, *l*, *m*, *K*.

See this matter more fully treated, in the fixth Section, of the Theory, Fig. 34.

EXAMPLE XXIV.

How to reprefent feveral Cylinders, in the fame Right Line; inclined to the Picture, at pleafure.

Fig. 97. Let X, Y, and Z, be the Seats of three Columns, on the Ground Plane, between the parallel Lines, AB and CD; inclined to the Picture in the Angle BAF, at difcretion. AF is the Interfection; and HL, the Horizontal Vanifhing Line; C is the Center, and CE the Diftance of the Picture.

Find the Vanifhing Point, H, of the inclination of the Line of the Columns, AB (Prob. 17.) and make HEL a Right Angle; which bifect by the Right Line EO.

L is the Vanifhing Point of the other Sides of the Squares; in which the Columns, i. e. their Plans are infcribed; and O of the Diagonals.

It would be fuperfluous, after fo many Examples of the fame kind given, to fhew how the Reprefentations of thofe Squares are obtained; either by applying their feveral Diftances from A to a, b, c, &c. (as in Ex. 4) or, as the Square *ABFD* is found (Prob. 19) feverally, at difcretion.

The doted Lines fhew how the laft is managed; and it is the beft, when there are Circles to be infcribed; as they are reprefented at *af*, *cg*, and *eh*.

Which being obtained, feverally, by the laft Example, draw AK, perpendicular, and equal to the height of the Columns.

Find the correfponding Squares of the tops of the Cylinders, in which infcribe Circles perfpectively †, and draw perpendicular Lines, *ai*, *cl*, *em*, &c. reprefenting the apparent edges of the Cylinders.

† Prob. 3.

If they were to reprefent Columns, diminifhing at the upper ends, the Squares containing the Circles muft be reprefented lefs, in proportion to their refpective Diameters; and the Lines, which are, in this Cafe, perpendicular, will be gently curved; having obtained the Diameters above and below, a curved Ruler is the beft expedient, for drawing the apparent Lines of the fides of the Columns.

Now, although this method of obtaining the apparent Diameters, if it be accurately performed, is ftrictly true; for, the reprefentations of their Bafes and Tops being true Ellipfes, there cannot poffibly be any error; yet, the performance of it is liable to great error, becaufe it is impoffible to defcribe the Ellipfis accurately; and therefore, the Diameters cannot, by this means, be truly obtained; efpecially at the bottom, being fo near the Vanifhing Line; but, much more accurately, by the laft Example.

Then, since EC is the Distance of the Picture, produce it to the Intersection, at G; make GS equal to EC, and conceive the Picture to stand erect, on its Intersection AF, and S to be the Station Point, at the foot of the Spectator; for SG the Distance of the Picture is equal to EC, from the Eye to its Center; the space between the Intersection and the Horizontal Line not being a part of the Distance, seeing they are both on the Picture. (See Fig. 37, No. 1.)

It is equally the same, if S be considered as the Eye, and AF the Horizontal Line; and suppose X, Y, and Z, sections of the Columns by the Horizontal Plane.

Right Lines being drawn to S, as the Station Point; or being, considered as the Eye, in the Horizontal Plane, are Visual Rays to the extreme edges of the Columns; which, by their Intersections with the Picture, at 1, 2, 3, 4, 5, and 6, give the apparent proportions of the Columns, on the Picture; from which, perpendiculars being drawn, give their true places, and representative Diameters, beyond any other means whatever.

EXAMPLE XXV.

How to represent the Tuscan Base, its Plinth parallel to the Picture, and seen obliquely.

C is the Center, nearly as in the last; but the Distance is greater, equal CH: the Horizontal Line is the same; and the Intersection, or Ground Line, is AB. Fig. 9

At X is a Profile of the Base, consisting of one Torus and Plinth.

Being parallel to the Picture, the Plinth, AabB, in Front, is geometrical; compleat the square of the Base, perspectively †; and, within that, another Square (abcd) equal, in width, to the Circle at the bottom of the Torus, in which describe an Ellipsis ‡; the nearer part, efg, only, will be seen.

Produce the corner of the Plinth, from A; AJ is considered as the Intersection of the Diagonal Plane. Make AJ equal to the height of the Fillet above the Torus; and having made a Perspective Plan below (the geometrical proportions, from the Profile, being set off, from A and B, as A*a*, B*b*) from the Angles *A*, *B*, and *D*, draw Perpendiculars *A*I, *B*K, and *D*L.

From J, on the vertical Intersection, draw JH, cuting *A*I at I; also draw IK parallel to the Ground Line (AB) and compleat the Square KL, of the Fillet; in which, describe the Ellipsis *def*, and another parallel to it, perspectively, equal to its width.

Make AG equal to BE; or, through E, the middle of the Torus, draw EG, and compleat the Square EF; in which describe an Ellipsis, as at *ghik*, which gives the greatest swell of the Torus; and, if greater accuracy be required, describe another Ellipsis, above that; and thus, as many representations of Circles as you please being described, the Curves, *dg*e, and *fk*g, described over their extremes, falling into the Ellipsis on the Plinth, at e and g, will be the true Contours of the Torus, in Perspective.

† Prob.

‡ Prob.

Although I can scarce suppose that any Person would go through this process, who would delineate a Base; yet is there no better or readier way of doing it, with certainty. Nevertheless, this dissection of it will greatly assist the imagination; and is the best method of acquiring a true knowledge of the nature of curvilinear practical Perspective.

hik, at the bottom of the Plinth, is an Ellipsis, which is the perspective Seat of the Shaft of the Column; from which may be obtained the edges of the Column, at MN; but it is of no other use, seeing the Curve of the Ellipsis, at MN, is very different from that below, on account of its being nearer to

Plate XXI.
Fig. 99.

EXAMPLE XXVI.

How to represent a Doric Capital, inclined to the Picture, casually, or at discretion.

S is the Center of the Picture, and C is the Vanishing Point of one side of the Abacus; the other is out of the Picture; the Distance is equal to SL.

E is the Eye, i. e. the Distance of the Vanishing Point of the other Side (found by Prob. 12) and F is the Vanishing Point of a Diagonal, bisecting the Angle.

Let A be the place of the nearest Angle, in the Picture; and let Z be the geometrical Profile. AB is the Intersection of the plane of the top; and it is the measure of the square of the Abacus.

A*B* being drawn indefinite, draw BE cuting it at *B*, and draw AC, and *B*C, and a Diagonal AF, cuting *B*C; compleat the square of the Abacus (Prob. 21.)

Draw AD the Intersection of the mitre Diagonal, and transfer all the measures from the Profile to AD; from which, draw Lines to F; and having got the Points *c* and *e*, on the Diagonal AF, the Seats of the Projectures of the Facia, Z, and the Fillet at x (obtained as usual, by seting off the measures at *a* and *b*) from *c* and *e* draw Perpendiculars, giving the Points a and e.

Compleat the Squares cd and fg, in which describe Ellipses; ab being drawn parallel to AB, may be considered as the Section of the under Face of the Abacus.

From E, or any other Point in the Horizontal Line, draw Ec to b; bisect ab, at *d*, and make a1, b2, each somewhat more than one seventh part of ab; or with the Radius a*d* describe an Ark of a Circle, and get the Points 1 and 2, as by Prob. 3rd, by which means, the Ellipsis, i. e. the Points *e*, *f*, *g*, *h*, *i*, by which it is described, are obtained.

After the same manner, in the Square, efg, of the fillet at x, describe the Ellipsis j, *k*, *l*, and another parallel to it, equal to the width of the fillet; and also, another above that, for the thickness of the small Bead, at x; which, being so little distant from each other, will not vary much in their Curves; otherwise, every different representation of a Circle must be severally described, as the foregoing.

As it is somewhat more difficult to get the necessary Points, when the Square is inclined than parallel; let the remainder of the Curves be thus obtained.

Through y draw a Line parallel to AB; and (because this Object is viewed centrally) where it cuts the Vertical Line, at i, set off if and ik equal; making fk equal to the Diameter of the fillet, at y, geometrical; draw fS and kS.

From the Point j, where fd cuts AD, draw jF, cuting iS at o, the representation of the Center of that Circle; and, through o, draw Lo, cuting fS and kS at h and m; through which, draw gh and lm parallel to fd; ghlm is the Square of that Circle, in which an Ellipsis must be described, as before; with the Fillet and Astragal, as was done above; particular care being had, to make them lead true, at the extremes, as if they were continued around.

The Square of the section of the Shaft (pq) is obtained in the same manner; and, an Ellipsis being described in it, from the Extremes of which, perpendicular Lines are drawn, for the Shaft or apparent edges of the Column. How they fall into the Cavetto is impossible to describe, seeing, the Line is lost insensibly.

The large Ovolo can only be described, truly, as the Torus, by means of various representations of Circles, and describing, carefully, a Curve over the extremes of them all; for it must be obvious, that the contour of such Mouldings, though circular, is not in a Plane, but is continually changing its place, from the upper Circle to the lower.

Let it be observed, that the Curves are the same, whether the Squares, in which they are described, be parallel to the Picture or oblique, in any inclination whatever.

EXAMPLE

EXAMPLE XXVII.

How to represent a large Crane, or Water Wheel, vertical.

Let ABCD (No. 1.) be the geometrical figure, and proportion of the Wheel; and S its Center, or Axis. Its Position is accidental, as the vertical Line, bg passing through the Center, indicates. Fig. 100.

Let C be the Center of the Picture, and CE its Distance, in the Horizontal Line.

Let AB be the Intersection, of the Plane of the Wheel, with the Picture; which, the Wheel is supposed close to; and let BD be the Ground Line.

Transfer the diameter of the Wheel to the Intersection, AB, at *FG*.

Draw CV perpendicular to the Ground Line, which is the Vanishing Line of the Face of the Wheel. Make C*E* equal to CE; draw *F*C and *G*C; and *FE*, cuting *G*C at *H*, and draw *HI*, parallel to AB.

FGHI represents a Square, containing the Wheel; in which, describe the Ellipsis, abcd; and within it another, representing the Rim of the Wheel.

Make B*K* equal to its determined width, and draw *KL* parallel to AB, by which describe the Curve *OPQ*, representing the cylindrical Surface of its convexity; and there may be two other Curves described, equal in width, representing the breadth of the Rims, and another, shewing the thicknefs of one.

The Levers, or Vanes, a, b, c, d, &c. (No. 1.) may be thus obtained.

Make A*G* and B*F* each equal to their width; and draw AC and BC; produce *EF*, cuting CB, at *C*, and draw *CD* parallel to AB, cuting CA, at *D*.

On *CD* describe, perspectively, the exterior Square, *CDHI*, in which describe an Ellipsis, by a Pencil Line; which, with the interior Curves, represent concentric Circles (as ABCD, abcg, No. 1) this last will limit all the extremes of the Levers. Their places are found, by drawing perpendiculars, from each, to the Ground Line; as ip, hq, &c.

Draw a Perpendicular from the Rim, at J (No. 1) cuting the Ground Line at P; from which transfer all the measures Pp, Pq, &c. (where perpendiculars from the Levers, at i, h, &c. cut it) to B*p*, B*q*, B*g*, &c. from which, draw Lines to E, cuting BC at 1, 2, 3, &c. from which draw Perpendiculars, cuting the outer Ellipsis at i, h, &c. as in the Figure, where the corresponding Characters particularize the several Levers.

Any one (as k) being obtained, a Line drawn through S, which represents the Center of the Wheel, gives its opposite, at *d*.

By the same Method the cross Timbers, &c. may be obtained.

This Method, though the same as the old Authors practised, is not repugnant to the new Principles, by Brook Taylor; and may do, accurately enough, in common Cases, and where room is wanting; but, as a contrast, I will also shew, how much more simple, masterly, and correct, they may be obtained, by means of their respective Vanishing Points; and, when they are not very remote, Intersecting Points.

AB was considered as the Intersection of one Plane of the Wheel, with the Picture. A Right Line, CV, drawn through C, the Center of the Picture, parallel to AB, is the Vanishing Line of them both, and C is its Center.

The Horizontal Line is of no use, in this operation; no regard being had to any other Vanishing Line or Intersection, whatever, but that of the Plane of the Figure.

CE, perpendicular to CV, the Distance of the Picture, is also the Distance of this Vanishing Line, which, is the Vertical Line of the Picture; and E is the true Place of the Eye, for that Plane. (Theo. 4.)

Find the Vanishing Points, D and V, of the Inclination of the Timbers, parallel to AC, and BD (No. 1) i. e. make the Angles CED, CEV, respectively equal to the known or determined Inclination of AC, and BD, with the Horizon †; cuting the Vanishing Line at D and V, the Vanishing Points of those parallel Timbers.

† Prob. 2. Sect. 3.

Or,

Plate XXII. Or, having found either, make DEV a Right Angle, it will give the other; becauſe the Timbers are at right angles with each other. (Prob. 4, Sect. 3.)

Biſect the Angle DEV by the Right Line EF; F is the Vaniſhing Point of the Timbers, E, G, &c. (No. 1.) which make half right angles with the other. In the ſame manner, i. e. by biſecting the Angles all the other Vaniſhing Points, G and H, &c. of the Levers are acquired; which, ſeeing they tend to the Center of the Wheel, may be drawn without more preparation; as GS and HS, &c. which gives the Levers, at i, *i*, and a, *k*, &c. and FS gives the Timbers at E and G.

The Timbers at I are parallel to them, and have the ſame Vaniſhing Point; the others, at O, F, and H, are at right angles with them. Make FEY a Right Angle, and they will tend to that Point, where EY would cut the Vaniſhing Line.

As the croſs Timbers, do not paſs through the center of the Wheel, they may be thus obtained; with the greateſt accuracy.

Let PJ, at the farther extreme of the Wheel (No. 1) be produced, and produce the Timbers, &c. till they cut it, at M, and N, &c. PQ may be conſidered at the Interſection of the Picture, coinciding with AB, or K *L*.

Transfer the interſecting Points M, N, &c. to AB and *KL*, at v, x, y, and z.

Draw v D and x D, giving the repreſentations of the Timbers, parallel to AC; whoſe thickneſs are obtained at v 1, x 2. y D and z D give thoſe on the other ſide of the Wheel, as they are ſeen between the hither Timbers.

From A draw a Line through the Center S, which gives the two Vanes, at n, and e, whoſe Vaniſhing Point is not in the Picture, nor neceſſary.

The Interſecting Points of the other croſs Timbers, parallel to BD, are below the Interſection, BD, which, if they were remote, could not be acquired; wherefore, from or through either of the extremes of thoſe which are obtained, draw *r* F and *s* F cuting the inner Curve of the Rim at *t* and *u*; and, through them, draw V *t*, and V *u*, which give thoſe Timbers.

The Levers, it is obvious, are all between the two Ellipſes, as their Originals are between the Circles, at No. 1.

The Points a, k, i, &c. in the hither Front, being obtained, draw a *a*, i *i*, &c. parallel to the Horizon; for the Front is, at right angles with the Picture.

The Supporters, or Stays behind them, are beſt deſcribed by the Figure; to particularize every minutia, would take above another Page to little purpoſe. The ſquare of the Axle being obtained, at S, the Sides are parallel; and the Timbers at I and O form a Square, whoſe Vaniſhing Points are F and Y.

Or, thoſe at I may be produced to the Interſection KL, cuting it at J and K; and one of the other, at Q. The reſt is obvious, on inſpection.

EXAMPLE XXVIII.

How to repreſent circular Steps, in Perſpective; with a round Pedeſtal on the Steps.

Fig. 101. Let CE be the Horizontal Vaniſhing Line; C the Center, and CE the Diſtance; and let AB be the meaſure of the firſt Step.

Deſcribe a Square ABDF (Prob. 19th) in which inſcribe the Ellipſis acbdefgh. Make c G equal to the height of a Step, as at X (No. 1) and by drawing Perpendiculars, a 1, b 2, &c. and, from *A* and *B*, Diagonals cuting them, the lower Curve may be deſcribed.

Draw AI or GH, perpendicular; and on them, from A or c, ſet off all the meaſures of as many Steps as are required, *viz.* *A*A, AC, or Gc, ci, equal (as in the Profile, for two Steps) and from them draw to E, or C, as in the Figure.

Find the Square of the next Step JKLM (Ex. 11.) in which deſcribe the Ellipſis *abcd*, &c. as before; and the lower Curve, *ghikl*, may be obtained, likewiſe, in the ſame manner. And thus, as many Steps may be repreſented as are required.

N.B. AB is not neceſſarily the Interſection of the Picture, but parallel to the Interſection; and AB the known or determined meaſure of the firſt Step, in that place.

3 To

To represent a circular Pedestal, on the Steps.

At No. 1 is the Profile of the Pedestal; from which, transfer all the measures of the Plinth, Mouldings, &c. to the vertical Intersection, GH or AI, or both; the one considered as the section of a vertical Plane perpendicular to the Picture, thro' the Axis of the Pedestal, and the other a diagonal Section. Fig. 101.

Make *CD* equal to the Plinth of the Base; *CI* to the height of the Cornice, and at 1, 2, 3, &c. set off the proportions of the Mouldings.

The square of the Plinth being obtained, at NO, and an equal one at PQ, in both which describe Ellipses; as, for the Steps below.

The other Curves must all be obtained after the same manner, by inscribing Ellipses in Squares; as pq is a Side of the square of the Facia, rs of the bottom line of the Cornice, and tu, of the upper Line of the Base Moulding.

It is obvious, that, as the plane of the Curve falls nearer to the Vanishing Line, the Curve is more excentric, i. e. flatter; as in right lined Objects, every section, parallel to the Base, becomes more obtuse and acute angled, the Figure being more contracted, as it approaches nearer to its Vanishing Line; in which it is wholly lost to sight, seeing that, the Eye is in the Plane of the Figure, and consequently, its whole representation is in the Vanishing Line. Yet, there is no possibility of describing them in another Plane either above or below, so as to be of any service, in respect of the Curve, as every Curve, the farther it is removed from the Vanishing Line of the Plane it is in, becomes more curved, i. e. it is more convex; or, as workmen phrase it, quicker, or more acute; which implies, that it falls off from a Tangent, more suddenly.

From what has been described above, in delineating round Objects, it appears nothing more than drawing a square one, of the same dimensions, and inscribing an Ellipsis in every representation of a Square.

It must be obvious to every Person of discernment, that, whereever the Eye is situated, in respect of a convex Object, as a Vase, &c. (Fig. 102) the Vanishing Line of the Plane of the Circles (CE) cuting the Object, the Curves abc, def, &c. or *abc*, *def*, &c. on either side of the Vanishing Line, are always concave towards it; and consequently, if a concave Object be represented (as the Abacus, of the Ionic or Corinthian Capital, Ex. 30 and 32) the Curves will be convex, towards the Vanishing Line. It is also manifest, that if the Curves, that is, if the Planes they are in, be equally distant from a Plane passing through the Eye, parallel to them; being equal and similar Curves, of any kind, and perpendicularly one opposite the other, their Representations will also be equal and similar. Fig. 102.

How is it possible then, for any Person, who has any pretensions to the Art of delineating, to run into such gross absurdities as are exhibited in No. 2. I have seen a Vase so represented, on the Pannel of a Coach Door, which was very well painted, and, in other respects passable.

The Plinth, AB, was parallel, and the Side, BCD, was seen; but, instead of the Lines, in that Face, tending to a Vanishing Point, they inclined to each other towards the Eye; and consequently, gave the Plane, ABC, the appearance of inclining, or ascending. The Mouldings, at E, where curved contrary to the Base, which plainly indicates the Horizontal Line, HI, to pass between them; and, the Curves, being regular, also evinces the Eye to be at E, and consequently the Face, BCD, of the Plinth, could not be seen. No. 2.

The Lines at FG, the greatest swell of the Vase, were Right Lines, which also evinces the Eye to be on a level with them; whilst the nulled, or fluted part, abc, was a parabolic Curve, rising as in the Figure; and the Top, at K, gently curved.

Is it not surprizing, that any Artist should be so regardless of his Reputation, as to suffer such a medley of inconsistencies to go out of his hands; and to palliate it by saying, in such things, it is not worth the while to stand considering about it. I grant it would not be worth the while, to delineate the whole by the Rules of

Plate XXII. Perſpective; but ſurely, the Lines in the Plinth are as eaſily made to incline the right way, as wrong; and if, by curving the horizontal Lines of the Vaſe, it was intended to give an appearance of roundneſs; would not that be better effected by curving them properly? it would take no more time; and, if the Delineator was not inclined to think much about it, I would adviſe all ſuch, to place an Object before them, and their Eye in the Point of View, as they intend the Object to appear; they will then aſſuredly ſee, how unpardonable ſuch miſrepreſentations are, becauſe they are as readily, without loſs of time, done right as wrong.

I have ſeen full as groſs abſurdities on the Stage; which ought to be regulated by the ſtricteſt Rules. In Theatres, Perſpective might be diſplayed to great advantage; and certainly, its Rules ought not to be diſpenſed with, on any account whatever. I do not mean in every minutia, but in the whole Deſign; for, if liberties be taken, in order to appear better in one Point of View, it will appear worſe in another; particularly, in Scenes repreſenting a grand Palace, &c. or other regular piece of Architecture, there is no poſſibility of departing from the Rules; in detached Planes, as in Scenery, they cannot repreſent one entire Building, in any Point of View, without adhering ſtrictly to Perſpective.

To the beſt of my remembrance, in that Maſter-piece, by Servandoni, in the Pandemonium (though I think, the grotesque Stile, of that Deſign, is not altogether conſonant to the grand deſcription of it, in Milton's Paradiſe loſt) the Eye, in that Scene, is on a level with the Baſes; and conſequently, the Ruſtics, on the Columns, appear curved. It is a long time ſince I ſaw it; but I believe, the firſt, from the Baſe, is as much curved as the upper one. The whole has a grand and ſtriking effect, chiefly occaſioned by the magnificent diſplay of Lights and Colours; which being left out, I am perſuaded that this Deſign would loſe much of the applauſe it has obtained.

EXAMPLE XXIX.

How to repreſent an Ionic Capital, perſpectively, having one Face parallel to the Picture.

This Example, and more particularly the 31ſt, of the Corinthian Capital, is a matter of real difficulty. To repreſent ſuch Objects as the Volutes (and other decorative parts, of which the latter is compoſed), will indeed baffle all Rules. I ſhall nevertheleſs attempt to lay down ſome, which will give their true places, with certainty; and, if the Practitioner will carefully obſerve how ſuch Objects appear, according to the poſition in which they are ſeen, and intended to be delineated, he will not be much at a loſs; if he has any hand at Ornament, in Foliage, &c. and carefully adheres to the Rules I ſhall preſcribe.

Fig. 103. At Fig. 103, No. 1, is the true geometrical Plan and Elevation of the Ionic Capital; not according to the Antique which is not ſo difficult, ſeeing that, the Volutes or Scrolls are in one Plane. On the left (at X) is the Plan of the Volute ſhewing its poſition in reſpect of the Abacus and Ovolo, &c. On the right (at Y) is the Elevation; which appears elliptical, on account of its Poſition, which is known from the Plan.

If ab be the Line of its direction (which is ſomewhat curved) to the front, AB; it is obvious, that it makes a conſiderable Angle with it; to which, perpendiculars being drawn give ec, its apparent width in front; which meaſures, being tranſpoſed to the other ſide, at d and g, and perpendiculars drawn, give its apparent geometrical Front; of which Z is the true.

Having deſcribed the geometrical Plan and Elevation, I now proceed to ſhew how it may be delineated, perſpectively.

Let AB be conſidered as the Interſection of the Picture, with the Plane of the top of the Abacus. Let VL be the Horizontal vaniſhing Line, and C the Center of the Picture. The Diſtance is greater than can be applied on the Horizontal Line, take C*E* half the Diſtance, and deſcribe the Square ABDG (Prob. 19) which contains the whole Abacus.

The Curve e f g, which is the fixth part of the Circumference of a Circle, may be eafily obtained; as thus.

From c, F, and d, any Points at pleafure, in AB, draw Perpendiculars, c 1, d 2, &c. to the Curve; and draw cC, F C, and dC, which reprefent thofe Lines, indefinite. Make c *c*, F *f*, &c. reprefent the finite Parts (Prob. 15) through the Points e, *c*, *f*, *d*, and g, defcribe a Curve, which will be a portion of an Ellipfis, and reprefents the Curve e f g.

The returning Side, BD, is obtained by means of Ordinates, at 1, 2, 3, after the fame manner, which are parallel on that Side. The interior Curves may be projected by the fame means, and the Plans *X*, *Y*, and *Z* of the Scrolls, as any other geometrical Figure (in Sect. 5) from which, their Reprefentations are projected, as below; the returning Angles at A and B, being perpendicular and parallel to the Diagonal A C, tend to the Diftance Points.

This preparation is fufficient for an extra Plan; I fhall next fhew how far it is ufeful in finifhing the Reprefentation below. This Plan may be fuppofed to be the top of the Abacus, as a Plane fimply; but I fhall fuppofe it to be above, as in Example 12th, and that the Reprefentation is below it.

Let *AB*, No. 3, be now confidered as the Interfection of the top, inftead of AB; to which Line transfer all the meafures from A B; and form the out Line of the Plane *ABD*, as before; by means of the fame Vanifhing Points.

To compleat the Abacus; bifect *AB*, at *F*, and draw a Perpendicular, *FG*.

From *F*, fet off all the meafures of the heights of the Mouldings, and draw Lines from them to C; fuppofing *FG* a vertical Section of a Plane through the middle of the Capital.

The Center (S) being obtained, draw a perpendicular, from S, cuting thofe Lines; through which, draw the parallel Lines *ce*, *df*, &c. till they cut Perpendiculars from the correfponding Mouldings, in the Plan above. Or, for greater accuracy, make *BC* equal to F f, the receding of the Curve from the Picture; where, make a true Profile of the Moulding, of the Abacus; from which, draw Lines to the Center of the Picture, cuting *ce*, *df*, &c. from the Axis, at *e* and *f*.

Having obtained the returning Moulding, at the Corner *B*, by means of the perfpective Plan, above, and a perfpective Section, at *FG* (obferving, that it is a mitre Angle, touching the Picture, and confequently, recedes more than the true Moulding, as may be feen by the geometrical Plan, No. 1, at A or B) from which, Lines tending to the Point of Diftance, on the right hand, cuting Perpendiculars from the Plan, above, give the other Mitre, *gi*.

From *g*, *h*, and *i*, draw Lines to the Center (C) cuting Perpendiculars from the Angle D, in the perfpective Plan; by which means, the feveral Angles at *D* are obtained; and the curve Lines may be drawn by a careful hand, as in the Figure, nearly parallel amongft themfelves; otherwife, if great accuracy be required, each Curve muft be got, feverally, in different Planes.

The Abacus being compleated, proceed to the Scrolls or Volutes.

Having carefully obtained the perfpective Plans (*X*, *Y*, and *Z*) of the Scrolls, which are regular Trapezia (as at X, No. 1) not regarding the hollowing.

In this Diagram, it is too near the Vanifhing Line; but may, for greater exactnefs, be at a greater Diftance from it (as in Ex. 13.)

Draw a b parallel to *AB*, the Interfection of a Plane paffing through the middle of the Scrolls (as at *ab*, No. 1) its Diftance from *AB*, is equal to the diftance of *ab* from the top of the Capital, geometrical.

By means of the Interfection ab, the reprefentations of thofe Trapezia, *x*, *y*, and *z*, are eafily obtained; transfering the geometrical meafures to that Interfection, firft; Perpendiculars, from No. 2, will cut Lines tending to C, the Center of the Picture, and give all their Angles, fufficiently correct; but, if greater accuracy be required, their Vanifhing and Interfecting Points may be had, as in other Figures, which are in the Horizontal Line; but, on account of their Inclination to the Picture, they are beyond its limits.

Now,

Plate XXII. Now, the ufe of thefe Figures is very obvious; by which means, the feveral apparent Faces, with their feveral divifions, are determined, paffing through the Eye of the Volute, &c. But, if an extra Plan be drawn, as above, Perpendiculars from *X*, *Y*, and *Z*, give the fame, in refpect of their widths.

Hence may be feen the unavoidable diftortion of the Volute on the left; which, on account of its oblique pofition to the Picture, and the Eye, is dragged out beyond its geometrical proportion; and confequently, cannot pleafe the Eye but in the true Point of View. It is true, the Diftance it too fhort; neverthelefs, every oblique parallel Reprefentation, of fuch Objects, muft be more or lefs diftorted, according to the diftance, and oblique fituation of the Eye.

Having got their dimenfions, of height, by means of a perpendicular, h i, from the middle of their geometrical Plan (X, No. 1) and the place where they fall on the Ovolo (which, muft be drawn as in the Doric Capital, without its Abacus, Ex. 26) the reft muft be delineated by a fteady hand, guided by a nice Eye; for, without great nicety in both, it is not poffible, by the Rules of Perfpective, to project fuch Objects; feeing that, the Curve is not only continually varying from the Center to its utmoft extent, but it is alfo continually growing forward, from the utmoft extreme to its Center, and is not defcribed on the fame Plane, or other Surface, which circumftance renders it utterly impoffible to be defcribed perfpectively, by rule; neverthelefs, by the Rules prefcribed and judgment in drawing, it may be projected fo as to pleafe the moft critical Eye.

Below this outline is a finifhed Capital, divefted of the neceffary Lines for projecting it. It is fomewhat farther removed from the Horizontal vanifhing Line, but, in other refpects, it is the fame; excepting the decorative parts, of foliage, &c. which, according to the pofition in which the Capital is viewed, will differ in Figure; they were omitted, above, for the fake of continuing the Lines of the Abacus, which they hide, in this. No Rule can poffibly be of any ufe in delineating fuch Ornaments; a real Object, before the Eye, is the beft and only means to effect it.

EXAMPLE XXX.

Is an Ionic Capital, according to the Antients, which is inclined to the Picture.

This Capital is delineated with much more eafe than the other, for feveral reafons; becaufe its Abacus is a Square, with only a Cima reverfa; and, the Volutes are both in one Plane, which is vertical.

Fig. 104. The Abacus, is fuppofed to cut the Picture, at A, through which, draw AB, parallel to the Horizon; V is the Vanifhing Point of the front Lines, its diftance is VD; the other Vanifhing Point is out of the Picture.

Make AB equal to a Side of the fquare of the Abacus, and project the Square *ABC*, perfpectively (Prob. 21) and finifh the Abacus, as in Ex. 13. of the Pedeftal.

The hither Scroll projects through the Picture; becaufe the corner of the Abacus, at A, cuts the Picture, and the Scroll projects beyond it, in this Capital.

The Interfection of the Plane of the Volutes may be determined, truly, from a geometrical Plan; as at X, an Angle of the Abacus, inclined to *AB* as the Object is fuppofed inclined to the Picture.

The whole Moulding projects before the plane of the Scrolls; therefore, produce *cb* to *a*, cuting *AB*. Make Aa equal *Aa*; draw ad perpendicular, and produce *ab*, cuting ad at c; all that part of the Scroll, on the right hand of ad, projects through the Picture.

Make cd equal to the height of the Scrolls, or a d of the Capital, and draw dV.

Defcribe a Rectangle, inclofing the Scroll, at each extreme, as cdef, *cdef*; and, within thefe, others may be defcribed, as in the Figure (dividing cd geometrically, at 1, 2, 3, 4, and drawing Lines to V; alfo, dividing b f, perfpectively, at 1, 2, 3, and drawing perpendiculars), or, feveral points may be determined, by means of which, the Scrolls may be very accurately delineated, perfpectively, but muft own it is a laborious procefs, and may be drawn accurately enough, within the large Rectangles.

The

The other Front, or rather the End of this Capital, is different, and is not easily delineated, truly. Having obtained the outline of the opposite Scroll; at *g h i*, bisect the bottom Line of the Abacus, at *k*; where, describe the representation of a Semicircle, *kl*, in a Plane parallel to the Front, but different in dimensions to the Scrolls. The proportion, and figure of it, may be had in Books of Architecture.

This Face represents a kind of Ballufter, spreading, at its extremes, to the width of the Scroll; but the Axis of it does not pass through their Centers.

The Ovolo is the same in this as in the modern Capital; save only, that it is not seen on the sides, at all, being wholly hid by the Ballufters.

EXAMPLE XXXI.

To represent a Corinthian Capital inclined to the Picture.

This Object is supposed to be direct before the Eye; and consequently, the Axis of the Capital passes through the Center of the Picture, at C. Fig. 105.

CE (perpendicular) is the Distance, VL the Horizontal Line, and V the Vanishing Point of one side of the Capital, or rather of its Position; for, the Object not being right lined, save the short returning Angles, it has indeed no Vanishing Point; yet they are necessary, in the Delineation.

Let a be the Point in which a Square, inclosing the Abacus, touches the Picture; through which, the Intersection of the top may be drawn; but it will be more eligible to take another, above, at BD, parallel to the Horizon.

The Abacus of this Capital is the same, in proportion, and every other respect, as the modern Ionic; therefore, the measures may be taken from Fig. 103, No. 1, being delineated by the same Scale; and excepting its supposed position, being different, the delineation of it would be the same.

Take A perpendicularly over a, and draw AV; and AY to the Point in which EY would cut the Horizontal Line (Prob. 13) indefinite.

Bisect the Angle VEY, by the Line EL, L is the Vanishing Point of a Diagonal.

Find F, the distance of the Vanishing Point Y (Prob. 12) and, because the distance of V cannot be in the Picture, take VG three fourths of VE (or any other part, at discretion.)

Take AB three fourths of AB (No. 1) which bisect at *c*, and take all the other measures, at *a*, *b*, &c on that side, each three fourths of the real measures on AB (No. 1) but, on the other, take *d*, *e*, *f*, &c. the full measures.

From all which Points, draw Lines to F and G, respectively, as in the Figure; cuting AV, and AY, at *g*, *h*, *i*, &c. from which, draw Lines to both Vanishing Points, V and Y, indefinite.

Take A 1, and A 2, equal c 1, Ff, &c. (No. 1) and draw 1 F, 2 F, cuting AY; from which Points draw Lines to V, cuting the Lines from *g*, *h*, &c. to Y, at *n*, *o*, *p*; and from *j*, where 1 V, 2 V, cut the Diagonal AL, draw Lines to Y, cuting *k* V, &c at *r*, *s*, *t*; through which Points, on both sides, draw the Curves of the Abacus.

At the Corner A, describe an Equilateral Triangle (x) perspectively, and the same at each Corner, *B* and *C*; giving the mitre Angles of the Moulding, as at A and B; (No. 1.) which, in this case, are not bisected, having a greater projecture at the Corners, than in front.

This operation done, draw a V, and a Y indefinite; to which, draw Perpendiculars from *g*, *h*, *i*, *k*, &c. also, from *n*, *o*, *p*, &c. and draw g Y, h Y, &c. cuting them at n, o, p, &c. through which Points, describe the Curves b n o p q, and a r s t u.

In respect of these Curves, the operation above is not absolutely necessary.

After the same manner, the Abacus may be compleated; *viz.* by projecting the curves of the Fillet, in the middle, and at the bottom, in their respective Planes.

The places of the Leaves may be obtained by describing an Ellipsis in the representation of a Square, inclined to the Picture, as the Abacus.

 This

Plate XXII. This may alſo be done above, at v, u, x, y, z. The Diagonals and the two Diameters, whoſe Originals are perpendicular to the Fronts, give the apparent width of each Leaf, in the firſt row, allowing a little ſpace between them, and they alſo give the middle of the upper Leaves; between each, is a kind of Balluſter, from which the Caulicola and Scrolls ſpring.

The true places and middle of the Heads may be obtained, by deſcribing repreſentations of larger Circles, concentric with the other. At Z is a Leaf, in front. Deſcribe two Ellipſes (cef and ghi) giving the greateſt projecture of the Leaves, in their true places; their heights above the Aſtragal (*FG*) are taken, from the Profiles (at X) to the greateſt projecture of the Heads (at *a* and *b*) which projectures (*ac*, and *bd*) reſpectively, are added to the diameter of the Column at the Aſtragal, for the diameters of thoſe Circles, repreſented by cf and gi.

Theſe Ellipſes being obtained, in their true places, inſcribed in Squares (as *HI*) inclined to the Picture as the Abacus, for the upper one; the Diagonals and parallel Diameters, give the true middle of each head of a Leaf, at c, d, e, &c. Thoſe of the lower row fall between them; as deſcribed, in the Plan above.

The Volutes, at the corners of the Abacus, are, except in dimenſions, the ſame as the Ionic, and muſt be obtained the ſame, by perſpective Plans, at *x*, *y*, *z*.

How they fall into the Caulicola, with the other decorative parts of the Capital, the Figure only can deſcribe; for, I fairly own that it is not in words to deſcribe it, ſo as to be of any ſervice in delineating. Being well acquainted with the ſeveral parts of the Capital, the Rules I have given are ſufficient; the reſt is beſt deſcribed by a careful and attentive peruſal of the Figure.

Thoſe who are not verſed in it, would do well to draw from a real Capital, firſt, in all poſitions; after which, they will be able to delineate it by Rule.

SECTION IX.

Shewing the application of the whole to entire BUILDINGS, and regular pieces of ARCHITECTURE.

IN this Section, I have applied the whole of what has been already done to compleat Objects, and particularly to Architecture; as being, of all other, the fiteſt Subject, and gives the greateſt luſtre to Perſpective; on account of the regular diſpoſition, and arrangement of the ſeveral parts of a Building, and being compoſed, chiefly, of plane Surfaces, which generate Right Lines.

It is, to me, ſurprizing, that Artiſts, in general, have no better notions of Perſpective; if they would give themſelves time to think about it, ſeriouſly, they could not commit ſuch palpable miſtakes and groſs abſurdities; even ſuppoſe they were not acquainted with the Rules, having a true Idea of the meaning and intent of Perſpective; which cannot but be obtained, from the Apparatus, to this Work.

Whatever we are about to delineate, it is ſuppoſed that the Object, itſelf, is on the other ſide of the Picture, generally (it may be on this ſide.) Is it not, then, rational to conſider, and thence to conclude, that the Picture we are delineating ſhould be directly between the Eye and the Object; or what reaſon can poſſibly be aſſigned that it ſhould not? and yet it is, almoſt generally, the caſe. Some, not conſidering it properly, would tell us, that it is in order to ſee the End or Side of an Object, as well as the Front; as if that could not be effected otherwiſe; true, it cannot, on their Hypotheſis, that one Face muſt neceſſarily be parallel to the Picture.

I ſhall illuſtrate the whole of this matter in few words.

Figure 106 exhibits the Plan and Elevation of a plain Building, geometrical, which is to be delineated; nearly the ſame as the Object, in the Apparatus; in the ſame Poſition, and at the ſame Diſtance, nearly, by a Scale of the proportion.

In the firft place, I fhall fhew how to determine the true Pofition of the Picture, the Station being previoufly determined. Plate XXIII.

Let S be the Station fixed on, from which it is intended to delineate the Building; X is the geometrical Elevation of the Front, Z is the Plan, or Seat of the Building on the Ground; in the Pofition, and at the Diftance, the Building is to be reprefented, as feen from the Station S. Fig. 106.

The Station, and confequently the Point of View being determined, it is evident, that the Building cannot vary in its appearance at that Station; but it is neverthelefs manifeft, that there may be a great variety of Pictures, or Reprefentations of the Object; feeing that, every different pofition of the Picture, or fection of the Pyramid of Rays (forming a folid Angle, under which the Object is fuppofed to be feen) will produce a different Picture. All which, Reprefentations, will affect the Eye alike, in the true Point of View; nor is it poffible it fhould be otherwife, feeing that, when the Eye is in the true Point of View, every Line, on the Picture, is feen under the fame plane Angle, and confequently, the whole Reprefentation under the fame folid Angle, as the Original.

Hence, it is eafy to account for the many diftorted and prepofterous Reprefentations which are frequently to be feen; and which, is moftly owing to the abfurd pofition of the Picture, in refpect of the Object and of the Eye; or frequently to the diftance of the Object, itfelf; a circumftance which ought to be particularly confidered; for, if it be too near, it is impoffible, in fuch cafe, to produce an agreeable Picture.

Then, fince every different fection of the Pyramid of Rays produces a different Picture (for all parallel Sections produce fimilar Reprefentations †) certainly, that Section which is perpendicular to the Axe of the Eye, or the Station Line, SL, will be the moft natural and agreeable Reprefentation; feeing that, the parts of the Reprefentation will differ but very little (if the Optic Angle ASC be not large) from the true Appearance, i. e. from a Section made by the furface of a Sphere, of which S is the Center.

† 1. 8. El.

Confequently, then, if the Station be fixed, from which it is determined to delineate an Object, the Pofition of the Picture is alfo determined, as follows.

ABC is the place of the Object, its Seat on the Ground Plane; and S is the Station determined on. Draw the Right Lines AS and CS, which determine the Optic Angle (ASC) of the Object, AC; for, S may as well be the Eye, and AS, CS Vifual Rays, the Eye being perpendicularly over S.

Bifect the Angle ASC, by the Right Line SL; SL is the Station Line for that Object; to which, the Picture muft (if placed properly) be perpendicular; all fuch Sections are fimilar, and are the moft juft Reprefentations.

VY, perpendicular to SL, is the true Pofition for a Picture of that Object, delineated from the Station S; and if Right Lines are drawn, from S, to the feveral parts of the Plan, as a, b, c, &c. their fections with the Picture, *a*, *b*, *c*, &c. determine the apparent width of each part on the Picture, perhaps with greater accuracy than by any other means.

Here, the Picture is fuppofed clofe to the neareft Angle of the Object, at B; and confequently, the perfpective fection of the Rays, by VY, is the largeft that can be made on this fide the Object. Now, if an Ark (*ABC*) be drawn, on the Center S (Radius SB*) it is obvious that the fection of the Rays by that Ark (or a Sphere of that Radius) differs but little from the fection by the Plane, on VY; becaufe the Optic Angle, ASC, is but fmall, and therefore the extreme Rays, AS and CS, cut the Plane not very oblique; which, being increafed, occafions Diftortion.

SV, parallel to AB, and SY, to BC, determine the Vanifhing Points of horizontal Lines, in the Building. 1, 2, 3, &c. may be confidered as Interfecting Points of feveral Lines in the Object.

* BS perpendicular to the Picture does not neceffarily fall on the Angle B (in the Apparatus it is fomewhat on the right hand) for, if the Station was ever fo little on either fide, it would fall differently, as muft be obvious, from the method given of determining it.

Having

Plate XXIII. Having thus determined the true pofition of the Picture from the Station S, anfwering to the Picture, MNOP, in the Apparatus; which is directly between the Eye and the Object; I fhall, next, fhew the great abfurdity of placing it otherwife.

Let every thing remain in the fame Pofition; S is the determined Station for the fame Object; and confequently it muft appear the fame, whatever Pofition the Picture is fuppofed to have, in refpect of the Object. But let it be obferved, again, that the Reprefentation may differ very widely; as will be fhewn.

It is ufual with all (who know a little of common Perfpective only) to make one Face of the Object parallel to the Picture; in which cafe (the Object being right angled) the Center of the Picture is, neceffarily, the Vanifhing Point of horizontal Lines in the other Side. Confequently, if two Faces of the Object are feen, the Picture muft neceffarily be obliquely fituated, in refpect of the Object and the Eye.

Now, can any Perfon conceive it rational, or eligible, to place the Picture in the Pofition ED, parallel to the End of the Building, to be feen from the Station S; and yet, this is the very fuppofition, in the Reprefentation exhibited by Fig. 107; whofe Center is C, and Diftance CE (equal twice SE, 106) which Pofition anfwers to the Picture MNO*P*, in the Apparatus.

How egregioufly abfurd is the Idea of viewing a Picture in that Pofition; and yet, in any other Point of View it cannot appear like the Object reprefented. The diftortion of the Front of the Building is obvious, but much more fo, is the Face X, of the Bow Window; whilft the other (Z) is geometrical, as well as the end of the Building.

Fig. 107. The Pofition being determined, and the Center (C) fixed; and alfo, the Interfection BG, of the front Plane (its diftance from the Center, equal twice EF, 106) the Reprefentation is delineated by Prob. 19 (as by Ex. 1 and 2) in which cafe, C, the Center of the Picture, is the only Vanifhing Point.

Now, thofe Artifts, who make a kind of neceffity of delineating their Objects after this method, cannot, I conceive, give a reafon, why, in a general View, fuch Objects muft neceffarily be fo fituated to the Picture, as to have fome Face parallel to it; or, why the Picture fhould not be placed parallel to the Front (AB) if it muft be fo to either; excepting that, they rather choofe the one to be parallel than the other; for there is as much propriety in one, as in the other, but the Reprefentation will be much more diftorted; becaufe the Diftance (SG) of the Picture ftanding on AG, is lefs than SE in the former; and, the much greater length of the Front (AB) occafions a greater obliquity of the Rays, with that Picture. For, SG being the Direct Radial of that Picture, ASG is but half the Optic Angle under which it is feen; but it is manifeft, that the Eye cannot take in as much on the right-hand, in which cafe, the whole Angle will be very obtufe.

CASE THE SECOND.

Fig. 108. Fig. 108 exhibits a Reprefentation of the fame Object and from the fame Station, in that pofition of the Picture; in which, the Front is geometrical; but the projectures of the Cornice, Steps, &c. are dragged out prepofteroufly; and the Face X, of the Bow Window, is worfe diftorted than before.

C is the Center of the Picture, and CE its Diftance, equal to twice SG (106.) The whole of the Object ought by no means to exceed the Point E, from the Center; whereas, it extends above twice CE, which is prepofterous.

KL is the Vertical Line; and the Angles, CEK, CEL, being made each equal to the Inclination of the Roof, determine the Vanifhing Points of the Lines FG, and F*D* in the Roof; K is alfo the Vanifhing Point of HI parallel to FG; the reft is determined as the former, in which, thofe Lines are parallel to the Picture.

In both thefe Pictures, the horizontal Lines, in the Face, X, of the Bow Window, vanifh in the Point of Diftance, in the Horizontal Line; equal CE, on the other Side of C, becaufe the Figure of it is a regular Octagon. (See Prob. 25.)

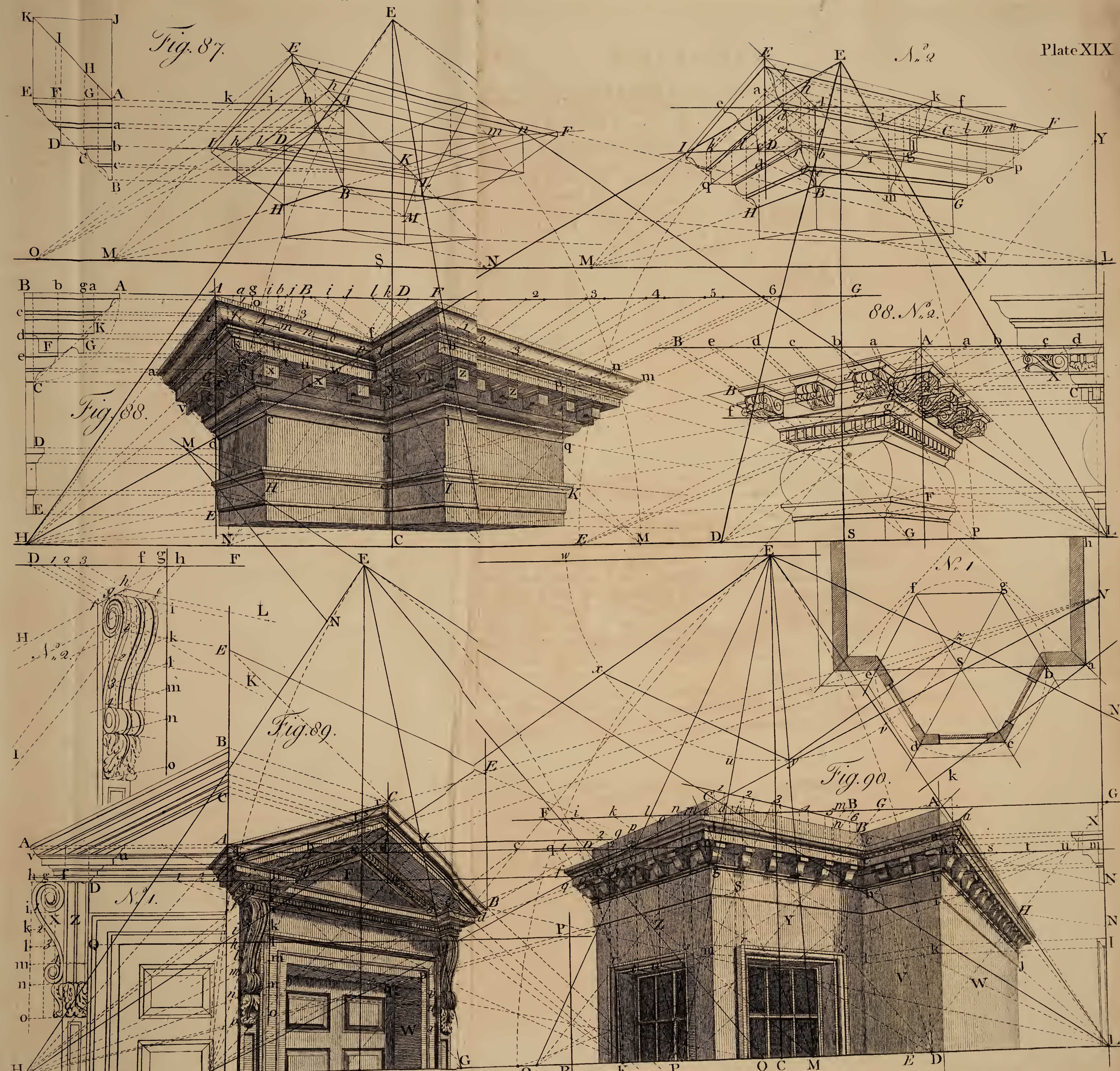

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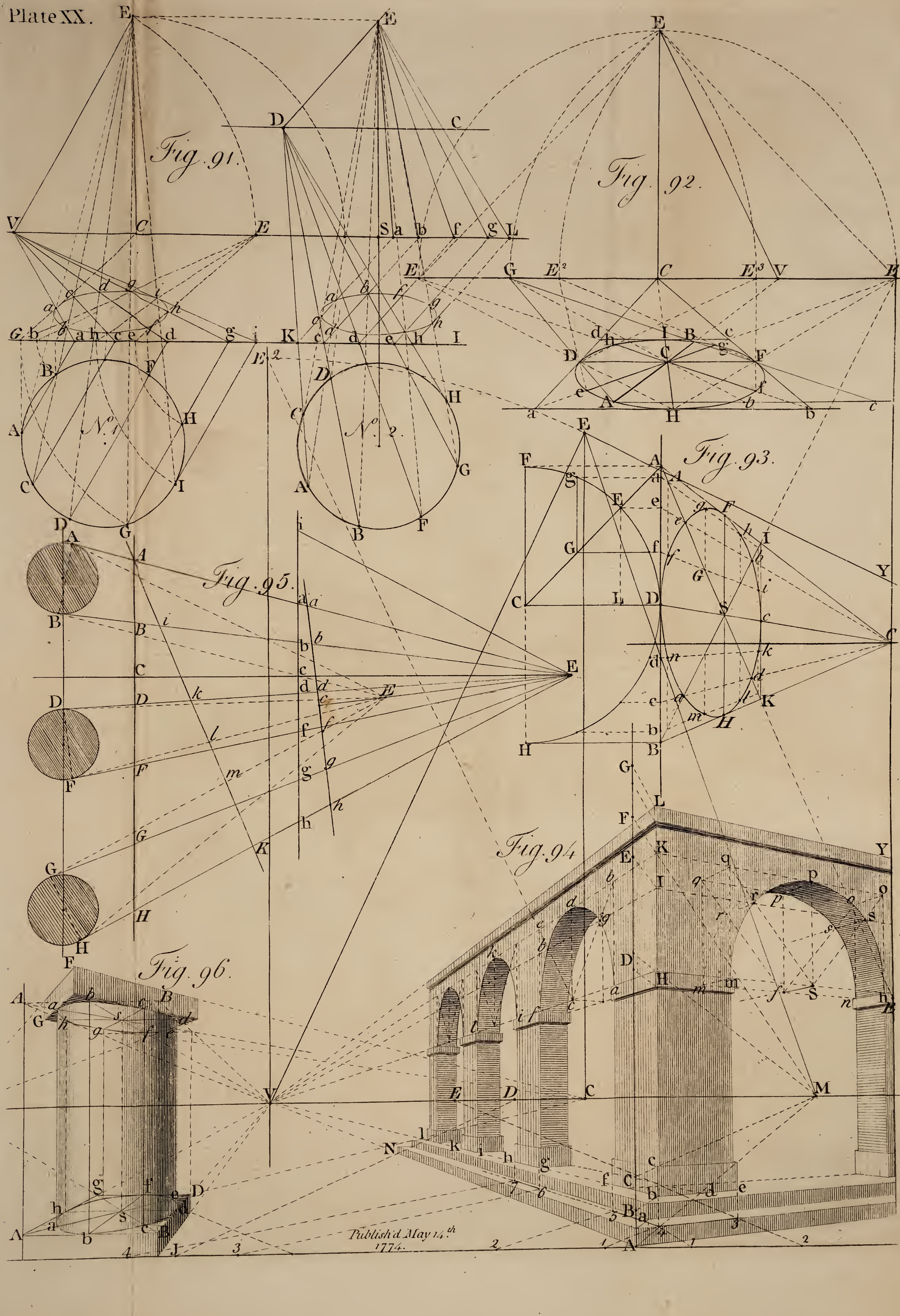
Plate XX.



Publish'd May 14.th 1774.

Plate XXI.

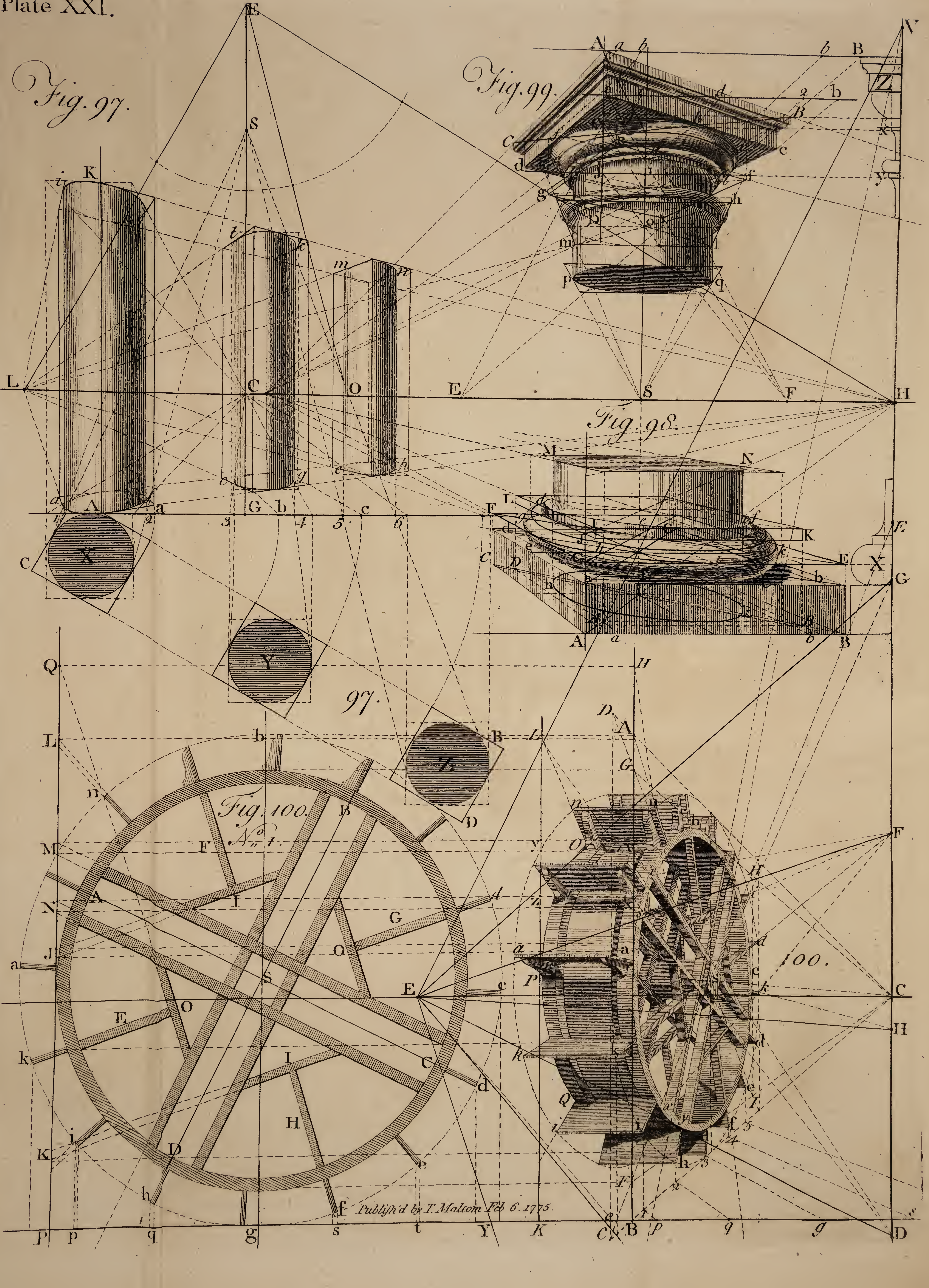
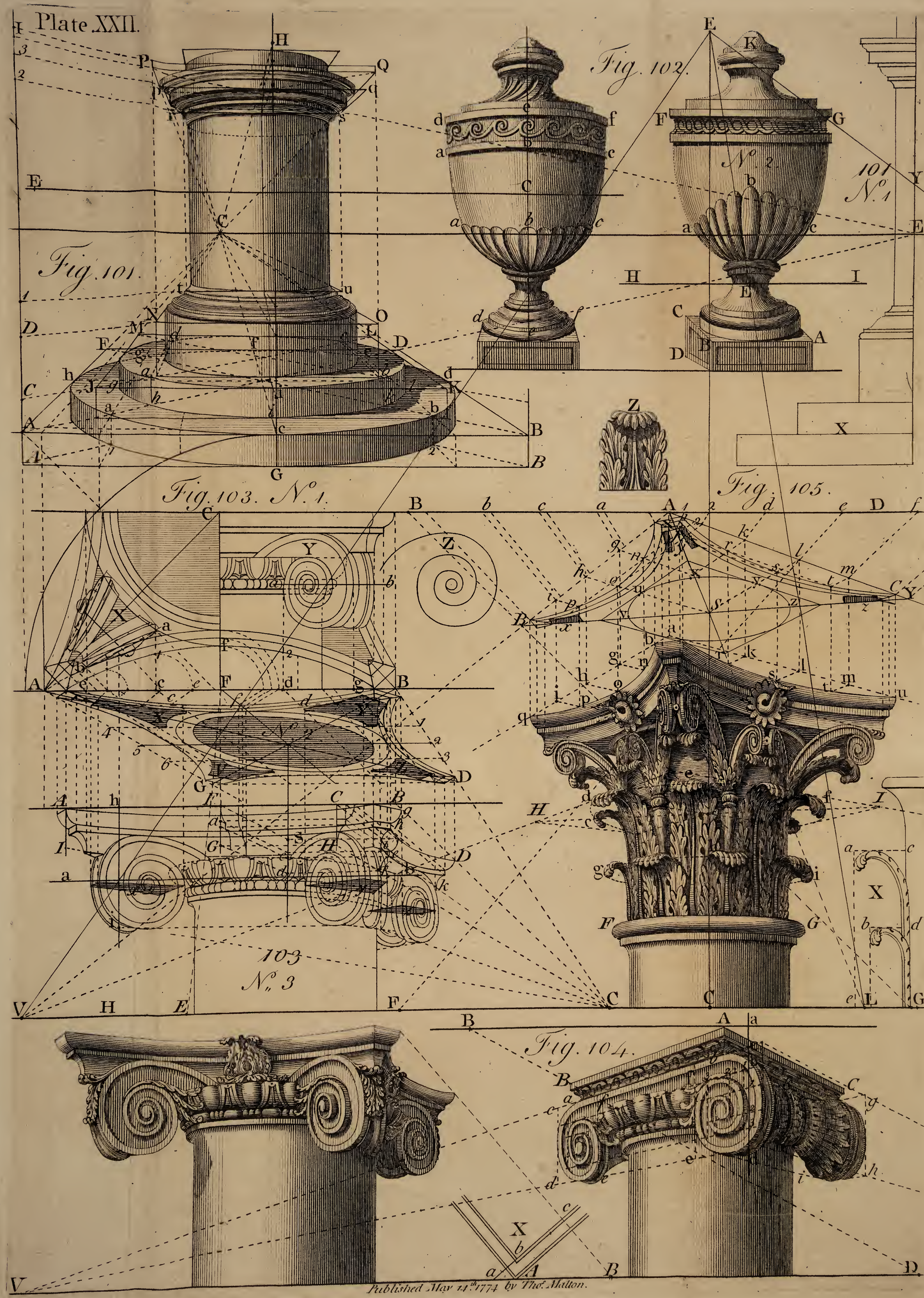


Fig. 97.
Fig. 99.
Fig. 98.
97.
Fig. 100.
No. 1.
100.
Publish'd by T. Malton Feb 6. 1775.



Published May 14th 1774 by Thos. Malton.

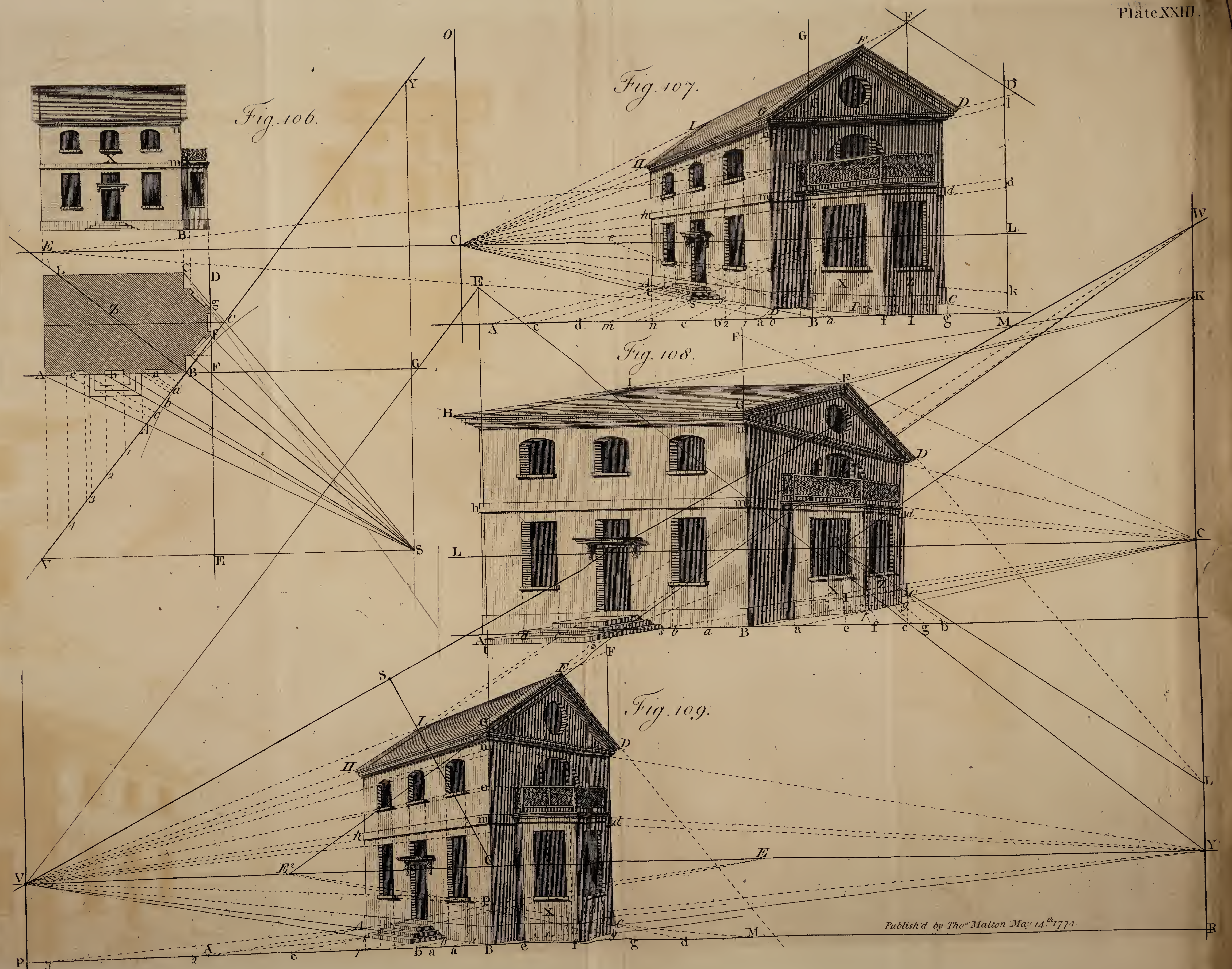


Fig. 106.
Fig. 107.
Fig. 108.
Fig. 109.
Publish'd by Thos. Malton May 14th 1774.

CASE THE THIRD. EXAMPLE XXXII.

Fig. 109 exhibits a true and natural Reprefentation of the fame Object from the fame Station, in the moft judicious pofition of the Picture; and, in order to fhew the affinity between this and the Picture MNOP, in the Apparatus, I have made ufe of the fame Letters for Reference, where they can be properly applied. Fig. 109.

VY is the Horizontal vanifhing Line, and C the Center of the Picture.

Draw CE perpendicular to VY, and equal to the Diftance (equal twice SB, 106) and make the Angles CEV, CEY equal, refpectively, to the Angles BSV, BSY (106) or, make CV equal twice BV, and CY twice BY; V and Y are the Vanifhing Points of horizontal Lines in the Front and End of the Building †. † Th. 10. and 11.

Then, becaufe the Angle B touches the Picture, BG is the Interfection of both Planes (in the Apparatus, the Object being at fome Diftance from the Picture, each Plane has a feparate Interfection, *BG* and *BG*) on which all the meafures, of the heights of the Windows, &c. are fet off, geometrically (as in the Elevation, X, 106; each being double) as Bm, mn, &c. from which, draw Lines to both Vanifhing Points, V and Y, indefinite.

Make V*E* equal to VE, and Y*E* 2 equal YE. From B, fet off Ba, ab, &c. as in the Plan (106) and draw a*E*, b*E*, &c. cuting BV, in the Points *a*, *b*, &c. the perfpective proportions of the Windows, &c. from which, draw Perpendiculars, cuting Lines drawn from the feveral heights, on BG to V, in the reprefentations of the Windows, &c. AB the length of the Front, gives B*A* its perfpective length.

Or, if B 1, 2, 3, &c. be fet off, as from B to V (106) from which, Lines drawn to Y determine certain Points; on BV, or aV.

By which means, they are projected in the Apparatus, *viz.* by the Interfecting Points; and the Steps by the fame, which may be finifhed by Example the 13th. Or, the place of each may be taken, from VY, 106, where the Lines, AS, &c. cut the Picture; which for the Bow Window, will be the readieft; obferving to take them double, becaufe it is but half the proportion.

Otherwife; the Bow Window may be projected, by Example 5th, obferving the difference in Figure; and, being too near the Vanifhing Line, another Interfection may be taken, at pleafure, at a fufficient Diftance (Ex. 12.) as it has been exemplified in various Cafes.

The Cornice, being plain, is managed by Example 18th, and the Pediment, by the 20th. The Vanifhing Points are determined by making the Angle Y*E*²W equal to the inclination of the Pediment; cuting the Vanifhing Line YW, of the Plane of the End, at W; the other will fall as much below the Horizon.

W is therefore the Vanifhing Point of *GF* and *HI* (as in the Apparatus) and *FD* tends down to the other, in WY produced.

The femicircular Window, and the Circle in the Pediment, are determined on BY, in refpect of their widths; the heights on BG; as the Arches in Ex. 22; for, the Plane being inclined to the Picture, they are confequently Ellipfes; as alfo in the former pofition; in the other they are Circles. (Theo. 9, Cor. 5.)

Thus, are delineated three different Pictures, or Reprefentations of the fame Object, from the fame Station and Point of View; and, notwithftanding they are fo very different, yet they are all true Perfpective. It is obvious, which is the neareft to the true Appearance, but, when the Eye is in the Point of View, of each, they will appear the fame.

In Fig. 107; let the Eye be placed oppofite to the Center, at C, at the Diftance CE; and having attentively obferved the Appearance, then let the Eye be moved to 108, viewing it, at its proper Diftance, CE; and laftly, to 109, at its refpective Diftance. I fay, that, each being feen in the true Point of View, exactly (which can only be done perfectly, by fixing a fmall Pin hole at the true place of the Eye) every part of each being feen under the fame Angle as the Original, muft neceffarily appear the fame.

Now, becaufe the Eye is perpendicularly on the Picture, at 109, and at the proper Diftance, the Angle it fubtends is equal to ASC (106) and, becaufe the Angle does not exceed 25 Degrees, the portion of a Sphere, of that Radius, will not differ much from the Plane of the Picture; and confequently, this Reprefentation is the neareft to the true Appearance.

SCHOL. This Leffon, properly digefted, will give the Student a clear Idea, of the caufe of Diftortion, in the Reprefentation, and teach him how to avoid it; for notwithftanding the Diftance may be fufficient for the Object (the Picture being judicioufly placed) yet if the Picture be placed according to the two firft, it muft neceffarily be diftorted. Although the real Optic Angle, refpecting that fingle Object, is not varied; yet the Angle under which the whole Picture is feen, may be varied and increafed infinitely, from 25 Degrees (the true Angle) to any quantity, lefs than 180, two Right Angles.

A brief and curfory retrofpect of the Theory; fhewing the immediate and abfolute dependance of Practice thereon.

Obferve, that No. 15 alfo refers to the Apparatus.

The firft Theorem is exemplified in the Picture MN*OP*, which is parallel to the Plane BGFDC. This is applied to Practice, by the 9th Theorem.

The fecond demonftrates, that the Vanifhing Line, and Parallel of the Eye, &c. are always parallel to the Interfection of the Original Plane.

This has been exemplified, in practice, throughout the whole; but chiefly in horizontal and vertical Planes. It is illuftrated generally, by the Diagrams.

Fig. 15 (No. 2 and 3) illuftrates the Theory for all Planes perpendicular to the Picture; or inclined in a certain pofition, refpecting the Horizon and the Picture; and if a Plane paffes through the Spectator's Eye (at E) parallel to the Original Plane HIFG, its Vanifhing Line, VW, is parallel to the Interfection, *FH*. (8. 7. El.)

The great ufe of this knowledge is obvious, for, if either be determined or found, as *FH*, and any one Point, as V) the Vanifhing Point of AB, &c.) the Vanifhing Line, VW, is determined; and confequently, the Parallel of the Eye is alfo determined, the Diftance of the Vanifhing Line being known; by Theo. 7.

Theorem 3rd has been frequently exemplified, and befpeaks its ufe and application fufficiently; particularly in Example 12. The Corollaries deduced from it, efpecially the firft, is derived from the fecond general Theorem, Book the firft.

The 4th is a generally ufeful Theorem; in common Practice, as by the old writers on Perfpective, more Planes are perpendicular to the Picture than in any other pofition. The Horizontal Line (the Vanifhing Line of the Ground, and of all other horizontal Planes) alfo the Vertical Line (of all which are perpendicular to the Picture and to the Horizon) both pafs through the Center of the Picture.

The Planes FIHG and FIKD, are perpendicular to the Picture MN*OP*; their Vanifhing Lines alfo, pafs through the Center (C) of that Picture.

The 5th, that Planes, whofe common Interfection is parallel to the Picture, have parallel Interfections and Vanifhing Lines, is exemplified in the Apparatus.

The Line BG, in the Object, is parallel to both Pictures, and it is the common Interfection of the two Planes, AHGB and BFC.

If thofe Planes are produced they will cut the Picture in *BG* and *BG*; which are their Interfections; the Planes being vertical and the Picture vertical, their Interfections are perpendicular to the Horizon, and confequently parallel. For the fame reafon, their Vanifhing Lines (OP and RW) are parallel; and, becaufe they are parallel to their Interfections, refpectively, by Theorem 2nd.

The ufe of this knowledge is evident; having obtained, by any means, one Interfection or Vanifhing Line; and alfo, by any means, the Interfecting or Vanifhing Point of a Line in any other Plane (whofe common Section, with it, is parallel to the Picture) the whole Interfection or Vanifhing Line is confequently determined.

In the laft Example the common Interfection (BG) of the contiguous Planes is in the Picture; confequently, it is the Interfection of both.

By the 6th Theorem, Vaniſhing Lines, of contiguous Faces of Objects, are determined according to the Inclination of the Original Planes, to one another.

In the laſt Example they were at right angles; but, being inclined in any Angle whatever, their Vaniſhing Planes being parallel to them, conſequently they make the ſame Angle with each other, at the Eye; and their common Interſection, which paſſes through the Eye, is parallel to the Interſection of the Originals; and, when it is not parallel to the Picture, produces its Vaniſhing Point (Def. 22) which, Vaniſhing Point, is the Interſection of the Vaniſhing Lines.

How the Vaniſhing Lines are found, in particular Caſes, is determined by Fig. 15, No. 2 and 3; *viz.* when the common Interſection is either parallel or perpendicular to the Picture. When it is perpendicular by Problem the firſt; and when parallel, by the third; and, when it is inclined to the Picture, in general, by the fifth. In all which Caſes, it is obſervable that the Interſection of the Vaniſhing Lines, is the Vaniſhing Point, and the Interſection, of the Interſecting Lines, is the Interſecting Point, of the common Interſection of the Original Planes; which is an eſſential thing to be well underſtood; for Practice.

Theorem 7th. A Line drawn through the Center of the Picture and the Center of a Vaniſhing Line is perpendicular to the Vaniſhing Line.

The uſe of this Theorem is to find the Center and Diſtance of every Vaniſhing Line, the Center of the Picture being determined, and Diſtance known.

To do which, there is only to draw a Perpendicular, as CS (109) from the Center of the Picture, to the Vaniſhing Line; and, forming a right angled Triangle, of the Direct Radial, or Diſtance of the Picture, CE, and the Diſtance between the two Centers; the Hypothenuſe is the Diſtance of the Vaniſhing Line. Def. 20.

In Theorem the 8th it is aſſerted, and proved, that the Repreſentation of every Right Line, in Perſpective, is a Right Line.

Any two Viſual Rays, EF, EI, or EF, ED, together with the Original Line, form a Triangle, which are all in the ſame Plane, paſſing through the Original Line and the Eye; which, by its Interſection with the Picture, generates the Right Line fi, or fd, its Repreſentation; and if the Original Line was extended, infinitely, its repreſentation will ſtill be a Right Line. (1. 7. El.)

The 9th Theorem contains the neceſſary knowledge for Practice, relative to Planes which are parallel to the Picture.

Firſt; all Lines in ſuch Planes, are parallel to the Picture; conſequently they have no Vaniſhing Point (Cor. 1. Th. 1.) but their Repreſentations are (by Theorem) parallel to the Originals; that is, to the Vaniſhing Line and Interſection of whatever Plane they are in; and their proportions to the Originals are alſo there determined; *viz.* as the Diſtance of the Picture, to that, of the Plane they are in.

The Corollaries, of this Theorem, are moſt uſeful Leſſons; which need only to be read over, with attention, to apply them to Practice. The fifth ſums up the whole; that the Repreſentation of every Figure, in ſuch poſition of the original Planes, is ſimilar to its Original; that is, it has the ſame geometrical Figure.

Theorem the 10th is ſelf-evident, being read attentively; and the Corollaries are uſeful practical Leſſons deduced from it, which are fully explicit in themſelves.

Theorem 11th is of the greateſt uſe; inſomuch that, without it, we cannot draw the repreſentation of one Line, inclined to the Picture, except by means of expedients; making uſe of Lines, otherwiſe, unneceſſary in themſelves.

Notwithſtanding this is the 11th Theorem, yet its application is prior to ſeveral others, reſpecting Vaniſhing Lines; ſeeing by its means, only, they can be determined; as in Prob. 2nd. It being impoſſible to determine any one Vaniſhing Line, before the Vaniſhing Point of ſome Line in the Plane be determined, through which the Vaniſhing Line muſt neceſſarily paſs.

For firſt, the Center of the Picture muſt be determined, before the horizontal and vertical Vaniſhing Lines can be drawn; or that of any other Plane, perpendicular to the Picture; or indeed of any other whatever; becauſe it is by means of Lines,

Lines, in horizontal Planes, others are determined; whofe Vanifhing Line is always determinable from its known pofition, the Center of the Picture being fixed; or, which is the fame thing, the height of the Eye being determined. Notwithftanding the Center of the Picture (being the Vanifhing Point of Lines perpendicular to the Picture) is fixed arbitrarily, yet it is as much fubject to the 11th Theorem as any other; and is only determinable from the fixed and invariable pofition of the Lines, which vanifh in it, to the Picture.

As this Theorem has been applied in almoft every Example, or Problem, where Vanifhing Points of Lines, inclined to the Picture, are required; I fhall only illuftrate it by an Example or two, from the Apparatus.

The Parallel of the Eye muft be imagined, paffing through the Eye of the Spectator parallel to the Vanifhing Line. Def. 9.

For the parallel Picture; EC cuts them both perpendicularily; therefore it makes equal Angles (i. e. Right Angles) with the Parallel of the Eye and the Horizontal Line; as the Original Lines, AB, HG, &c. make with the Interfection of a horizontal Plane paffing through them; or any other, whatever.

In the direct Picture; EV, producing the Vanifhing Point of the fame Lines, alfo makes equal Angles with the Parallel of the Eye and the Vanifhing Line, of either Plane, AHGB, or FGHI, as the Original Lines, make with the Interfection.

And laftly; EW*, makes equal Angles with the Parallel of the Eye of the Plane FGHI, and the Vanifhing Line, VW, of that Plane, as FG, or HI, makes with *FH*, the Interfection of that Plane with the Picture, or, with WY, of the Pane BFC, as FG makes with BG, its Interfection.

This is practically exemplified, in Problems 5, and 21, Method 3; 23rd, &c.

By Theorem 12th, the Indefinite Reprefentation, of a Right Line, is a Line drawn through its Interfecting and Vanifhing Points.

This Theorem has been exemplified frequently; but I fhall in this place illuftrate it further, it being moft effential in Practice.

Whatever Plane any Right Line is in, if it be not parallel to the Picture, it will cut it, fomewhere, in the Interfection of the Plane it is in. IF, HG, and FG, are all in one Plane; which being produced, the two firft cut both Pictures, at *F* and *G*; the other (FG) is parallel to one Picture, and therefore cannot cut it; but it will cut MNOP at *H*; *F*, *G*, and *H* are, therefore, the Interfecting Points of thofe Lines; and confequently, the Plane FIHG, being produced, would cut that Picture, in the Line *FGH*; and the other in *FG*; as by the 10th.

But, EV and EW being parallel, refpectively, to thofe Lines, the Points V and W are their Vanifhing Points; wherefore, the Lines *F*V and *G*V, alfo *H*W, are the indefinite Reprefentations of thofe Lines; by this Theorem

The fame thing is applicable in all Planes whatever; as *B*V, *G*V, &c. in the Front and End, which are vertical; and aV, *B*V, MV, &c. on the Ground Plane.

By changing V for C, it is all applicable to the parallel Picture (MN*OP*) fave only, FG is parallel to it, and therefore has no Vanifhing Point; for the Indefinite Reprefentations of all Lines, in fuch cafe, are infinite, feeing they have neither Interfecting nor Vanifhing Point; and are, confequently, parallel to the Vanifhing Line of the Plane they are in.

In Theorem 13th, is contained the whole knowledge of proportioning Right Lines, indefinitely drawn on the Picture (by the 12th.) It may, to fome, appear too mathematical for Practice; but, notwithftanding that Theorem is more ftrictly

* Let the triangular Piece be fixed, by the two Pins, to the Picture MNOP, of which it is a continuation; and bring the piece, which folds back, to the fame Plane; the continuation of the Vanifhing Lines, &c. on them, fhow their affinity with Figure 109. If the thread, at W, be drawn through the Eye of the Spectator, EW will be parallel to the Lines, FG and HI, in the Roof of the Object; and confequently, W is their Vanifhing Point (Def. 22.)

By the fame, the thread EV determines the Vanifhing Point of AB, GH, &c. on both Pictures; to the firft, it is perpendicular; therefore C is its Center, or Point of View; whilft another, EC, perpendicular to the other Picture, gives its Center (Def. 17) directly in the middle of the Reprefentation, as it ought always, when there is but one Object.

and

and rigidly demonſtrated than any other, it is nevertheleſs very applicable to practice. For, whether it be conſidered mathematically or not, 'tis certain that it is put in practice, by every one, in proportioning Lines; even without its ever entering into his Head what Proportion means; on the contrary, if he really underſtands what is there demonſtrated, with how much more pleaſure, ſatisfaction, and certainty, would he proceed in the application of it to Practice.

The Theorem tells us, that the Diſtance of the Repreſentation of any Point (in the indefinite Repreſentation) from the Interſecting Point (of the Original Line) is in that proportion to the whole Indefinite Repreſentation; as the Diſtance, between the Original Point, and the Interſecting Point, is to its Diſtance from the Directing Point of the ſame Line.

Now when the Premiſes of this Theorem are clearly underſtood (for which purpoſe, the words are adapted, for brevity and perſpicuity, as much as may be) what is there of myſtery in it? For the firſt, in reſpect of the Proportion, ſimply. If the Diſtance of the Original Point, from its Interſecting Point, be to its Diſtance from the Directing Point, as 2 to 3, 5, &c. or as 3, to 5, 7, 8, &c. in any Ratio, whatever; then, the Diſtance, of the Repreſentation of that Point from the Interſecting Point, will be in the ſame ratio to the whole Indefinite Repreſentation*.

Can any thing be more ſimple and eaſy? nor does it depend on the real meaſures, but on the ratio of one to the other. Wherefore, ſince the Diſtance of ſome Points, in original Lines, cannot eaſily be obtained, from their Interſecting and Directing Points; I have ſhewn, further (which Dr. Brook Taylor has not) that the Diſtance of the Original Point, either from the Interſection of the Plane it is in, or from the Picture, is in the ſame ratio to its Diſtance from the Directing Line or Plane, a circumſtance of great utility; becauſe we may, with eaſe, have the meaſures of one, when we cannot, without applying Geometry, or by Calculation, have the other. For, the Diſtance of any Point in the Object, from the Picture, may be had; and the Diſtance of the Directing Plane, from the Picture, is nothing more than the Diſtance of the Eye from it; hence it is, with the greateſt facility imaginable, reduced to Practice, in any poſition of the Lines whatever. It is moſt frequently done by the laſt; and commonly, by its Diſtance from the Interſection of the Plane the Original Line is in.

And further; the repreſentation of any Point in a Line being obtained, the repreſentation of any other Point in the ſame Line, or any other, cuting the former in that Point, may be obtained by the ſame Theorem; when the Interſecting Point cannot be had, nor is in the leaſt degree neceſſary.

The 14th Theorem, which is more theoretic than practical, or really uſeful, is fully explicit in itſelf; and the Corollaries, deduced from it, need no further Comment, in order to a clearer elucidation.

Thus I have, briefly, taken a tranſient retroſpect of the whole Theory: and I think it cannot fail of anſwering the End I aimed at, which is moſt obvious. As the Reader, who has advanced thus far, muſt now be perfectly acquainted with every Rule given: yet, being eager in his purſuit of what he deemed really and only uſeful, he may not have given that attention to the Theory (which is frequently refered to) as I could wiſh, for his more perfect knowledge in it.

He cannot now be inſenſible of the great advantage reſulting from a well-founded Theory, the very eſſence of all uſeful Knowledge; for, without it, like a blundering Workman in any mechanic Art, he may continually be runing into error; which, for want of a juſt knowledge of the Theory, he muſt, unavoidably; nor can he, with eaſe and certainty, extricate himſelf from the difficulties he will frequently be immerſed in.

* See this fully and practically illuſtrated in Prob. 6. Sect. 4. and indeed in almoſt every Problem and Example, throughout the whole Work.

Plate XXIV.

EXAMPLE XXXIII.

Is a Tuſcan Arcade, with Pilaſters; having one Face parallel to the Picture.

Fig. 110. The ſcale of Proportion being determined, and a Profile, of the Order, proportioned to that Scale (at AD) as in the previous Examples, let AB, the Interſection of the upper face of the Step, be conſidered as the Ground Line; and, equal to the height of the Eye (in proportion to the Object) let CE be the Horizontal vaniſhing Line, C its Center, and CE the Diſtance.

Then, the Object being right angled, the horizontal Lines in the returning Side vaniſh in the Center of the Picture †; in the other Face, every part of it, which are in Planes parallel to the Picture, are repreſented by Figures ſimilar to their Originals, according to the 9th Theorem, and its Corollaries.

† Cor. to Theo. 4.

The place of the Sub-plinth, at X, being determined, by its diſtance from A, the Step (*AB*) is firſt repreſented; then, the Plinths, X, Y, and Z, are equal, and have equal ſpaces between them; their places are obtained by ſeting off their true meaſures from *A* towards B, as a, b, c, &c. on the firſt Step; and after the ſame manner, the Piers are obtained, and the Plane of the Arches, whoſe Centers are got by biſecting bc and de, at *a* and *b*; and *ac* being made equal to the Diſtance of the Plane they are in, from the Picture. Draw *c*E, cuting *a*C at f; and a Line fg (parallel to AB) cuts *b*C, at g; f and g are the Seats of the Centers of the front Arches, on the Plane of the firſt Step.

Draw FG parallel to AB, and equal to the height of the foot of the Arches; from *a* and *b* draw perpendiculars, *a*F and *b*G, and from F and G, draw to the Center, C; then, Perpendiculars, from f and g, cuting FC and GC, at h and i, give the Centers of thoſe Arches: which, being parallel to the Picture, are Semicircles. (Cor. 5, Theo. 9.) The Centers of the inner Curves, are in the ſame Lines, FC, and GC, at k and l, and determined as the other.

The Mouldings in the Cornice, Pilaſters, &c. may be proportioned by the general method (Ex. 15) as the Doric Entablature in the 16th.

The true Profile of the whole Order being deſcribed geometrical, at AD, and the Picture being ſuppoſed cloſe to the Plinth, at A, draw AD perpendicular; which is conſidered as a vertical Section of the Picture, and CE its Diſtance.

Then, E being the Eye, and EA, ED, &c. Viſual Rays from every Part to the Eye, it is obvious, that the parallel Mouldings have their places and proportions transfered from the Profile, to AD (as in Ex. 15 and 16) and, becauſe the Cornice projects beyond the Picture, as Db, it is projected to the Picture, at D; the difference is only in the proportion of the whole; for if the Picture was at the greateſt extreme of the Cornice, as ab; it is manifeſt, that the proportion of all the Mouldings, &c. would be leſs than thoſe on AD. (See Example 15th.)

The returning Face (*A*HIK) being perpendicular to the Picture, the Vertical Line (AD) is the Vaniſhing Line of all Planes in that Front. (Theo. 5.)

The meaſures of the Piers, Arches, &c. being ſet off, at *d*, *e*, &c. are proportioned, by drawing Viſual Rays *d*E, *e*E, &c. as in the foregoing Examples; and the Arches, are managed as in the 22nd Example, their Repreſentations are ſemi-Ellipſes; to particularize the whole would be ſuperfluous.

The Mouldings, in all ſuch Caſes, *viz.* when a true Section of them is parallel to the Picture, may be proportioned, by making a geometrical Section or Profile at S, where the Picture is ſuppoſed to cut the returning Mouldings, if they were continued beyond the Picture.

In this Example, the Diſtance is conſiderably too little, for taking in ſo much as is repreſented; in this poſition of the Picture. The deſign, of which, is to ſhew how diſtorted the returning Mouldings of the Pilaſters, &c. are, beyond the firſt Arch; which is neceſſarily the caſe in all ſuch Views, being parallel, except a ſufficient Diſtance be taken; in which Caſe, the returning Front would be greatly contracted. On the contrary, being both inclined, they may be ſhewn to much greater advantage, at a leſs Diſtance, without any ſenſible diſtortion; as may be ſeen in the next Example.

EXAMPLE

E X A M P L E XXXIV.

Is the Representation of a Doric Arcade, with Columns, inclined to the Picture.

If the Problems and preceding Examples are underſtood, there is nothing more requiſite to the delineation of this Object; the Situation, Diſtance, and Poſition to the Picture being previouſly determined. Fig. 111.

In Prob. 21, the Rectangle may be conſidered as the Plan of ſuch an Object, promiſcuouſly ſituated; or the Plan of the Building in Fig. 106, inclined to the Picture, on VY. The foregoing Example ſuppoſes the Picture in the poſition DE; the Repreſentation, of which, is as Fig. 107.

The height of the Eye being determined, and the Center of the Picture (C) draw the Horizontal Line; and, at a convenient Diſtance, take AB for the Interſection of the Ground Plane, on which ſet off the ſeveral meaſures, at a, b, c, &c. and, having obtained the Vaniſhing Points of the Sides (both which are, in this Caſe, off the Picture) and their Diſtances, D and E, being fixed (by Prob. 12) compleat the perſpective Plan; as in Example 16, of the Doric Entablature.

The direction of the Lines, in the Figure, ſhew how it is effected, a repetition of it is almoſt unneceſſary; ſeeing that, the rectilinear part is managed as inclined Lines, in general; and, the plans of the Columns are repreſentations of Circles inſcribed in Squares, by Prob. 3rd.

Let the Reader obſerve, once for all, that, either a true geometrical Plan of the Building, muſt be drawn out, or its meaſures known; which are applied to the Ground Line, or other Interſection, as there is found Occaſion. The Profile, and meaſures of Elevation, muſt alſo be known; for it muſt be obvious, that every Object delineated by the rules of Perſpective, are projected by the true and real proportion of one part to another, which muſt be known, or conceived.

A a is the width of the Step, a b, b c, &c. being made equal to the Sub-plinths of two Columns, draw a D, b D, &c. cuting AL (drawn to the Vaniſhing Point of the front Lines, indefinite) at H, K, L, &c. b h and k c are each equal to the break at the Pier, and h k to the opening between the Piers. Being firſt projected to AL, by means of the point D; from which they are drawn to the Vaniſhing Point of the Lines in the End, till they meet the returning Lines of the Front (as at I) which are obtained on FQ, drawn to the Vaniſhing Point. A d is the meaſure of the break at the corner of the Step; and, by drawing d E, the internal Angles of the Step and Sub-plinth are obtained, at G. From *a* and *b* (making A *a* equal to the receding of the Arches, and *ab* to the thickneſs of the Wall) draw Lines to D, cuting AL, and from thence to the Vaniſhing Point of the End; where they cut the Diagonal GE, they are returned at the Front; the reſt is obvious, on inſpection. The Columns at MN recede from the Front, being in a Line with the firſt returning Column at the End, at G.

The perſpective Plan being compleated, or ſo much as is neceſſary (for thoſe Parts which cannot be ſeen, it would be uſeleſs to particularize, except for the affinity of thoſe which are) take *A*, at the proper Diſtance from the Vaniſhing Line (perpendicular over A) equal to the determined height of the Eye; and from it draw indefinite Lines to the Vaniſhing Points (as *AB*) and, from the Plan below, cut off ſuch portions as are the repreſentations of the ſeveral Parts, and proceed as in the foregoing Examples.

Here, the neceſſity of a Plan being formed, or, which is the ſame thing, the Lines proportioned below, by an imaginary interſection, AB, is evident; on account of the great Diſtance, and the Eye not being far out of the Ground Plane; otherwiſe, that operation would be unneceſſary; for, if the height of the Eye was equal to the Diſtance of AB, from the Vaniſhing Line, then, the Plan formed, by means of that Interſection, would be the real Baſe, or Seat of the repreſentation, of the Object, and no other would be wanted; in which, the expediency of the 12th Example is remarkably obvious. Being farther removed from the Vaniſhing Line, the ſeveral parts are more diſtinct, and may be done more correct.

Plate XXV.

The Step being firft drawn, the Sub-Plinths, next, are but fo many equal Parallelopipeds of a certain proportion and diftance from each other (as in Example 4.) The places and proportions of the Columns are determined from their Plans below, by drawing Perpendiculars from them (or by Example 22) the Bafes and Capitals, being inclined, by Example 26; and the Entablature, by Ex. 17, 18, and 9.

The Balluftrade, may be planed on the top of the Cornice, or at fome diftance from it; the Pedeftals, or plane parts, are directly over the Columns, and equal to their Diameters at the top; and the places of the Ballufters are got by dividing the Line *AC*, perfpectively (by Prob. 8) into the number of Ballufters and fpaces between them (equal or otherwife.) The Plinths, at their tops, are perpendicular over the Bafes; and central Lines (h i) being drawn, through each, the Ballufters are eafily formed, from the known geometrical figure of them.

The Doric Entablature is defcribed at large in Example 16 (but parallel to the Picture) how it may be inclined is fhewn in the 18th and 19th Examples.

The Plane of the Arches being obtained, the Arches are reprefented by Example 22, and the Confoles by the 20th; the reft, it would be needlefs to defcribe, as the previous Examples contain every neceffary Leffon for complex Objects.

EXAMPLE XXXV.

Is the Reprefentation of a compleat Building, with a Portico in the Ionic Order; having a fpherical Dome, and Cupola.

Fig. 112. At No. 1 is half the geometrical Plan, and, at No. 2 is the Elevation; in proportion to the Reprefentation, as one to four.

The Defign is intended as an exterior Building, adapted to St. Stephen's, Wallbrook; but of a different Order; having a Front towards the Manfion Houfe.

This Example fums up all the foregoing, in one entire Building.

The Horizontal Line being drawn, at difcretion, and the Center of the Picture fixed, as ufual, let AB be the Ground Line, and D the Place of the neareft Angle of the Building. The Diftance of the Picture is 15 Inches; C is its Center.

The Inclination of the Front and End, to the Picture, is determined by the Line ab (No. 1) on which the Picture is fuppofed to ftand.

The Diftance of the real Ground Line from the Horizontal, being equal to the height of the Eye, which is elevated above the common height, yet is too low to form a correct perfpective Plan, in its true Place; therefore, at any convenient Diftance take another Line *AB*, parallel to AB, and proceed as in the laft Example.

Having laid down the diftance of each Vanifhing Point, at E and C, as ufual; take the known meafures of the feveral parts of the Building and apply them on each fide, from the Angle at *D*. Compleat the exterior Plan, *CDEF*, as in the laft Example; from which, the feveral parts of the Building, above, are proportioned; as the perpendicular doted Lines fhew. F is the Vanifhing Point of Diagonals; i.e. of a Line bifecting the Angle of the Building.

It muft be obvious, that any Line (AB or *AB*) being confidered as the Ground Line, will give the fame proportions; which, being properly confidered, there is nothing more in it, than a fuppofition of the Eye being more elevated above the Ground Plane; and confequently it is more feen and better defined.

The exterior Plan being compleated, from which the corners of the Steps are determined, and drawn in their true places, by Example 13. Alfo the Sub-plinths of the Columns, v, u, x, &c. from their correfponding Plans below; the Diagonals of which, being drawn, give a central Line, the Axis of each Column; the heights of the feveral parts are determined on the diagonal Interfection, at D, F being the Vanifhing Point.

The

The Columns with their Bafes are drawn and finifhed as in the laft Example; and the Capital, being the antique Ionic, by Ex. 31ft. The interior part of the Portico has a Pilafter oppofite each Column, which are alfo determined from the Plan, below; fome of which are hid behind the Columns; perpendicular Lines from the Plan, fhew how much, and where they are feen entire.

The Plane of the Front, of the Pediment, over the Portico, being much inclined to the Picture, its Vanifhing Line is very remote; the Diftance of the Vanifhing Point of horizontal Lines, in that Face, may neverthelefs be afcertained, (as E) by Ex. 12, and the Lines drawn by the Expedients, in the 13th.

As the Diftance of the Vanifhing Point of horizontal Lines on the left Hand, would fall off the Picture, it is taken half, at C; and confequently, the meafures applied on the Interfection or Ground Line (*AB*) for that fide, are alfo, each half of the real meafure, agreeable to the 13th Theorem, and Prob. 6 and 7.

The front Pediment is thus determined. If the Cornice projected equal to the Steps, which the Picture is fuppofed to touch, a Right Line drawn through D, perpendicular to the Ground Line, would be the Interfection of the Plane of the Pediment; but, as its projecture is lefs, it will fall to the left hand of it; as the Line *ab* (No. 1) which is a continuation of the front Line of the Cornice, cuting a b, at *d*, indicates fufficiently.

Or, having determined the perfpective Plan of the Cornice, at a, b, c, produce ab, till it cuts the Ground Line at d, and draw de perpendicular, which is the Interfection, of the Plane of the Cornice in the Pediment.

Then, having made J e equal to the known height, and determined the front Line of the upper Moulding, as fg, fet up the height of the Pediment, from e to i, and draw i k, tending to the Vanifhing Point (by Prob. 13.)

Bifect fg (perfpectively) at h, and draw h l perpendicular, cuting i k, at l, the true pitch, or middle of the Pediment, which may be compleated, by Ex. 20.

The Pediment in the other Front, on the left hand, may be projected by means of its Vanifhing Points, by the fame Example.

To this Front is added a Pedeftal which is continued around the Building; more for the fake of diverfifying the Leffon, than propriety in the Building; the heights of the Pedeftal, Windows, &c. are proportioned on the Interfection, DK, of that Front, at *a*, *b*, *c*, &c. for the Niches and Windows over them.

The Body of the Building being compleated, the feveral parts being determined from the Plan below, with the breakings or returnings of the Cornice, and other Mouldings (as in former Examples) let the Circle of the Dome be planed, at GH, perpendicular Lines from the extremes of that Ellipfis, give the extreme apparent edges of the cylindrical part, IK, below the Dome; of which Cylinder, the Ellipfis, on the Ground Plane, may be fuppofed its Bafe.

The curve Lines, in the Cornice, around the Dome, are defcribed in their refpective Planes; which are thus determined.

From the Vanifhing Point on the left, draw through the Center, S, of the Plan below, cuting the Ground Line at J; which reprefents a Line paffing through the middle of the Plan (No. 1) fuppofing a Section made by a vertical Plane through the middle of the Building.

Draw J L, perpendicular, the Interfection of fuch a Plane with the Picture (Prob. 3) on which, all the meafures of the heights of the Dome, &c. are applied, in the fame ratio as on d e, in the Elevation (No. 2) at M, N, &c.

From M, the height of the Cornice, draw a Line parallel to the Ground Line, which is the Interfection of the Plane of the Cornice, with the Picture. A Right Line, drawn from M to the Vanifhing Point on the left, will cut a perpendicular Line from S at f, the reprefentation of the Center of the Circle of the Cornice.

From C, the Center of the Picture, draw C *S*, through f; and through f, draw m n, parallel to the Interfection; make *S*R and *S*T each equal to half the Diameter of the extreme Circle of the Cornice; and draw RC and TC, cuting mn, at m and n. m n is a Diameter of the Reprefentation, parallel to the Picture; the whole Circle may be compleated, by Prob. 2, Sect. 8th.

 After

Plate XXV. After the fame manner, all the other Circles, of the Corona, &c. may be projected, in their refpective Planes; as the feveral Circles in parallel Planes, by Ex. 28.

The Cornice, &c. being compleated, fet up, from N, the height of the Dome, and take feveral Divifions *a*, *b*, &c. anfwering to the fame ratio on d e, in No. 2.

Through thefe divifions, parallel Lines being drawn, as r s, each may be confidered as the Interfection of a Plane, cuting the Dome (as *r s*, No. 2) in each of which, an Ellipfis being defcribed, reprefenting a Circle of the true diameter, made by that Section; a Curve defcribed over all their extremes (as in Ex. 25 and 26) is the true exterior contour of the Dome; which may be confidered as an Ovolo, inverted; as in the Doric Capital.

By any other means (the Dome being plain) I cannot conceive it poffible to obtain the true perfpective Figure of a Dome; which is generally higher than a Hemifphere; efpecially in a lofty Building, which would otherwife appear flat, below; befides, it is a more graceful Figure; as the magnificent Dome of St. Paul's fufficiently evinces, to a judicious Eye.. But, being a Hemifphere, or other Segment of a Sphere, the Eye being below its Center, the Contour of the exterior Curve, on the Picture, will be elliptical. (Th. 3rd, of curvilinear Perfpective.)

By this method, it may be truly projected, let the Figure be what it may, whether a Hemifphere or leffer Segment. Neverthelefs, a judicious Eye, and judgment, guided by a thorough knowledge of Perfpective, may direct a fteady Hand to trace a Curve, fufficiently correct, in common Cafes; the height and Vertex being truly afcertained.

The Dome in the Frontifpiece is octagonal; and, by means of feveral Octagons, reprefented perfpectively, in various parallel Planes, the true Figure of it may be projected. Or, by means of feveral Points, in an Angle; for, if the true Curve of an Angle be obtained (as at Y, No. 3) from the Curve, X, of a vertical Section through the Dome, perpendicular to two oppofite Faces (by means of perpendicular and horizontal Ordinates, *ab*, *cd*, &c. having made h A equal to the angular projecture, and divided it at a, c, e, as *Ah* is divided, at *a*, *c*, *e*; perpendiculars, a b, &c. being drawn, from the points *b*, *d*, &c. in the curve at X, draw lines, parallel to *A*A, cuting them at b, d, &c. through which the angular Curve A dg may be defcribed) then, having drawn any one, perfpectively, they may be transfered from one to the other, by the Vanifhing Points of horizontal Lines in the Cornice around it, at the foot of the Dome.

By the fame means, the Ribs, in this Dome and Cupola, may be defcribed.

After this defcription of the Dome it would be lofs of time to defcribe particularly the projection of the Cupola. From the Plan below, its place and dimenfions are truly afcertained, together with the place of each Pilafter and Window; from which perpendicular Lines are drawn; and, the height of each part fet up, on the Vertical Interfection, HN, as the Dome, and other Parts, below; of which, the Cupola is a fimilar Figure, nearly.

EXAMPLE XXXVI.

Is a Reprefentation of that much famed Building, by Inigo Jones, of St. Paul's, Covent Garden; and the Buildings adjacent.

Plate XXVI. As the laft Example was a Leffon for an entire Building, fo this is intended for a Leffon in detached Buildings, or diftinct and feparate Objects; which may be confidered as a Street-View.

Having, in the laft, given a compleat perfpective Plan of the whole Building, it would be multiplying Examples to little purpofe to do the fame thing here; let it fuffice, therefore, to fuppofe, that the Plan of the whole, in this piece, is truly defcribed on the Ground Plane, on which the Objects ftand; their meafures being applied to the Interfection of the Picture or Ground Line, as in the preceding Leffons, which it would be quite fuperfluous to repeat; I fhall, therefore, only make fome general Remarks.

It may be obferved, that the Church is nearly in the fame Pofition as the laft Object, but a much fimpler, and confequently an eafier Figure to reprefent.

The Portico is of a Tufcan Order; which, for its fimplicity and fingular Stile, is much efteemed by the admirers of the antique Grecian Temples, of which, this is efteemed a perfect Model. The great projecture of its Cornice (if it may be fo called) has a bold and ftriking appearance; owing to the great effect of Shade it occafions. The attached Parts, at Y and Z, which is the ftaircafe into the Gallery, feem not to belong to the Body, but are adapted merely for conveniency; I am of opinion that it would make a better Figure without them. The other, at V, is the Veftry Room; which, with a fimilar Object, on the other Side, adds to the appearance of the back Front.

How thefe feveral parts are projected, may be feen in the laft Figure; having formed a Ground Plan of the whole; their feveral heights being fet off, on the vertical Interfection of the front Plane, or proportioned to the Pilafter, at the corner.

The two Gates, or entrances into the Church-Yard (X) are a great ornament to the Church, and graceful in themfelves. After what has been done, there is nothing fingular in their Conftruction, the geometrical Figure being known.

The Houfes, one on each fide, are equally diftant from the Church; 'tis pity they are not both in the fame ftile, as that on the Left, for the fake of uniformity. The Hut, adjoining to the Houfe on the Left, at W, is the Parifh, Watch-Houfe. I could wifh fome other place had been deftined for it, as it blocks up an Avenue, and breaks the regular Order.

The Fronts of the Houfes and the Gates are in a Right Line, much inclined to the Picture; the Portico projects before them. There is no occafion for a Ground Plan of the whole, for the meafure of each Part being known, the Line may be proportioned, by the Problems in the 4th and 5th Sections; and, the heights are determined on the Line AB, which may be ufed as an Interfection of the whole, confiderd as one Plane.

The Pediment has been frequently defcribed; and, the Bell-Turret may be determined by the length of the Roof; or, more correctly, from a Plan formed below; as the Cupola in the laft Example.

On the Left is feen the opening into Henrietta-Street, at P; and one of the Green-Sheds, at Q; the place, of which, is obtained by its known diftance from the Church, and the line of Direction in which it ftands, refpecting the Church, projected if neceffary. Or, the Ground Line may be fuppofed on this fide of the Shed, parallel to the bottom edge of the Picture. The Horizon is about the natural height of the Eye; the Center of View is at C, in the middle of the Picture, and the Diftance is about 16 Inches.

EXAMPLE XXXVII.

Is the Reprefentation of another well-known Building, viz. The Royal Hofpital, of Invalids, at Chelfea.

The Stile, of this Building, is fomewhat fimilar to that of St. Paul's, Covent-Garden, in refpect of its fimplicity. There is, in the Gufto of the whole, fomething remarkingly pleafing; the Symetry of the feveral parts, in refpect of each other and of the whole, is finely preferved, in a regular and gradual fubordination, from the Principal to the feveral Offices; the agreeable arrangement, of which, is uniform, and perfectly harmonious. Here, the attention is not attracted by the richnefs of the drefs, or mafterly execution; the mind is not captivated by grandeur and magnificence, but moft agreeably entertained, with an elegance of Defign; fimple, yet majeftic; and comely, though unattired; perfectly agreeable in its native fimplicity, and almoft naked Beauty, devoid of every luxuriant Ornament. Plate XXVII.

In the laft Example, the Line of the Building was much inclined to the Picture; in this, it deviates little from a Perpendicular; which is far more agreeable than to be perfectly fo.

In

Plate XXVI.

In this View, many, who pretend to know Perſpective, would immediately fix on the Point V, in which all the horizontal Lines, of the Front, converge, to be the Center of View; but it is not ſo; for, according to my general maxim, it is at C, in the middle of the length; and, notwithſtanding there is not one Line vaniſhes in it, yet, it governs the whole.

If V had been the Center, or Point of View, the Lines BD and HI, &c. would be parallel amongſt themſelves, and to the Horizon; but, they converge to a Point, at a great Diſtance, on the left Hand.

The Diſtance of the Picture is about 19 Inches; at which Diſtance (the Eye being perpendicularly oppoſite to C) the whole will appear as the Original, at the ſtation intended; and, I am perſuaded no Perſon will ſuppoſe that it would have a better effect, or ſo good, if V had been the Center of View; for, the Optic Angle would then be almoſt double of what it now is; and conſequently, the Repreſentation would differ more from the true Appearance; which, now, is inconſiderable, the Optic Angle not exceeding 34 Degrees.

The Inclination of the Building to the Picture, and the Diſtance, being determined, the Vaniſhing Point, V, is fixed by Prob. 12; and the indefinite Line, GV, being drawn, it is proportioned as uſual, by ſetting off the real meaſures, or their ratio, from G, at K, L, &c. and drawing Lines, which repreſent Viſual Rays, to E, in the Horizontal Line.

The meaſures are taken half, becauſe EV is but half the Diſtance of the Vaniſhing Point, V; as the whole Diſtance exceeds the bounds of the Picture.

The horizontal Lines on the returning Sides, whoſe Vaniſhing Point is at a great Diſtance, may be drawn by the Expedient, No. 5, Prob. 13. Or; having proportioned the Line of the Angle, at AB, into the ſeveral Diviſions, at a, b, c, &c. for the Windows, &c. and having drawn one Line to its Vaniſhing Point, as BD (by the 12th) the Diſtance of the Vaniſhing Point being determined*; produce BD, and, at a convenient Diſtance, draw FG parallel to AB, and proportion FA in the ſame Ratio as AB (by Prob. 33, Geom.) draw a*a*, b*b*, &c. which will tend to the ſame Point.

But, when the Inclination is not very great, the beſt Expedient is to fix a Lath to the Board and make uſe of a long Ruler; or, in large work, fix a ſmooth String to the Vaniſhing Point, and ſtrain it, in a Right Line, to the ſeveral Points a, b, c, &c. and then apply a Ruler to the direction of the Cord.

The Hip of the Roof, at H, may be determined from a Plan, on the Ground Plane; or by the known Inclination of BH to the Picture, and its Vaniſhing Point; and, the Line HI by the ſame, as in the 13th Problem.

The Window Frames, in the Roof, being drawn, as the other Windows, and the ſtreight Cornice, fh is obtained, by which, the Pediments over them may be completed, as in Example 20; by finding the Vaniſhing Points of fg and gh, in the Vaniſhing Line of the Front, paſſing through V, perpendicular; the beſt method for the front Pediments. The Vaniſhing Line of the ſide Planes is too remote to uſe for thoſe Pediments; which, in ſuch caſe, differ but very little from their geometrical form and proportion.

The Cupola is beſt done by a Ground Plan, as in the 35th Example.

The Columns and Pediment over them have nothing particular in the delineation; the middle Pediment is more contracted than the hither one, ſo that the Mouldings are ſeen on one ſide only; but is done after the ſame manner; and, if they have the ſame inclination to the Horizon, the Lines in them have the ſame Vaniſhing Points as thoſe over the Windows; becauſe the Pediments are all in parallel Planes, and conſequently they have the ſame Vaniſhing Line †.

† Theo. 5. and Cor. 1.

The Trees (on the right hand) excepting their diſtances from each other, are not ſubject to the Rules of Perſpective; becauſe no proportion of the Boughs can be taken. The Rails and Poſts have nothing ſingular in them; as the Rails are

* This Vaniſhing Point is about ſeven Feet Diſtance, from the Center of the Picture.

parallel to the front of the Building, they have consequently the same Vanishing Point (V) as the horizontal Lines, in the Front; and the Posts, being equally or otherwise spaced, are determined as in all other similar Cases.

The heights of the Figures, in the Walk, are thus determined.

Take any Point, L or M, in the Ground Line, and make MN equal to the height of a Figure (by the Scale of proportion, of the measures of the Building) and draw MO and NO, to any Point in the Horizontal Line; then, wherever you intend a Figure, as at *a*, *b*, or *c*, draw *a*m, *bm*, &c. parallel to the Ground Line cuting MO, at m, or *m*; and draw m n, or, *mn*, parallel to MN, which is the height of the Figure in that place, representing an equal height to MN.

Or, wherever you intend a Figure of any kind, draw a Line through the place of the Feet, cuting the Ground Line and Vanishing Line, obliquely; and, where it cuts the Ground Line, draw a perpendicular, and set up the height of the intended Figure; draw another Line to the same Point in the Vanishing Line, between which, a perpendicular being drawn is the height of the Figure.

By the same means, the proportion of any other Object may be determined.

EXAMPLE XXXVIII.

Is a View of the Queen's Palace in St. James's Park, and adjacent Buildings.

This neat and elegant Building was formerly the Town Residence of the Dukes of Buckingham; which Title becoming extinct, it had for some Years been occupied by a distant branch of that ancient Family, without the Title, who made no Figure, suitable to such a noble Mansion; insomuch, that it was gone greatly out of repair when purchased by his Majesty. It has, since, been much enlarged, repaired, and beautified, and made the Winter Residence of the Royal Family. The octagon Building, on the left hand, was added at the same time, and is the Library. Plate XXVIII.

Since which, there has been great improvements made; the semicircular Area, in front, has been enlarged; but I think it would have been more advantageous, had the Pallisadoes stood on a dwarf Wall; for want of which, they are not distinguishable, at some Distance from it. The Gate leading to Chelsea, and the Lodge are also new; and, several old Trees were cut down, to make a more spacious and open View to the Canal, &c. in the Park.

This Building would, with propriety, admit of a full Front View; but such a one is by no means picturesque; as, one Side, in such case, is but a duplicate of the other, which is not the case in this View; the Center, of which, is in the middle of the Picture, as usual.

The Front is inclined, and consequently, the horizontal Lines vanish, but at a great distance, because the Inclination is very great; the Distance of which Vanishing Point is determined by the 12th Prob. The Vanishing Point of horizontal Lines in the other Faces is at V, in the Pavilion on the left hand; the Horizon is high, for the conveniency of seeing the Area, more commodiously.

The Distance of the Picture being determined*, and the Vanishing Point V, the Distance of the other is a third Proportional, to C V and the Distance of the Picture, found by squaring the Distance, and dividing by CV. (Pr. 12.) (Pr. 31. Geo.)

As the octagon Building has two Faces parallel to the Fronts of the House, the adjacent Faces vanish in the Vanishing Points of Diagonals; i. e. all the horizontal Lines, in one Face of the Octagon, vanish in that Point where a Line bisecting the Angle, made by the Radials of horizontal Lines, in the Front and End of the Building, cut the Horizontal Line; and the other, on the Left, where a Line, making a Right Angle with the Line of bisection, cuts it; as it has been exemplified in various cases, in Mouldings, &c. both which fall out of the Picture.

* In this Picture it is about 12 Inches.

Plate XXIX. On account of the great Diſtance, and the Horizon being high (between the Mouldings in the upper part) their inclination is not perceptible; as is frequently the caſe, in Views taken at a tolerable diſtance, and perhaps from an eminence, which is generally made choice of in Landſcape Views; otherwiſe, the Ground would be too much contracted, unleſs diverſified with riſing Grounds, &c. The diſtant Buildings, in ſuch caſe, are repreſented nearly geometrical; for, unleſs they are ſituated conſiderably above or below the Eye, in proportion to the Diſtance, the inclination of the Lines is not diſtinguiſhable; and, if they are obliquely ſituated to the Picture, their Faces are contracted, almoſt geometrically; ſo that, diſtant Objects, of all kinds, are not cognizable to the Rules of Perſpective, otherwiſe.

It is unneceſſary to ſay more, in reſpect of this Object, as Rules have been given for projecting all kinds of Figures of which it is compoſed; and finding the Vaniſhing Points, &c. at any given Diſtance and poſition of the Picture.

Figure 106 exhibits a general Method for contracting the Faces, &c. of an Object, and for obtaining the true place of each part, in reſpect of its bearing with others (being detached) from any determined Station, which, in many caſes, is the beſt method of proceeding, and the leaſt liable to error.

EXAMPLE XXXIX.

How to repreſent Doors and Window Shutters, open, in any given Angle; or at pleaſure.

Fig. 113. Let AV be the indefinite Repreſentation of the Side of a Building in any horizontal or vertical Plane; which, from the point A, is required to be perſpectively divided into certain finite parts, repreſenting Windows, Piers, &c.

By the 8th Problem, Ac and VE are drawn parallel, between themſelves (VE may be conſidered as the Vaniſhing Line, and Ac, the Interſection, of whatever Plane the Original Line is in) make VE equal to the Diſtance of the Vaniſhing Point (V) of the Line AV; and, on Ac, take Aa, ab, &c. equal to the known proportions of the Piers and Windows, or Apertures of any kind.

Draw aE, bE, &c. cuting AV in the Points *a*, *b*, *c*, &c. which are the repreſentations of the Original Points, a, b, &c. Wherefore, if ab be ſuppoſed the geometrical proportion of a Window, then, *ab* repreſents the ſame perſpectively; and ſo of any other diviſion, as *bc* of a Pier, &c.

N. B. Let it be obſerved, that it does not depend on the real meaſures being applied on Ac, but, that they are in the ſame ratio, reſpectively, as EV is of the diſtance of the Vaniſhing Point; agreeable to Theorem 13th, which is frequently exemplified in the preceding Work; and, whether AV repreſents a Line perpendicular or inclined to the Picture, there is no difference in the proceſs.

Fig. 114. Let AB repreſent the aperture of the opening of a Door, in the ſide of a Room, &c. which is required to be ſeen open, in any poſition, at pleaſure.

Now, AB, the width of the Door, is the radius of a Circle, which it would deſcribe if the Door revolved quite around; and conſequently, the Door itſelf (being a Rectangle) would deſcribe a right Cylinder, of which, BC is its Axis.

By Prob. 10th, Sect. 4th. or, Pr. 2, Sect. 8th, deſcribe the repreſentation of a Semicircle, Aa*a*F, whoſe Radius (given or found) is AB.

It is manifeſt, that AB the perſpective width of the Door, would, in its ſemi-revolution (from AB to BF) deſcribe the ſemi-Ellipſis Aa*a*F; that is, the point A (B being fixed) would deſcribe the ſemi-circumference of a Circle: and conſequently, in whatever poſition the Door is open, the point A will be ſomewhere in that Semicircumference, as at a or *a*; wherefore, aB, *a*B, &c. are alſo repreſentations of Radii, of the ſame Circle.

Take the Point a or *a*, in the Circumference, at pleaſure (according as you require the Door more or leſs open) and draw aB or *a*B, which produce to its Vaniſhing Point, H or *H*. Draw the Perpendicular ad, or *ad*, and, through C, draw HC, or *H*C till it cuts ad or *ad*, at d or *d*; then is aBCd, or *a*BC*d*, the repreſentation of the Door, open in the poſition required.

Or,

Or, if the Angle of the opening, on either side, be known or determined; produce A B to its Vanishing Point (V); S being the Center of the Picture, draw SE perpendicular, equal to its Distance, and join EV; make the Angle VEH, or VE*H*, equal to the Angle determined; which will give the Vanishing Point H, or *H*, of the top and bottom Lines of the Door, in the position required.

If the Door be required more open, as *A*B, making an acute Angle, *A*BF, on the other side; then, the Vanishing Point will be on the other side of V.

The process is the same, in every Position.

After the very same manner, Window Shutters, being vertical, are determined; the Radius, of each, being half the width of the Window, Ad or dD. Fig. 115.

ABCD is the aperture of a Window, to which there is required the representations of folding Shutters, open in any position or angle, at pleasure.

Produce AD and BC, both ways, and bisect either, perspectively (as BC) and, make B*a* and C*g* each represent half BC. On *ad* and *dg* describe representations of Semicircles, *abcd*, and *defg*; in which, according to the determined angle or opening of the Shutter, take *b* or *c*, *e*, *f*, or *g*; from which, draw perpendicular, and draw *b*B, or *e*C to its Vanishing Point, at S or V; from which points, draw SA, or VD, and produce them cuting the Perpendiculars, at b or e, which compleats the Shutter AB*b*b, or C*e*eD.

AB*c*c represents another, less open; and C*f*fD is one parallel to the Picture, consequently, it has the true figure and proportion of half the Window.

AB*a*a, and C*g*gD represent others, quite open, against the Wall.

The Shutters Y and Z describe Semicircles in vertical Planes, as abcd, or *abcd*; or, more properly, they describe half horizontal Cylinders. Fig. 116.

The first, being parallel to the Picture, are Semicircles†; the others are perpendicular to the Picture; or they may be inclined, in any Angle whatever; the Vanishing Points of the describing Lines are in the vertical Vanishing Line (VL) of the Plane of the Circle, as at V and L, &c. the front Lines are parallel.

† Theo. 9. Cor. 5.

In the former they are parallel to the Picture; the front Lines vanish in the Center.

SECTION X.

OF INSIDE VIEWS, IN GENERAL.

THE application of the Rules of Perspective, whether to the interior or exterior parts of Objects, is the same, for, Planes and Lines, of which Objects are composed, are still the same however situated; whether they form internal or external Angles, of Objects seen externally; or, forming Rooms or Concavities of any kind. Nevertheless, at first Sight, there appears a difference; for I believe that, many who have practised Perspective with tolerable success, in exterior Objects, are somewhat puzled, at the first attempting an Inside View; not knowing, rightly, where to begin or where to leave off; how to place their Picture, or determine its Distance.

I have seen an attempt to represent an Octagon Building, internally, in which Picture were introduced seven Sides or Faces, out of the eight; it was a misfortune that the other could not be seen also, quite around, and then it would have been a master-piece, indeed. However, from what was seen, and the tendency of the Lines, it was manifest, that the Distance of the Picture was not more than one fourth part of its width, or length; and consequently, the Optic Angle was above 120 Degrees, which ought not on any account (except when the Eye is confined to the true Point of View, always) to exceed 60.

Plate XXIX. 'Tis the ſame thing, if, in order to ſee the whole of a Dome and Cupola, internally, we advance ſo far into the Building, that the repreſentations of ſuch Subjects are only fit for horizontal Pictures, or Cieling-Pieces; for, when they are repreſented on a vertical Picture, they appear (at a proper Diſtance to take in the whole) as if falling, or not upright. Yet are theſe things to be met with in Pictures and Prints, by Men of ſome diſtinction in the Arts.

That the Perſpective Repreſentation of the inſide of a fine Building is more difficult to manage than the exterior is certain, and is manifeſted in theatrical performances; which, being repreſented on ſeveral detached Planes, it is impoſſible, by the Rules of Art, to make them correſpond in every Point of View; but, they are very rarely connected in any one Point. There are, undoubtedly, many fine Performances of the kind; yet, without attempting to diſparage their Authors, I am confident, that, were they better acquainted with Perſpective, they would produce better and more natural Repreſentations; even that famed Scene in Cymon is a jumble of inconſiſtencies, both in point of Deſign and Execution; although it was a bold attempt out of the common mode of Repreſentations.

Few Artiſts have made Perſpective ſo much their Study, to know how to proportion one part to another, on detached Scenes, ſo, as to make the whole unite in the proper Point of View, whether the Repreſentation be internal or external; indeed, from the conſtruction of the Theatres, it is ſcarce poſſible to be done; nor are the Rules of Perſpective ſufficient, for the purpoſe, without a tolerable knowledge of Lines, geometrically. It is the leaſt qualification of a Scene Painter to be excellent in Landſcape, in which a ſmall knowledge of Perſpective is requiſite; but, in order to execute Deſigns in Architecture with correctneſs, and a juſt proportion of the ſeveral Parts, requires a thorough knowledge of Perſpective. Is it not ſurprizing, that all who are concerned, or any way engaged in Scene Painting, do not make Perſpective their immediate ſtudy; being the baſis, the very ſoul and exiſtence of their Profeſſion? yet, to my certain knowledge, ſeveral Artiſts, employed in it, are not only totally ignorant of it, in Theory, but they are, almoſt, wholly unacquainted with its Rules, which, to me, is moſt unaccountable.

In inſide Views, the bounds of the Picture limit the whole, every way, which renders the operation, in ſome caſes, more difficult than external Views.

In order to ſhew the inſide of a Room, Temple, &c. properly, a Section is ſuppoſed to be made; that is, the hither Wall (or incloſure of any kind) is ſuppoſed to be removed, and the whole Inſide laid open to View; ſo that, a proper Station and Diſtance may be taken, from which, the whole, or as much as is required, may be ſeen under an Angle not too large; for, to ſuppoſe that we can be within a Room and exhibit the whole, or nearly, is truly abſurd; yet it has been attempted, by thoſe who have not a juſt notion of Perſpective.

In reſpect of determining the places and proportions of Doors, Windows, &c. it muſt be obvious, there can be no difference whether they are interior or exterior; the whole of which is contained in Prob. 8th, and has been univerſally applied throughout the Work; particularly in Ex. 2nd and 4th, alſo in the 16th, 19th, and 21ſt, in finding the places of the Mutules, Modilions, Blocks, &c.

EXAMPLE XL.

How to repreſent the inſide of a plain Building; the Section, by the Picture, being parallel, to the Ends of the Buildings.

In repreſenting the Inſide of any Room, Church, &c. it is uſual to take the Station in a Line drawn through the middle of the Building, which indeed appears the moſt rational; but in ſuch caſe, in a regular Building (one ſide being a duplicate of the other) it is not ſo pictureſque, as when the Station is towards either
Fig. 1. ſide, as at S; from which, the inſide of the Room is to be viewed; by which means, either Side may be ſhewn to greater advantage, being more oppoſed to the Eye than the other.

Plate XXIV.

Published May 14th 1774. by Thos. Malton.

Plate XXV.

Published by T. Malton, March 4th 1775.

T. Miller Sculp.

Plate XXVI.

Published by T. Malton May 21st 1774.

A. Smith sculp.

Plate XXVII.

Plate XXVIII.

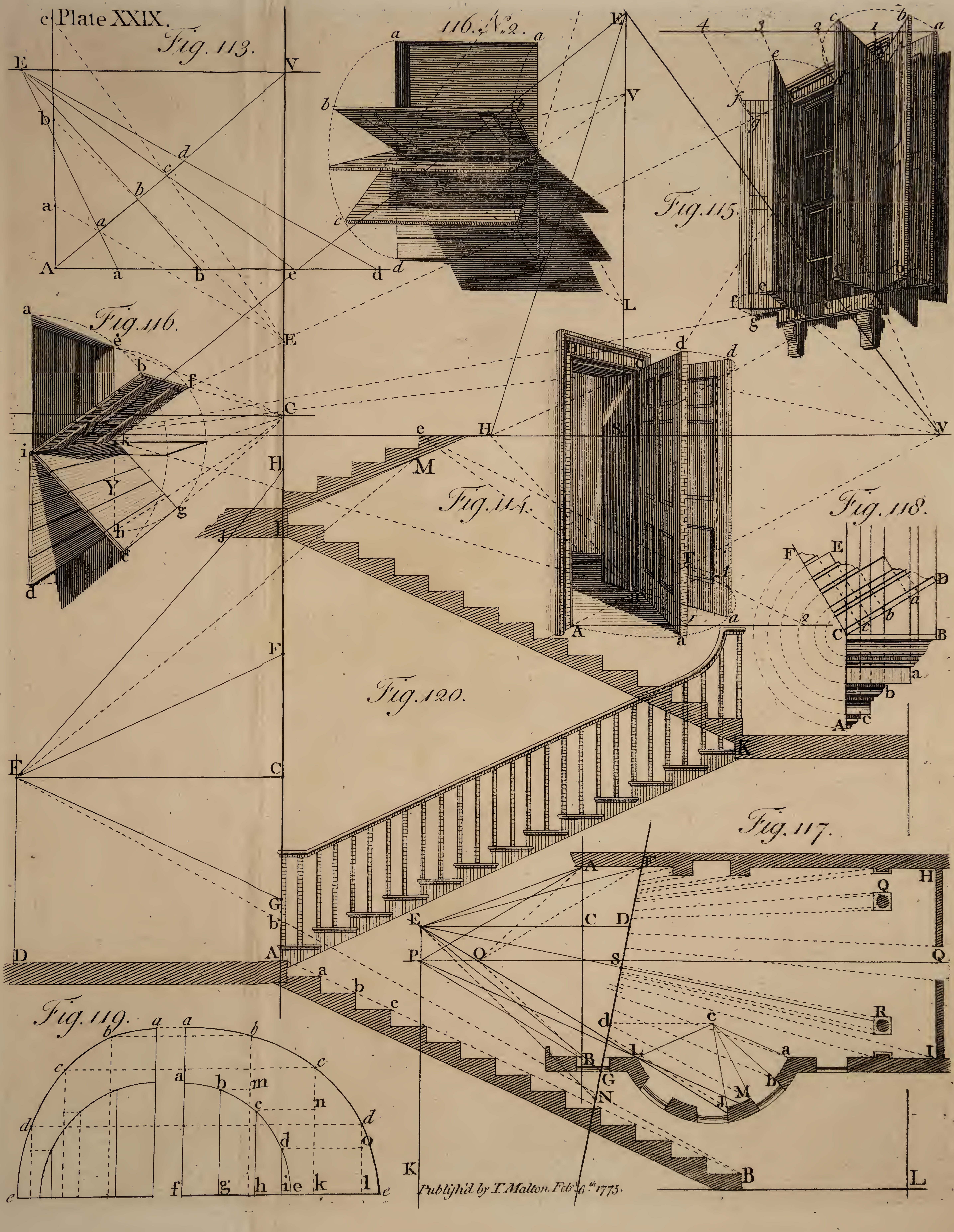


Plate XXIX.
Fig. 113.
116. No. 2
Fig. 115.
Fig. 116.
Fig. 114.
Fig. 118.
Fig. 120.
Fig. 117.
Fig. 119.
Publish'd by T. Malton Feb. 6th 1775.

No. 1. is the Plan of a Prifon; on the left hand are Doors into feveral Cells, &c. and on the right are Doors and a Window, into the Area, or open Yard; at the farther end is a paffage, at R, leading to the Keeper's Apartments, and, at the End is a Stair-cafe; a few Steps of which lead, through a Door, to the Courts of Juftice. V, U, &c. are Piers fupporting the Roof. The Elevation, and Sections, are fuppofed to be underftood. Plate XXX. Fig. 1.

Let AB be the place of the Picture; by which, a fection of the Building is fuppofed to be made, and the hither Wall removed; fo that, the whole of the Infide, beyond it, being open to view, is intended to be reprefented on the Picture AB, from fome determined Station, as at S; as feen through the Picture, being fuppofed a tranfparent Plane. SP, SQ, &c. may be fuppofed Vifual Rays cuting the Picture, which give the true place of each part of the Building thereon; indicating alfo, what Parts can be feen and which not, being obftructed by the Piers.

Let AB be produced to b, ab is the Ground Line of the Picture; but for the conveniency of making a correct perfpective Plan, take AB at a proper diftance, below; in which, let A B be taken equal AB (No. 1.) or in any other proportion to AB; it is, here, taken double, or twice.

Let the Horizontal Line (ECG) be drawn, at a proper diftance from ab, equal to the height of the Eye above it; in which, take DC double AC (No. 1. where a perpendicular, from S, cuts AB) C is the Center of the Picture; alfo, make CE double SC (No. 1.) E is the Eye, or Point of Diftance, by which the feveral parts of the Picture are proportioned; C D and C F are each half the Diftance. HI, drawn through C, is the Vertical Line of the Picture; which determines the fituation of the Eye, in refpect of the Building.

Let the perfpective Plan be drawn, making I*A*, I*B*, &c. each double the meafures in the Plan, for the thicknefs of the Piers, and their diftance from the Station Line. How it is perfected, it would be impertinent to explain; the Point E being ufed, where the meafures *Cb*, *Cc*, &c. are the full meafures (that is, double thofe at ab, bc, &c. in the Plan, No. 1.) and, when it is more convenient to apply the half meafures, make ufe of the Point D, or F, on either fide, as occafion requires; as A *D*, for the Door at P; the full meafure being applied from A to *b*, gives the fame Point p, as the half meafure, at *D*.

A*Q* is equal to AQ (No. 1.) which, by drawing a Line to D, cuts AC at *Q*, the place of that Corner, in the perfpective Plan; making A*Q* reprefent twice AQ (No. 1.) feen at the Diftance CE; or equal AQ, at the Diftance CD.

AC, or *C*C, being proportioned at b, c, d, &c. perfpectively, for the feveral Piers, &c. Lines parallel to AB, being drawn, determine thofe on the other fide, at g, k, &c. and againft the Wall, at n, o, &c. by which means the Plan is compleated, in Perfpective, from which the Picture is delineated, as follows.

The meafure, on AB, being transfered to ab (that is, refpecting the fpace between the Wall and the Piers, the thicknefs of the Piers, and diftance between them, in front) draw aC, c*C*, &c. and, at *A*, draw *AB* perpendicular, the vertical Section of the plane of the Piers, on the right hand, with the Picture; which, being fartheft from the Vertical Line, is beft, for proportioning the heights of the Piers, Arches, &c. which are fet off, from *A*, at a, b, c, &c. according to known, or determined, meafures of their heights, in proportion to the Plan.

Draw *A*C, aC, bC, &c. and, from the Plan below, draw perpendiculars from g, i, &c. cuting them, at *g*, *i*, &c. Perpendiculars being drawn from all the other correfponding parts in the Plan, as l, m, n, o, &c. parallel Lines drawn through *g*, *f*, &c. determine the true places of all the Plinths, and Borders, at the tops of the Piers, from which the Arches fpring.

The Arches, on each fide the Avenue, in front, are Semicircles; wherefore, their Centers being obtained, at S, by bifecting the Diameter RT, they are readily defcribed; and, by drawing Lines, from S, on each fide, to C, the Centers of all the receding Arches are determined, in thofe Lines.

Plate XXX.

In the returning Fronts, the Arches are ſemi-Ellipſes; their heights are the ſame as the other, determined on *AB*, at d; Cd being produced to the crown of the Arch, at *D*, obtained from below, at N, projected. For, ſeeing that the Picture, on A B (No. 1.) is nearer to the Piers than the Wall, conſequently, it does not cut the Arches in the middle; wherefore, all the Wall, with the eliptical Window, on this ſide of *AB*, projects through the Picture and is projected again to it. *C* is the Center of that Arch; the Curve, *Def*, is determined after the ſame manner as if they were circular, it being obvious, that the ſame lengths, from *i* to *k*, or from *f* to *l*, between the Piers, may repreſent any length of the Indefinite Repreſentation, *A*C, according to the diſtance of the Eye (Cor. 5. Th. 12.) *A*C remaining the ſame. Hence it is undeterminable whether thoſe Arches repreſent Circles or Ellipſes, of any proportion, the Diſtance of the Picture being unknown; for the two middle Arches are more excentric than the other two, ſeeing they have the ſame height, and the Piers are wider aſunder*.

The eliptical Windows, over the Arches, are deſcribed after the ſame manner; as the firſt on the right hand, half a Window, inſcribed in the Rectangle *abgh*, indicates ſufficiently.

The groined Arches, over the ſide Avenues, have an appearance of difficulty without the reality. If a Line be drawn through the crown of the Arch, at c, to C, and, diagonal Lines being drawn between the corners of oppoſite Piers, in the perſpective Plan, as r t and m s, interſecting at u, from which, a perpendicular, being drawn, cuts cC at *c*, the center of the Groin. Or thoſe Diagonals may be drawn, at the tops of the Piers, and having drawn perpendicular Ordinates to the circular Arch, at *A*, *B*, they may be transfered, to the Diagonals; where, perpendiculars being drawn, and, to C, Lines drawn through the points a, b, c, &c. in the front Arch, cuting the diagonal Ordinates at *a*, *b*, *c*, &c. through which the Curve may be accurately deſcribed, by encreaſing the number of the Ordinates at diſcretion.

The Arches in the middle Avenue are alſo ſemi-Ellipſes, geometrically deſcribed, being parallel to the Picture; the tranſverſe Diameter of the firſt is *FG*, and *CD* half the Conjugate. Having determined ſeveral Points, 1, 2, 3, at equal diſtances, or otherwiſe draw the Ordinates *o* 1, *A* 2, *B* 3 parallel to *CD*; from which Points, *A*, *B*, *C*, &c. draw Lines to the Center, giving the Diameters of the other Arches, *HI* and *KL*, cut in the ſeveral Points a, b, c; *a*, *b*, *c*; at which Points, Ordinates being drawn, and, from 1, 2, 3, and *D*, Lines drawn to C cuting them at 1, 2, 3, through which thoſe Curves may alſo be deſcribed.

The Apertures through theſe Arches, being Circles parallel to the Picture; their Centers being determined, and their Diameters, are only neceſſary.

The large Beam, at O, is in the middle of the opening, at MN, and tends to the Center, on which the Joiſts reſt, which decline to the Wall on each ſide, geometrically; their meaſures and diſtance from each other being known, are readily determined, by Prob. 10, drawing a Line through *B*, parallel to MN.

The Stairs, at the far end, are managed by Example 8, in front; the returning flight being parallel to the Picture are inclined, geometrically, to the Horizon. The front Steps may be proportioned to the opening between the Piers; as at Y and Z, in the geometrical Plan; the Hand-rails, of which, tend to a Vaniſhing Point, in the Vertical Line, at H; as the Kirbs in Example 8th. The Door is geometrically proportioned, on the Steps.

The middle Avenue of this Building, being loftier than the Sides, the Repreſentation is limited by a Section of the Walls, and Timbers of the Roof, over the Sides, geo-

* At No. 2. is half a circular Arch, that is, a Quadrant, or fourth part of the circumference of a Circle, in a Square abcd; c is the Center. *abcd* is a Rectangle circumſcribing a fourth part of an Ellipſis, of the ſame proportion as the middle Arches to the circular ones at each ſide; in which, it is obvious that, the Diagonals, ac and *ac*, cut the Curves at the ſame height, at e and *e*; and if the circular Curve be divided into more parts, at 1 and 2, thoſe being transfered by parallel and perpendicular Ordinates, cut the Diagonals of both in the ſame proportion; and conſequently they give correſponding Points in the Curve.

metrically drawn; the lower part is bounded by the limits of the Picture in length which, on account of the Station not being in a central Line, has its Center towards the left hand, out of the middle; by which means, it becomes diſtorted, on the right, being extended too far from the Center; by which the optic Angle is enlarged, and the Viſual Rays cut the Picture too oblique, as SB in the Plan (No. 1.) in which caſe, no more ſhould, properly, be taken into the Picture than the Piers, the optic Angle A S f, being about 50 Degrees.

EXAMPLE XLI.

Is the Inſide of a Church, in the ſame poſition, as the foregoing Example.

At Figure 2, No. 1, is a half Plan of the Building, as far as the Section by the Picture, on AB. The Piers ſupport the Roof as in the laſt Example, having Columns on them, of the Ionic Order, with an Entablature all around the middle part of the Building, and a coved Cieling. At the far End is a ſemi-octagonal receſs, in which is the Altar and communion Table.

It would be wholly ſuperfluous to form a perſpective Plan, as in the preceding Example, or to fix the Station, with all the other Preliminaries to the Ground Plan; ſuffice it to ſay, that the ſituation of the Eye is nearly the ſame in reſpect of the Building and Picture. C is the Center, and CE half the Diſtance by which the whole is proportioned; the full Diſtance cannot be contained within the Picture, on either ſide.

The places of the Piers, the Windows, &c. it is obvious, are obtained here, as in the foregoing, by applying the half meaſures (that is, the full meaſures of the Plan) to the Ground Line, AB, ſufficiently correct, inſomuch that, an extra Plan would be unneceſſary. The Piers are ſquare, having a Sub-plinth with a Baſe Moulding, and Impoſt at the top; on which is a regular Pedeſtal, which forms the front of the Gallery, receding ſomewhat from the Pedeſtal; all which need no other deſcription. The Picture is ſuppoſed to cut the Galleries, by which, the Seats, &c. are laid open to view; the Section is drawn geometrically, from which, all the Seats and Backs tend to the Center of the Picture.

The Columns, are ſuppoſed to be planed at the bottom of each Pier; but, if a central Line be drawn (as KL) from the middle point of each, and the height of the Column be there aſcertained, all the reſt are determined by Lines drawn to the Center, giving each its proper Diameter, in proportion to its height.

The Baſes of the Columns, being above the Eye, are curved contrary to the uſual order when below it; the Curves are ſtill concave towards the Vaniſhing Line. The Capitals are according to the modern Ionic, in Example 29.

The receſſes of the lower Windows, on the Floor, being leſs than the window Frames, cannot be a difficulty; the ſloping part makes up the difference. The Windows above are perpendicular over the other; the heights of which are determined, on the vertical Interſections of the Walls, AD and BG; alſo the Gallery, and other parts of the Building.

The Nich at the far End, with the Doors and circular Windows over them, are all parallel to the Picture, conſequently they are all geometrically proportioned, in front. The receding of the Nich, that is, the perſpective widths of the Faces within, are beſt determined from a perſpective Plan, formed below, the figure being determined; or, if the whole be drawn geometrically, and the Station fixed, as in the foregoing, Viſual Rays may be drawn, cuting the Picture and giving their proportions thereon.

The Mouldings in the Impoſt, runing around it, have their Vaniſhing Points, for two Faces, in the Points of Diſtance. The middle Face is parallel, and in one, ſeen on the right hand the Lines tend to the Center. The Compartments in the Sides and in the Head are beſt deſcribed by inſpection; the Lines, forming the Head, are drawn after the ſame manner as in an octagonal Dome, ſeen externally, as deſcribed in the 35th Example.

The

Plate XXX. The semicircular Heads of the upper Windows have been frequently described; those below have a flat Arch, which is but little seen, by reason of the spherical recesses in the Cieling of the Gallery, which are represented by semi-Ellipses, in the Plane of the Cieling.

The Entablature, supported by the Columns, is formed by means of geometrical Sections, at X and Z; the true place being acquired from a Plan below; or from the Ground Line and vertical Intersections, at H and I.

The Profile of the Mouldings, in the Entablature, on one side, with the large Cove, and a smaller on the other side, being drawn, it would be impertinent to describe the delineation of every Line from each Angle to the Center of the Picture, and how to determine the mitre Angles at the End, where they return, over the Arch; for which, see Example 16, or 19.

For the large Cove, draw Lines from M and N, the internal Angles of the Building to C, cuting Perpendiculars from the Angles of the Frize, or the Pilasters, below the Capital, at O and P; from which, draw Lines to the Eye, at the full Distance on each side, and produce them; then, Lines drawn from Q and R, to C, will cut those Diagonals at S and T, and determine the returning Line or Moulding, which bounds the plane Cieling.

The mitre Angles of the Cove may be truly projected by means of Ordinates from various Points (as *a* and *b*) in the geometrical Curve; but, as it comes so small, at the place, it may be done correctly enough, without that process; taking care that it does not curve too much, but leads truly into the Perpendicular and Diagonal, both which are Tangents to the Curve, as R N and N Z, in the geometrical Section, and must never cut it.

The Compartments, in the Cieling, are but simple Figures, as Rectangles, with Ellipses inscribed, (see No. 1.) which are managed the same as Circles, as it has been shewn in the last Example, in respect of the Arches.

The recesses in the Cieling, over the Galleries, are no more than a repetition of the same thing by a less Scale, having only an Architrave below the Cove, which returns at every Beam from the Columns to the Wall; whose mitre Angles are determined by the same means, from a Profile of the Cove, at G.

Thus, without the real process, I have described every necessary step to be taken in the delineation of this Object; for, as the last is, in respect of its Plan, the same kind of Figure, the same kind of Process will answer for both, in the general parts; and, for particular ones, the Examples refered to give particular Instructions.

EXAMPLE XLII.

Is the representaticn of an elegant Room, having a large, circular, Bow Window, in the Side, and a Cove-Cieling, inclined to the Picture.

In the two former Examples, the center of View, not being in the middle of the Picture, it may be imagined is wholly owing to the Station being towards one Side more than the other; but I shall shew, here, that it is not; for, let the Station be taken where you please, the Point of View, or rather the Center of the Picture may still be in the middle, and yet, take into the Picture as much of the Object; a circumstance extremely simple in itself, though to some it may appear a mystery.

In Plate 29, Fig. 117, is a Plan of the Room intended to be delineated. Let E be the Station determined on; from which, if Right Lines are drawn to F and B, the extreme on each Side of the Room, intended to be delineated, FEB is the Optic Angle under which it is seen; and, if AB be the Intersection of the Picture, EC, perpendicular to A B, determines its Center, at C, and EC is its Distance; in which case, the Center of View is not in the middle of the Picture. But, if the Optic Angle be bisected by the Right Line ER, then, FG, perpendicular to ER, is the true Position of the Picture, S is its Center, and ES its Distance; and the farther End (HI) is consequently inclined to the Picture; as it is represented in the 31st Plate. See Figure the first.

Let AB be the Ground Line, which is not neceſſarily the bottom of the Picture; but, whatever falls on this ſide is ſuppoſed projected to the Picture.

Draw AD and BK, perpendicular (the vertical Interſections of the Sides) on which, ſet up all the meaſures of the heights (by the Scale of Proportion) for the Dado and Mouldings, the Windows, Chimney Piece, the Entablature, Cove, &c.

On AD, deſcribe, geometrically, the true Profile-Section of all the Mouldings; as DEF for the Cornice, &c. the Lines of which (in this Caſe) not being perpendicular to the Picture, the Section of it, with the Picture, is not the true Profile; the deviation, in this, is inconſiderable. But, when the Lines are more inclined to the Picture, an Expedient for truly proportioning the Mouldings may be neceſſary.

Let ABC (Fig. 118, Pl. 29) be the Profile of the Cornice to be repreſented in the Picture.

The ſection with the Picture, being vertical, makes no variation in the heights of the Mouldings; but, according to the inclination of the Picture, to the Building, their projectures are varied conſiderably.

From each projecture a, b, c, &c. draw Lines perpendicular to BC, conſequently parallel amongſt themſelves; make the Angle BCD equal to the complement of the inclination of the Cornice to the Picture; CD will be the extreme projecture of the ſection of the Cornice, whoſe Inclination to the Picture is ECD; and, FDC is the true Section, in that poſition of the Picture; the projecture of each Moulding being taken from CD, where the parallel Lines cut it, at *a*, *b*, *c*, &c.

On the other ſide, the heights of the Windows, &c. are ſet up from B to I.

Having drawn the Horizontal Line (VL) and fixed the Center of the Picture, the Vaniſhing Point V being determined as uſual; which, on account of the inclination of the Sides of the Room, is not in the Center, as is cuſtomary in Inſide Views; the Room, or Building of any kind, being right angled.

In this Caſe, theſe Preliminaries are beſt determined by the geometrical Plan.

If the Ground Plan of any Building which we intend to delineate be drawn, truly geometrical, the Station may be determined, ſo, as to ſee ſuch parts of the Objects as we require, whether internal or external.

If it be required to ſee part of the ſecond Window in the Bow; draw a Right Line, from J, through the Angle at L, allowing ſo much of the receſs of the Window, &c. as you wiſh to repreſent. It is manifeſt, that the Station muſt be ſomewhere in that Right Line, produced; but if the View was to be central, it would be too near, as at O, for, the Optic Angle, AOB, is too large for the Eye to take in, at one View. Fig. 117.

At a further Diſtance, in the ſame central Line, at P, if PL be drawn, it is plain; that the Window cannot be ſeen at all, as it cuts the Pier at M; whereas, any where in the direction of JL, as at E, if the Station be fixed ſo, that, drawing EF and EB, the Optic Angle does not exceed 50 Degrees, it may be repreſented from that Station, without much diſtortion. Then, making EG equal EF, FG is the poſition of the Picture; and ES, biſecting the Angle FEG, is its Diſtance; wherefore S is its Center (Def. E) for ES is perpendicular to FG†, and SG is equal to SF, conſequently it is in the middle of the Picture. † 9. 1. El.

Now, E, is the Station, or Point of View, and FG is the poſition of the Picture.

Draw ED parallel to AH and BI; conſequently, D, and not S, is the Vaniſhing Point of thoſe Sides. (Def. L.)

The End (HI) vaniſhes in a Point where EK, parallel to HI, would cut the Picture, FG, produced; its Diſtance, from the Center, S, will be to SE, as SE is to SD‡, (Prob. 12.) And, if the Angle DEK be biſected, by the Line EN, N is the Vaniſhing Point of a Diagonal, or mitre Angle. ‡ 7. 6. El.

Then, if AB (Fig. 1, Pl. 31) be equal to FG, make VC equal to DS; or, in whatever ratio AB (Fig. 1) is to FG, ſo make VC to DS, and V will be the Vaniſhing Point of the Sides of the Room; that is, of all the horizontal Lines in thoſe Sides, and of all other Lines parallel to them, whether in the Cieling, or on the Floor, or elſewhere. (Cor. 1, Theorem 3.)

Being thus prepared, we now proceed to the delineation.

Plate XXXI. Fig. 1.

Draw the indefinite Lines AV and BV, and proportion them, by feting off the meafures of the feveral parts from A or B, and drawing Lines to the Diftance Point of the Eye, placed on either Side (at E, or E^2) EV being half, and VE^2 one third part of the Diftance of the Vanifhing Point V, there not being room, on the Picture, for the full meafure.

Make Aa, ab, &c. each one third of the meafures of the Originals, and draw aE^2, &c. cuting AV at *a*, &c. which give the Angle of the Chimney, &c.

For its projecture, make Aa equal to, or fomewhat more than the real meafure (becaufe, AB being inclined, it cannot be equal, but is the Diagonal of the Inclination; i. e. as CD to CB, Fig. 118) and draw aV, cuting a Line drawn from *a* to the other Vanifhing Point, at *b*. Make a*d* in the ratio of one third part of the front of the Chimney (as CD to CB, 118) and draw *d*V, cuting a parallel Line from *b* at *c*. Divide *bc* in the proportion of the Truffes, &c. and draw Lines to E^2, cuting *b*V in their perfpective Proportions.

Draw perpendiculars from *a*, *b*, and *e*; and, from all the Angles of the Cornice, &c. draw Lines to V; and return the Mouldings, at the feveral Angles of the Chimney, internally, at *gh*, and externally, at *ik* and *lm*, as defcribed in Example 19; alfo, at *no*, the length being obtained by a Perpendicular from *f*.

To defcribe every particular would be fuperfluous and unneceffary, previous Leffons having been frequently and particularly explained.

Having obtained the mitre Angle at *no*, return the Mouldings of the Entablature, at the End, by means of the Vanifhing Point of the End; or by Prob. 13.

The Angle of the Cove may be obtained, with accuracy, having made a geometrical Section, as in the laft Example, feting up its height from D, and making its projecture as CD to CB (Fig. 118) in refpect of the true projecture; then, draw a Line from the internal Angle to V, cuting the Perpendicular *fn* at *p*, and draw the Diagonal (*pq*) of the Angle, indefinite; and, from the extreme projecture of the Cove, draw a Line to V, cuting it at *q*.

But, becaufe the Vanifhing Point of that Diagonal is not determined, and is out of the Picture; from *p*, draw a Line tending to the Vanifhing Point of the End; and, where it is cut, at *r*, make *rs* equal *pr*, and draw a Line from the Vanifhing Point of the other Diagonal; which will give the Point *q* nearly *.

The other Angle, *x*, is determined by the Vanifhing Point of the Diagonal *xy* †.

After the fame manner as in the foregoing, *viz.* by drawing feveral Ordinates to the Curve, at *t*, *u*, &c. the perfpective Curve at the Angles may be defcribed, and transfered from one Angle to the other.

On the other Side, VE being half the diftance of the Eye, the meafures on the Ground Line, AB, are applied half the real meafures, for proportioning the Piers, and the opening of the Bow Window, &c. from B, to e, f, &c. BI is the height of the Windows. Draw Ih tending to the Vanifhing Point of the End of the Room (as for the projecture of the Chimney) and Bd being made equal to the recefs of the Window, draw d h; regarding the projectures of the Mouldings.

Having obtained the opening of the Bow Window at i k, which is a Segment of a Circle (by drawing eE and fE, cuting BV) find S the reprefentation of its Center (bifecting i k perfpectively, at 2, and drawing S2n to the Vanifhing Point of the End, and 3V, cuting it at S; B3 being fomewhat more than its Diftance from the Wall, *viz.* as CD to CB, Fig. 118) and draw Si, and Sk, which reprefent Radii; by means of which, other Radii may be obtained, as Sl, Sm, &c. making certain Angles with Sk perfpectively (Prob. 10, Cafe 3rd.) and thus, not only the Curve (klmn) but alfo, the true place of each Window, &c. is acquired.

The Curve at the Top may be defcribed by the fame means.

If the Lines, on the Floor, in the receding of the Windows, tend to the Center S, draw SR perpendicular, and divide it, geometrically, into the feveral di-

* This is not ftrictly true, *prs* not being parallel to the Vanifhing Line; but, being fo little inclined, the deviation is inconfiderable. *rs* is lefs than *pr*; they may be truly proportioned by Prob. 8, Cor. 1.

† This Vanifhing Point is alfo out of the Picture, being diftant from the Center as SN to SE (*Fig.* 117.) the Diftance of the Picture.

visions for the Mouldings, &c. and producing Sl, &c. to the Vanishing Line; then, Lines drawn from each division, on RS, to the respective Vanishing Point of each Jamb, will divide each Window into its several divisions of Mouldings, Sash Squares, &c. and, by the same means, all the Curves, in the Mouldings, &c. at the top, may be described with the greatest accuracy.

If the Jambs of each Window be parallel between themselves, then, a Right Line, So, bisecting the Angle lSm, &c. perspectively, will produce the Vanishing Point of each respectively, and each Jamb must, in that Case, be divided geometrically, on the Angle, into the several divisions required.

The Entablature may also be proportioned on BK, the same as on AD, on the other Side; which will be more accurate, than to depend entirely on the proportions being carried around, from the other Side; which, as they diminish so much, at the far End, are liable to error.

The places and proportion of the Columns, may be obtained by Prob. 8th, thus. The Angle *f* at the foot of the Pilaster being obtained, draw *f*j parallel to the Ground Line, which divide, geometrically, in the Proportion required; that is, *f* 1, equal j 4, is the distance of the Column, at the Plinth, from the Wall; and 1 2, equal 3 4, is the width of the Plinth of the Column; from all which, draw Lines to V, cuting the inclined Line, in which the Columns stand, in the perspective proportions of the Columns and Spaces; which may be compleated from the known proportions of the Order, as in former Examples.

If r s be taken, geometrically, the width of the Door, &c. in proportion to the Columns and spaces, Lines drawn to V will give the place of the Door, at the farther End, which may be compleated, by Example 20.

The Cieling has nothing of difficulty in it, save the Ornament, being composed of Right Lines, regularly disposed; which, from the geometrical Figure, may easily be determined. GH, may be considered and used as the Interfection of the Picture with the Cieling, on which, the divisions are geometrically disposed; from which, draw Lines to V; and the several divisions, in the length, being found perspectively, on GV or HV, Lines drawn to the other Vanishing Point will cut each Line, drawn to V, in the ratio required; by which, the perspective Figure is formed on the Picture.

EXAMPLE XLIII.

Is the Representation of the inside of the Piazza, Covent Garden, from the farther corner of the entrance into the Playhouse.

The Position of the Picture being determined, and consequently, the Inclination of the Lines, in which the Piers stand, is known.

Let C be the Center of the Picture, EV is the Horizontal Line, and V is the Vanishing Point of one Face of the Piers, found, or determined at pleasure; the other is out of the Picture, on the Left, found as usual; the Distance of the Picture is six Inches and a half, nearly*. Fig. 2.

Let AB be the Interfection of the Picture, i.e. the Ground Line, and, let S be the determined Seat (on the Picture) of the corner of the Pier, on the Ground, its Distance from the Picture being known.

Because there is not room, on the Picture, to set off its whole Distance, take CE half the Distance; and make SD half the distance of the corner of the Pier; draw DE cuting SC at a, the true place of that Corner, on the Picture.

Draw aV, the indefinite Representation of one Side of the Piers, and aY, to the other Vanishing Point, whose Distance, from C, is nine inches and three fourths,

* Let it be observed, that the Distance, here used, is too little; but, being a true Portrait of so public a Place, I thought proper to dispense with it; by representing it as it appears, from the Station determined above. See Preliminary Observations, Page 111.

Draw

Plate XXXI.

Draw a*d* parallel to AB; and having transfered the full meaſure of the Piers and Arches from AB, the Ground Line, to a*d* (by means of the Point C, or any other in the Vaniſhing Line) at *b*, *c*, and *d*, draw Lines to *E*, the Diſtance Point, of V, giving their places, at ab, and cd, &c. the reſt may be obtained to any length, required, by Example 4th. Note, a*d* may be the Ground Line, by a leſs Scale.

On the returnining Angle, they are obtained by the ſame meaſures, applied on the other Side, by the Point F the diſtance Point of Y.

Produce Va to the Ground Line, cuting it at A, and draw AH perpendicular, on which, ſet up, from A, the meaſures of the heights of the Piers, &c. at G and I; from which, draw Lines to V. Perpendiculars, from a, b, c, &c. cuting them at e, f, g, &c. give the Piers, on that ſide.

From, ae, the common Interſection, they are returned on the other Side, to the Left.

The circular Arches are all conſtructed by Exam. 21, as the one on the Left; and the Curves over them, by means of Ordinates *a*b, *c*d, &c. their heights being ſet off from e to i and k, &c. the true Curve being determined (ſee Fig. 119) and as many Ordinates drawn as are requiſite, they are returned on both Sides.

The eliptic Arches and Borders, from the Piers to the Wall may be thus deſcribed.

Produce Ya and Ye, and having found the Point J (Prob. 8.) in the middle of the whole width; from J draw a Perpendicular, cuting Ye and Yk, produced, at l and m; m is the middle Point in the Crown of the Arch.

Draw lV and mV; and, from the Vaniſhing Point Y, produce Lines through g, j, &c. from each Pier at the foot of the Arches, cuting lV at n and p, &c. from which, draw Perpendiculars, cuting mV at o and q, &c. the height of each Arch, reſpectively; as lm, no, and pq.

Then, by means of ſeveral Ordinates, as *ab*, *cd*, &c. perſpectively found, on the Baſe Line of the Arch, at *a*, *c*, *e*, &c. (ſee Fig. 119, Pl. 29, for the geometrical conſtruction) their ſeveral heights are ſet off on ek, and projected, by the Vaniſhing Point Y, cuting Perpendiculars from *a*, *c*, *e*; by which means, half the Curve of the firſt Arch is deſcribed, as e*bdf*m. The other half is determined by the ſame means, and all the other Arches after the ſame manner, by drawing Lines from the Seats, *a*, *c*, *e*, &c. of the Ordinates, to V, cuting the Baſe Line of each Arch, as gn, at 1, 2, 3, &c.; from which, Ordinates being drawn, and Right Lines from *b*, *d*, *f*, &c. to V, cuting them at 4, 5, 6, &c. then, through their Interſections, the Curve g5ovu is deſcribed.

Having obtained all the Arches, or Borders, the Groins are next to be determined. The middle Points, r, ſ, t, &c. are in the Line mV (allowing ſomewhat for the thickneſs of the Border) and by means of horizontal Lines, whoſe Vaniſhing Point is V, drawn from *b*, *d*, *f*, &c. their lengths, from thoſe Points, as *bg*, *dh*, *fi*, &c. may be determined, by the Point *E*, as in all other ſimilar Caſes whatever. Or, perhaps more readily, by perpendicular Lines from the Diagonals, fu and gx, (at the foot of the Arch) where they are cut by Lines drawn from *a*, *c*, and *e*, to V, cuting the former at *g*, *h*, and *i*, reſpectively.

The Plan of the Plinth of the Column, being determined, from its known meaſure and place (by its Seat on the Ground Line, or otherwiſe) there is nothing ſingular in its conſtruction; ſave the Blocks or Ruſtics (W) which are each equal to a Diameter, in height; the ſpace between is the ſame; they project equal to the Plinth of the Baſe. Y and V are the Vaniſhing Points of the horizontal Lines, in the Cornice, &c. The Picture is ſuppoſed to cut the Column, conſequently its full meaſures are applied, on BL or ſomewhat more, ſeeing it is projected.

The Piazza, on the right-hand, is ſeen to the End; and, through the Arch, at K, is ſeen the front of the next, on the other ſide of James Street.

The diſtant view of the Church and adjacent Buildings, being ſo little ſeen, are beſt ſketched from the place; as it would be attended with unneceſſary trouble to find their places, &c. perſpectively, from their true geometrical proportions and poſitions. The Ruſtics, in the Piers, are a Scale for proportioning them.

EXAMPLE XLIV.

Is the Representation of a Staircase, internal; shewing the descending Stairs, direct. Plate XXXII.

This Lesson, is intended, not merely as an Inside View, but in order to shew that a descent may be represented by the Rules of Perspective, on the same Principles as any other Subject, whatever; in which I have reduced to practice, what was heretofore treated on (Art. 8. Sect. 6. B. 1. P. 97) respecting a down-hill Representation.

In this Example the Window Side is parallel to the Picture, and consequently, the Sides of the Staircase are perpendicular to it; therefore, the Center of the Picture is the Vanishing Point of horizontal Lines, in the Sides.

C is the Center, and HL the Horizontal Vanishing Line; and VL, the Vertical Line, is the Vanishing Line of the Sides of the Staircase.

The Vanishing Lines of the ascending and descending Planes, i. e. of the Stairs, are determined by Prob. 2, making CE equal to the Distance of the Picture, and the Angle CEF equal to the inclination of the Stairs to the Horizon. Make CG equal CF, and through F and G, draw Lines parallel to the Horizon; one is the Vanishing Line of the ascent, the other of the descent; and because they are, in this Case, parallel, the Vertical Line, VL, is common to them all †, and consequently, F and G are the Centers of those Vanishing Lines, respectively, and their Distance is EF, equal EG. (Theo. 7, and Def. 20.) † Theo. 7. Cor. 2.

As this is a circumstance which has occasioned some disputes amongst Artists, I shall display it in the best light I can, and doubt not I shall do it satisfactorily.

In order to which, let Fig. 120 represent a section of the Staircase, geometrical, by half the Scale of the Picture; AB is the descent, from the Landing, at AD, on which the Spectator is supposed to stand, at ED; E is the Eye or Point of View, and EC is the Distance of the Picture*, of which AH is a section. AK is the flight of Stairs, immediately ascending, as AB is descending, and KI is the under side of the next flight, over them, parallel to AB. Fig. 120.

Now, because the Vanishing Point of every original Right Line is where a parallel from the Eye cuts the Picture ‡, and EC the Direct Radial, produces the Center, C is the Vanishing Point of the sides of the Half pace BL, and of the extremes of the Steps, as may be seen in the Picture; also, EF, and EG, being parallel respectively to the Ascent, AK, and Descent, AB, consequently, F and G are the Vanishing Points of all Lines, on the Picture, parallel to them (as the Hand-Rails, the Wainscoting at the Sides, &c.) and consequently, Planes passing through the Eye and those Lines, respectively, parallel to the Stairs, must cut the Picture in the Lines IK and MN passing through F and G, respectively, and therefore they are their Vanishing Line.. (Def. G.) ‡ Def. 22.

This I presume is intelligible and clear; and, if Visual Rays are drawn to the several parts of the Staircase, they will determine what can be seen and what cannot; as EB determines how much the Descent rises on the Picture, which it cuts at b; Ab is, therefore, its whole apparent width. The ascending Flight, parallel to AK, it is evident, is seen on the under side, as the Visual Ray EM evinces; and consequently, it descends on the Picture.

Let AB be the Intersection of the Picture with the landing of the Stairs, which, in this Case, is the Ground Line, the Picture being supposed close to the Stairs; all that lies on this side is projected to the Picture; as the Door on the Landing, and Cieling, as well as the Floor; and it is obvious, that, whatever appears to descend, below the Landing, has its place on the Picture above the Ground Line; consequently, it rises on the Picture.

Make BD equal to the known width of the Stairs, the width of the whole, being represented by AB; the Station is determined by the Vertical Line, VL.

* The Distance of the Picture, it must be observed, is too little, for taking in so much as is here represented; but it is manifest, that, if the Distance was half as much more, or one third lower, we should not see the descending Stairs at all, as the Eye would be either in the Plane of the Stairs, or on the other Side.

Plate XXXII. Draw BG and DG, to the center of the Vaniſhing Line MN; the Originals of thoſe Lines being perpendicular to the Interſection, AB; in which Lines are all the upper edges, or noſeings of the Steps; whoſe places are obtained by Prob. 8th.

Make G*E* equal EG,* the Diſtance of the Vaniſhing Line MN, or IK (Def. 20) Make Ba, ab, &c. double Aa, ab, &c. (Fig. 120) and draw a*E*, b*E*, &c. cuting BG at *a*, *b*, *c*, &c. giving their places on the Picture; and parallel Lines, *ad*, &c. being drawn, give the edges of the Steps; the Ends, being at right angles with the Pictures, vaniſh in its Center; therefore, draw C*a*, C*b*, &c. and produce them to the next Step; and thus the deſcending Stairs are compleated.

The Half paces, below, at X, and above, at Y, being horizontal; their Sides, conſequently, vaniſh in the Center; alſo, the ends of all the Steps, every where. The width of the Half pace being equal to the Steps, draw *e*C and *f*E, cuting *e*C at *g*, which gives the Angle *gh*, at the farther end of the Staircaſe.

The height may be determined on a Perpendicular from B, as BJ, and drawing JC.

The aſcending Flight is managed by Example 9th in every reſpect. The Vaniſhing Point of the Hand Rails, Wainſcoting, &c. of that Flight is at F, as well as of the Lines de and fg, of the Plane defg, being parallel to thoſe below; the other, with the Lines in the Plane Z, is at G.

The height of the Hand Rails, &c. are ſet up on the Interſection BJ, as at h; or from D, as Di, allowing for the ramping Curve, equal ik.

The panneling of the Wainſcot is divided in the ratio required, as in any other Caſe, by Prob. 8th, making uſe of the Diſtance EG (not EC) for the inclined part.

The Windows may be proportioned on *gh*, at the farther end, geometrical.

The Cornices, and other Mouldings, are managed by the Leſſons in Sect. 7.

The Architrave around the Door, being projected, is thus done.

Make Bl, lm, equal, reſpectively, to its diſtance from the Steps, and width of the Architrave, and draw El, Em, projecting thoſe meaſures to CB, produced, cuting it at *i* and *k*, and giving *ik* for the Architrave.

The Cieling is projected in the ſame manner; and all beyond BP.

AOPB is the Section of the whole by the Picture; OP is the Interſection with the Cieling, and AO and BP with the ſides of the Staircaſe; on which Interſections, all the meaſures are applied, for each Plane reſpectively.

OF HORIZONTAL PICTURES.

Horizontal Pieces are conſidered, by many, as performed by Rules different from vertical Pictures; whereas, they are the ſame in every reſpect, being founded on an univerſal Theory, which regards no particular poſition of the Picture; but only, the poſitions of the Planes and Lines in Objects, to the Picture, and to the Eye. For, however the Picture be ſituated, the Eye being conſidered as a Point, it muſt either be in the Plane or out of it, and cannot vary in its poſition, otherwiſe; conſequently, its Center and Diſtance, being determined by a perpendicular from the Eye to the Plane, are the ſame in all poſitions of the Plane, reſpecting the Horizon, to which we are naturally partial; but, being properly conſidered, we ſhall find it of no conſequence in the Theory of Perſpective.

Let the Picture be ſituated as it may, all Original Lines perpendicular to the Picture vaniſh in its Center (Cor. to Th. 4.) Planes, or Lines, being parallel to the Picture, have no Vaniſhing Line, or Vaniſhing Point; but, in ſuch Caſe, the repreſentations of all Figures, in Objects ſo ſituated, are ſimilar to the Originals (Theo. 9, Cor. 5) alſo, all Original Lines, however inclined to the Picture, vaniſh in that Point, where a parallel Line from the Eye would cut the Picture; and conſequently, all Lines, which are parallel, amongſt themſelves, have the ſame Vaniſhing Point (Th. 3, Cor. 1.) with every other circumſtance in the Theory given, which is quite general, and cannot poſſibly regard any particular poſition of the Picture, to the Horizon.

* Suppoſe GN produced and made equal to GE.

The Reprefentations of moft Objects (in common Cafes) on a vertical Picture, are managed by means of the Horizontal Vanifhing Line, only, but, in many Cafes, other Vanifhing Lines are requifite.

A horizontal Plane, paffing through the Eye, which, in other Cafes, produces the horizontal Vanifhing Line, is, in this Cafe, parallel to the Picture, and confequently cannot cut it; for it is the Directing Plane, of Pictures in fuch pofition. (Def. 4.) The theoretic conftruction of the elementary Planes is the fame, however the Picture be fituated; all the difficulty feems to be in fixing the Vanifhing Lines, in this Cafe, having no particular bias in refpect of the Horizon.

As, in all common Cafes, the Horizontal Line and Vertical Line cut each other, at right angles, in the Center; fo, in this, two Lines cuting each other, perpendicularly, in the Center, anfwer the fame purpofe, either of which may be confidered as the Horizontal Vanifhing Line; for, if the Picture be changed to a vertical pofition, and, the Objects being fuppofed changed with it, keeping their pofition to the Picture, it is then a common Cafe, in every circumftance; the placing of thofe Lines, moft conveniently, is therefore the principal bufinefs.

Now, if the Piece be viewed centrally, it is immaterial how they are drawn, for reprefenting a Dome only; but, if we would reprefent the Infide of any prifmatic Object, whatever, it is moft eligible to draw the Vanifhing Lines parallel to its Sides, as AB and DE; becaufe they are, then, the Vanifhing Lines of thofe Planes, refpectively. But, if the View be not central, then, one of the Lines (DE) generally paffes through the center (C) of its Bafe and divides the Dome equally; which may be confidered as the Vertical Line of the Picture; the other (FG) as the Horizontal Line; S being the Center of the Picture, whofe Diftance is known. Fig. 121.

The chief difficulty, in Cieling pieces, is the reprefenting Objects in fuch uncommon Pofitions, they being feen on the under fides, inftead of looking direct at them, as ftanding before us, upright, in common Cafes; and are reprefented as if lying horizontal, in refpect of a vertical or upright Picture; in which, the difference confifts, wholly. The Cielings, Soffits of Doors and Windows, Plancceers of Cornices, &c. are, in this Cafe, parallel to the Picture, as, in other Cafes, they are perpendicular to it; being thus reverfed, and feldom practifed, makes the difficulty greater.

There is one particular circumftance attending Cieling pieces; the Center of the Picture is the only Vanifhing Point, generally; not owing to the pofition of the Picture, but of the Objects to be reprefented. For the Bafe of the Dome, or whatever elfe is reprefented, is always parallel to the Picture; and confequently, all Lines perpendicular to the Bafe (being alfo perpendicular to the Picture) vanifh in its Center; and, fince the Objects to be reprefented are always upright, there is no occafion for any other Vanifhing Point; as, all Right Lines, in the Objects reprefented, are either parallel or perpendicular to the Picture. Nor can it be otherwife, unlefs reprefented on an inclined Cieling; in which Cafe, there may be as much variety of Vanifhing Lines and Points, as in any other.

Thefe Preliminaries being well confidered and digefted, the difficulty will vanifh, and leave the whole, fubject to the fame common Rules already laid down; for there are no other ufed, or neceffary.

EXAMPLE XLV.

How to reprefent on a Cieling, or horizontal Picture, the reprefentation of a Gallery of Communication; with a Dome and Cupola, as feen from below.

ABCD is a geometrical half Plan of the Gallery, &c. to be reprefented, which is a Square, fuppofe of 30 feet; and BFHC is a Section of the fame. The height, BF, to the Cove, is 18 feet, the Cove 3 feet, and the Bafe of the Dome (which is a Hemifphere) is 24 feet. Through the Arches, in the middle of three Sides, Galleries are fuppofed to communicate to various Apartments, above. Fig. 122.

The

Plate XXXII. Fig. 123.

The Baſe of the Gallery being a Square, and the Picture horizontal, conſequently parallel to its Baſe, its repreſentation is therefore a Square.

Deſcribe a Square, AHIB, of the dimenſions you intend the Picture, which is here, double that of the Plan.

The Station being determined, and the Diſtance of the Picture, which, in this Caſe, cannot admit of much variation, from the true, to any Eye which views it. For, I ſuppoſe, every Cieling Piece is calculated to be ſeen from ſome particular Station, below; and every Perſon who would ſee the Piece, truly, muſt ſtand in the ſame place; conſequently, the difference, in Diſtance, is equal to the different heights of the Spectators.

Suppoſe the Hall, or Saloon, to be a Cube of 30 feet; then, allowing 5 feet for the height of the Eye, its Diſtance from the Cieling is 25 feet, which is the Diſtance of the Picture; upon which, is to be repreſented, a continuation of the Walls, &c. with the Entablature, from B to F, near 20 feet, with the Cove, Dome, and Cupola, covering the whole.

Now, the Diſtance of the Picture is 25 feet, which is not equal to its width, but cannot, here, be more, unleſs the Spectator be ſuppoſed to ſit or lie down. Then, as, in common Caſes, the height of the Eye is next to be determined, ſo, in this, the place of the Eye in reſpect of the Picture; which, I would never adviſe to be in the middle of the Piece, as it cannot produce an agreeable Repreſentation; ſeeing that, the longitudinal Lines in the Dome (which in that Caſe are repreſented by Right Lines) will be but ſo many Radii of a Circle tending to its Center, as aC, bC, &c. (Fig. 124) and, the latitudinal Circles will be ſo many concentric Circles, as abc, *abc*, &c. alſo, every Side of the Room will advance equally towards the Center.

Therefore, let the Station be determined ſomewhat towards one Side, where the Cieling may be ſeen moſt advantageouſly; nor is it neceſſary to be ſituated centrally, in that Side; although it is a deviation from my general Maxim, of fixing the Center of View in the middle of the Picture; but, ſince the poſition of the Picture cannot, in this Caſe, be varied, I had rather diſpenſe with it than have duplicate Repreſentations.

Let C, then, be the Center of View; ſomewhat to the right hand. Let AB be conſidered as the Ground Line; and if, through C, a Right Line, EF, be drawn, parallel to AB, it may be conſidered as the Horizontal Line of the Picture; and DG, perpendicular to AB, alſo paſſing through C, is the Vertical Line, i. e. they are the Vaniſhing Lines of the Sides of the ſquare Room reſpectively.

The Galleries, on three Sides, which are ſupported by Truſſes from the Walls, are firſt to be repreſented; which I have made to project but three feet, as they would hide the Walls too much if they projected more.

Now, ſince the Picture is parallel to the Truſſes, and they are ſpaced equally, divide the three Sides AH, HI, and BI, geometrically, into as many ſpaces and Truſſes as are to be repreſented, at X, X, and deſcribe their Seats on the Picture, truly geometrical. Then, the Lines which meaſure their thickneſs or height, being perpendicular to their Baſes, and to the Picture, vaniſh in its Center, as in all common Caſes whatever.

Make CE equal to the Diſtance of the Picture, in the Vaniſhing Line EF; draw the indefinite Repreſentations, aC, bC, &c. and proportion them in height, making a*a* equal to the height, and draw *a*E, cuting aC at *b*; which, by drawing parallel Lines, to AH, HI, &c. will determine them all around.

The Cieling of the Gallery, between each Truſs, is conſequently determined. The under ſide of the Truſs is like an Ionic Modilion, having, alſo, a Cimma reverſa, around the top; which, the Figure ſufficiently deſcribes.

The Cornice of the Gallery is deſcribed as any other; ſave only, that the Facias, or upright Fillets, are contracted, in this Caſe, as the under Sides, or Planceers, in common; the difference is owing to the poſition of the Picture only, the appearance is the ſame; as it is illuſtrated in Fig. 125.

Let

Let A B be the Profile of a Cornice, feen from the Point E, by means of the Vifual Rays A E, B E, &c. If the Rays are cut by a vertical Plane (A b) the reprefentations of the feveral Angles, at C, D, &c. are where the Rays are cut by the Plane, at c, d, &c. and a horizontal Plane, B a, cuts the fame Rays at a, *b*, *d*, &c. It is manifeft, that the proportions of the Members, on tho'e Planes or Pictures, differ confiderably, yet it is obvious, that each appears the fame, in the Point of View, E; the difference, in the Reprefentations, arifing from the different Sections of the Rays, as in any other Cafe.

For the iron railing which is in the plane of the Facia of the Cornice, from each internal Angle, k, draw kC. Make Ad equal to its width, and de equal to its known height, from the Cieling (equal twice *de*, Fig. 122) and draw eE, cuting dC at f; draw fg parallel to AH, and gh parallel to HI; alfo fi, parallel to AB, cuting cC, where it joins the Wall, on the other Side.

The divifions and proportions of the Railing are determined, geometrically, on dk*, &c. at *c*, *d*, &c. from which, Lines are drawn to C.

The Gallery, with the Railing, being compleated, we next proceed with the decorations of the Walls, feen beyond or over it.

Having drawn, geometrically, the Plans, or Seats of the Pilafters, with all the projectures of the mouldings of the Pedeftal, &c. at Y, Y; from each exterior Angle, draw Lines to C, as jC, lC, &c. and from l, fet off, firft, the height to the Plinth; then, the meafures of the Plinth, Mouldings, Dado, &c. of the Pedeftal, at m, n, &c. and draw mE, nE, &c. cuting lC in their feveral perfpective Proportions, at *e*, *f*, &c. from each of which draw Lines parallel to their Seat, j l, cuting j C, and return them at right angles. The Diagonals, or mitre Angles are, in this Cafe, parallel to the Picture, i. e. to A I and B H; by which means the Mouldings are determined, every where.

The Bafe Moulding is hid by the projecture of the Plinth, and alfo the Bafes of the Pilafters, on this Side, being greatly forefhortened; which, where they can be feen, are managed after the fame manner.

From o, p, &c. draw Lines to C; make pq equal to the height of the Capital, from the Cieling, and draw qE, cuting pC, at n, and compleat the fquare of the Abacus; which being parallel to the Picture, the curviture of it is geometrical.

The Mouldings of the Capital are managed as the other (except in the Abacus, which are all circular curves, being parallel to the Picture) their meafures of height, being obtained, from pq, at r, &c. and their projectures from the Plans below, drawing Lines from each, to C.

The Volute may be determined, from its Plan, below, in refpect of its projectures; but to defcribe it, particularly, would baffle all defcription.

The Windows, in this Side, and Niches, &c. in the others, are determined geometrically, on AB, &c. the perpendicular Lines of which, vanifh in C, and the circular heads are femi-Ellipfes, defcribed, by Prob. 3, Sect. 8.

The other Sides are managed after the fame manner, from Plans below.

For the Entablature. Let M, N, O, and P, be the Seats of its projecture, on the Picture, in the Diagonals AI and BH. Draw MC, NC, &c. Make MQ equal to its height from the Picture, and draw QE, cuting MC at *n*; draw *no*, *op*, &c. parallel, refpectively, to the Sides of the Picture, *nopq* is the extreme projecture of the Cornice, around.

Make QR, &c. equal to the meafures of the Cornice, Frieze, and Architrave, with their feveral Mouldings; and draw RE, &c. giving them perfpectively, on MC. Their projectures are determined on the Diagonal, AM, geometrical; or, on Diagonals from *n*, *o*, &c. parallel to AM, &c. m*n*, &c. being divided in the fame ratio, as A M, (Cor. 2. Th. 9.)

* dk is the Interfection of the Plane dfgk of the Railing, diftant from AH, equal Ad, i. e. equal to its known diftance from the Walls; confequently, d is the Interfecting Point of that Angle.

For, becaufe the Picture is parallel to the Bafe of the whole, the fection of every Plane with it, being perpendicular, is geometrically determined; therefore, the Interfecting Point of every Perpendicular Line in the Objects, as d or k of the Angles of the Railing, is readily found, its diftance from each Wall being known, or determined; the whole Bafe being a true geometrical Plan.

3 M The

Plate XXXIII. The Square of the Cieling above the Cove is thus determined.

From T and U, the Seats of the projecture of the Cove (in the Diagonals AI and BH) draw TC, and UC; and, TV being made equal to its height, draw VE, cuting TC at *r*; draw *rs* parallel to the Cornice, and compleat the Square *rstu*; in which, inscribe a Circle; and, concentric with it, another, leaving a border around the Dome; and also, compleat the same within the Square *rstu*.

For, because the Cieling is parallel to the Picture, its perspective form is truly geometrical; consequently, one Side being perspectively found, the rest are also determined, in the ratio of *rs* to the Original.

The Angles of the Cove are determined by Ordinates, on the Diagonals (if it were necessary) geometrically proportioned, alike on each; and others, on AC, &c. perspectively; (See the geometrical construction, Fig. 126, which determines three Points, *a*, *b*, and *c*, in the Curve, equally spaced.) but, as they approach so nearly to right Lines, here, it is an unnecessary process.

To represent the circular Cornice, at the foot of the Dome.

The Frieze or Facia, below the Cornice, is perfectly cylindrical, its height is determined on AC, which represents a Perpendicular from that Angle; on which, every measure of height may be transfered from AB, geometrical, as s, t, u, &c. of the several divisions in the Dome.

From S, the Center of the Base, draw SC, in which Line are the Centers of every Circle in the Dome, &c. for, the representations of all Circles, in Planes which are parallel to the Picture, are Circles; and, because SC represents a Line perpendicular to the Base, in its Center, SC is the Axis of the Dome, and passes through the Center of every Circle in it; which being determined, on SC, or transfered from AC, by lines parallel to the Diagonal AI, as s 1, t 2, &c. draw lines through 1, 2, &c. parallel to AB.

Now vx is the Diameter of the Base of the Dome, cuting the Axis at o; and 1, 2, &c. are the representations of the several Centers, of Circles in the Cornice.

From v, set off the projectures of the Cornice, in proportion to the Radius vo, at *a*, &c. from which draw Lines to C, cuting parallel Lines from 1, 2, &c. which give the Radius of each Circle in the Cornice, as 1 *k*, 2 *e*, &c. by which the Cornice may be compleated; being composed, wholly, of Circles.

The parallel Circles in the Dome are determined after the same manner; which are necessary, for describing the longitudinal Borders.

No. 2. In Fig. 122, draw as many Ordinates parallel to its Base, GI, as are necessary, from the several points a, b, c, equally spaced in the Curve, to the Axis, or central line CH, cuting it at d, e, f. Then, make o*a*, o*b*, and o*c* in the same ratio to ov, as ad, &c. to the Radius GI; from which draw lines to C, as *a*C, *b*C, *c*C; and, from the perspective Centers, 2, 3, 4, and 5, obtained as above, draw parallel lines cuting them at *e*, *f*, *g*, and *h*, which give the Radius of each Circle in its true place; *viz.* *e* 2, *f* 3, *g* 4, and *h* 5, which, last, is the Base of the Cupola.

Divide the Base of the Dome into eight equal parts, at a, b, c, &c. so that, ah and de are parallel to AB, &c. and set off the widths of the Borders regularly from those Points (half on each Side) draw oa, ob, &c. and, from the Centers 2, 3, 4, draw lines parallel to oa, ob, &c. cuting their respective Circumferences at i, k, l, m, through which, Curves being traced, they are the central Lines of each Border *. Their widths may also be set off at each Circle, in proportion to its radius, as at the Base, by which they are compleated.

The Compartments are done after the same manner, from those central Lines, respecting the Margins and Mouldings which are longitudinal; the parallel Mouldings are in Circles parallel to the Base.

* The Eye being in the Plane of those passing through d and h, they are, therefore, represented by Right Lines; those passing through b and f, being farthest removed from the Center, are the most curved; for, being viewed centrally, they all appear Right Lines (as in Fig. 124) seeing the Eye is in the Plane of each Curve.

The

The femicircular Windows, at the Bafe of the Dome, are but little feen, and may be truly determined by Ordinates, tending to the Center, geometrically divided at the Bafe, and parallel Circles cuting them, as at *j*, *i*, *k*, *l*, *m*. The Margins about them may be done by the fame means.

The Bafe of the Cupola being obtained, as above, the Windows in it, are geometrically divided at the Bafe, and Lines drawn to the Center.
Their apparent height may be obtained on SC or AC, as the other parts.

Thus have I defcribed the whole procefs of this Cieling Piece, from the begining to the End; to fay more concerning it would be fuperfluous, as it muft be obvious, that the Rules made ufe of are the fame as have been applied in all other Cafes, and Subjects whatever.

SECTION XI.

Is applied to Houfhold Furniture, Wheel Carriages, Machines, &c.

THE foregoing Sections contain the Elements and Rules for the practice of Perfpective, in all common Cafes; I have alfo fhewn how to apply them, in familiar Leffons and Subjects, adapted to any capacity, in the 6th, 7th, 8th, and 9th Sections; and, in the laft, to particular Subjects, and in a particular fituation of the Picture; fo that, there remains nothing more to be done in refpect of the application of it, to all ufeful Subjects, almoft whatever.

In this Section, which relates to Objects in particular Profeffions, yet ufeful to Artifts, I fhall only make fome curfory remarks, in refpect of the reprefentations of fuch Objects as are here treated on. The former Leffons, being well underftood, will be found applicable to whatever Subject the Artift choofes, or has occafion for. To multiply Cafes and Subjects, in which the application of the Rules of Perfpective may have the appearance of fomething particular or fingular, would be endlefs, as, in almoft every different Object, there is occafion for fome variety in the application of its Rules; which, neverthelefs, it is obvious, are all founded on the fame univerfal Principles, though the Rules deduced from them are varioufly applied.

It is a neceffary requifite, in many Profeffions, to be able to give a Defign, to a Gentleman, of whatever may be wanted, out of the common run of things; and although a perfect Drawing may not be required, yet to give a flight fketch, with propriety, is certainly an Accomplifhment which cannot well be difpenfed with. In an Architect it is abfolutely neceffary, to give a correct Defign, but they are generally contented with geometrical Reprefentations, which does not give the true appearance and effect; that can only be done perfpectively. Geometrical Drawings to a Workman are neceffary; but, to give a true Idea how the Object will appear, when executed, from any particular Station, requires fomewhat more; to give fuch an Idea, without knowing fomething of Perfpective, requires a much greater fhare of genius than falls to the lot of many.

Next to the Architect, the Cabinet-maker and Upholder will find his account in a knowledge of Perfpective; nay, I am of opinion, that it is full as neceffary to the Furnifher as to the Builder; having almoft as large a field, and as various. There is nothing influences a Gentleman more in the favour of his Workman, when he is pleafed to want fomething whimfical and out of the way, than to take his Pencil and fketch out the Idea the Gentleman had conceived, and was big with, yet could not bring it forth without affiftance. He, who can do that, and, at

Plate XXXIV. at the ſame time, diſplay a little modern taſte, in Ornament, being known, is certain of ſucceſs, or of employ, at leaſt. I have, therefore, given ſeveral pieces of Furniture, of various kinds, for his practice, which will alſo, frequently, be found uſeful to the Artiſt, to furniſh his Family, or Converſation Piece.

The Organ Builder, and Coach-maker, may ſometimes find it neceſſary to give their Deſigns in Perſpective; a ſpecimen is given for each Profeſſion. And, laſtly, the Mathematical Inſtrument-maker, Mill-wright, or Engine-builder of any kind, by Perſpective, may be able to ſhew to much greater advantage, the ſeveral movements, and mechaniſm of his Inventions.

There are but very few Perſons, who have not had much practice, in Perſpective, know any thing more of it, than the application to Objects, whoſe Planes are parallel and perpendicular to the Picture, and conſequently, have ſeldom occaſion for any other Vaniſhing Point than the Center of the Picture; they always, therefore, adapt every thing to that poſition, without thinking about the propriety of it. Some cannot think a Drawing is Perſpective, unleſs an End of the Object is repreſented; for which reaſon, we ſeldom ſee any Object delineated in Perſpective without having an End ſeen, which, in a Building, or other Object of a tolerable length, is abſurd to repreſent, the Front being parallel.

Notwithſtanding I have endeavoured to explode the abſurdity of the Picture, being neceſſarily parallel to the front, or ſome other face of the Object, as not being pictureſque; yet, in the following Caſes, I would rather prefer it; ſuch as giving a Deſign of a piece of Furniture, &c. to a Gentleman; becauſe, the Gentleman, not having judgment in Perſpective, might be greatly deceived in his Ideas of the proportion of the ſeveral parts to each other; whereas, the front of the Object being parallel to the Picture, the true geometrical proportion is preſerved, which is moſt eſſentially requiſite, in order to be clearly underſtood.

Fig. 129. Figure 129 exhibits a repreſentation of a Pedeſtal and Vaſe direct in front; the Eye is directly oppoſite to the Ball at the top, being about the height of a Perſon ſiting; for, to ſtand, when we view ſuch Objects as are wholly below the Eye, although we ſee no imperfection or diſtortion in the Object, itſelf; yet, unleſs the Diſtance be greater than we uſually ſtand at, to ſee ſuch Objects, it will be greatly diſtorted in the Repreſentation, particularly in the lower parts, ſeeing that the Diſtance, in ſuch caſes, is frequently leſs than the height of the Eye.

C is the Center of view, for this Object, the Diſtance is CV, which is leſs than the height of the Eye; but, as the far feet, are not ſeen, and as there are no receding parts ſeen, below, no diſtortion is obvious; the Object ſtanding in little compaſs, the inſide of the feet are ſcarce ſeen at all, owing likewiſe to their tapering downwards; otherwiſe they would be much diſtorted, on the returning ſides.

AE is a Scale of Proportion, on which the heights of the ſeveral parts are ſet up, from A, to B, D, and E, for the Pedeſtal. EH is the geometrical width, by the ſame Scale. From E and H, if Lines be drawn to the Center, and FH be made equal to its depth, in proportion to the front, FV being drawn, or a Ruler applied to F and V, cuts HC at I; from which a parallel Line is drawn, cuting EC, and determines the width of the top.

The Mouldings in the front of this Object are the chief difficulty; but the geometrical figure being known, the place of the parallel ſtreight part is eaſily determined, on EH; and the curved parts, whether convex or concave, are flatter or quicker, as they are nearer to or farther from the Vaniſhing Line, and may be done ſufficiently correct by hand; indeed all attempts at accuracy, by rule, in ſuch minutias, being ſo ſmall, would be fruitleſs and loſt labour; experience and judgment are the beſt and only guides.

The place of the foot of the Vaſe being got, on the Pedeſtal, at FG, let the height of the greateſt ſwell be ſet up, from E to J; where, draw a Line parallel to the Horizon; which, if the ſwell be equal to the ſtreight part of the Pedeſtal,

2 may

may be ufed for the fide of a Square, enclofing the Circle, making KL equal to its diameter. If the Vafe fwell more, it will project on this fide KL, if lefs it will fall within it; the real meafure, in either cafe, is ftill the fame, on KL, whether it be projected larger, on this fide, or be diminifhed on the other.

This curve being determined, all the others will be more or lefs curved, as they are higher or lower; each of which muft be done by the fame means, if accuracy be required; finding the true place and proportion of each, by drawing a parallel Line, as NO, according to its height, above or below the other. The height of the top is beft determined by a central Line, and adjufting the fcale of proportion to its diftance by means of the Vanifhing Point (V) of Diagonals, and a perpendicular Line drawn through K; but, as it is, here, on a level with the Eye, it is determined by the Horizontal Line; and thus, this Object may be compleated.

Figure 130 exhibits the reprefentation of a round Pier Table according to the prefent tafte, with tapering, term Feet. Fig. 130.

No. 1, is half the Plan of the bottom, for the place of the Feet, and No. 2 is the figure of the Top, geometrical, in inlaid work; both together form a Semicircle; but, if it has not a folding Top, which, when open, form, together, a complete Circle, I would rather it fhould be fomewhat broader, than half the length; equal to the narrow margin, around; becaufe, the figure within it fhould be an entire Semicircle, which is not fo here; confequently the Figure is not fo regular.

Let the Center of view be determined, at C, and take FG equal to the diameter of the Top; as much below C as the Eye is fuppofed above the Table. The Eye is, here, fituated fomewhat to the left hand, fo that this Object is not viewed direct in front, that is, not centrally, as the foregoing.

Bifect FG, at *E*, and draw *E*C, FC, and GC, and from *E* or F draw a Line to the diftance Point (which is not here fixed) cuting GC or *E*C, at K or *D*; through which, draw IK parallel to the front Line. In fhort, complete a Semi-ellipfis (I*E*K) reprefenting a Semicircle, within the Rectangle FIKG; and, within it others, reprefenting the feveral Borders, &c. in the Top of the figure; which may be obtained by drawing perpendiculars from a, b, and c to FG, and, from *A*, *B*, *C*, draw Lines to C, cuting the inner curve of the border at *a*, *b*, *c*, from which draw Lines to *D*. On account of the inner Curve, *abc*, not being a complete Semicircle, *bD* does not fall in the Diagonal G*D*.

The Top being completed, take *FG* parallel and equal to FG, at the diftance of the breadth of the Frame, and within it, take a lefs Diameter, allowing for the projecture of the Top; on which, defcribe the curve at the bottom of the Frame, and the fmall Aftragal; from the extremes of the inner curve, draw perpendicular, to the lower curve Line expreffing the thicknefs of the Top.

For the place of the Feet, take fg below, at the diftance Ff, equal to the height of the Table, in proportion to its width (FG) and having defcribed the reprefentation of a Semicircle, of the diameter of the Frame, the place of the Feet, at i and k, are in the Angles (but let it be noticed, that, the Top not being more than a Semicircle, the curve i*xx*k is lefs, becaufe the Top hangs over, behind the Table; fo that, d reprefents the Center)

From x (No. 1.) draw perpendiculars to HE, cuting it at Z; which meafures transfer to fg, and draw Lines to C, cuting the curve, at *x*, which give the place of the front Feet. The Plans, at *x*, *x*, being completed by means of their proper Vanifhing Points, in the Horizontal Line, draw perpendiculars from each corner, to the Frame; and, within the fquare of the Foot, defcribe another, anfwering to the thicknefs at the bottom, from all the corners of which, draw Lines to the Frame, where their full dimenfions are obtained. The Plinth, at a little height from the bottom, is equal to the thicknefs of the Foot, at the Frame, the Lines of which tend to proper Vanifhing Points, as the Plans, below.

The Compartments in the Frame, and the margins of the Feet, &c. it would be trifling to defcribe, being divided in the middle, and at each Foot; or otherwife, at pleafure, from the Plan, below; the Figure muft fupply the reft.

Plate XXXIV. Fig. 131. Figure 131 reprefents a Chair, direct in front; C is the Center of view, the Diftance is CE. GH is the Ground Line, in which, take G and H for the places of the fore Feet; where, make their Plans, and defcribe the front, GABH, truely geometrical.

Find V and X, the Vanifhing Points of the Sides, AF and BD, refpectively (Prob. 2.) and having fet off, from V, the diftance of that Vanifhing Point, on the Horizontal Line, make B*a* equal to the depth of the Seat, and draw a Line from *a* to the Eye, cuting BV at D, and draw FD parallel to AB, cuting AX which gives the Figure of the Seat, in the naked Frame; but being ftuffed, they muft be gently curved at the fides and behind, as on the left fide.

The place of the back Feet, on the Floor, muft be obtained by the fame means, obferving that they fplay, backward, from the Seat, and confequently, are farther diftant from the front; alfo the line of direction is thereby fomewhat varied.

To determine the height of the Back, and the Elbow, draw GB perpendicular, and equal to its height. Draw BC or BX, cuting a perpendicular Line from K or k, at a or c; through which, a parallel Line, ab, determines its height. As to the Figure, 'tis at each Perfon's option, and depends wholly on the Eye.

The height of the Elbow is fet up from A to d, and by drawing a Line to X, its direction, and where it is joined to the Back are afcertained; alfo, its place on the Rail may be determined, as the whole length, at D. The reft depends entirely on the Hand and Eye; for, no Rules can poffibly determine its apparent Figure, perfpectively.

Fig. 132. Figure 132 is the reprefentation of a Lady's dreffing Table.

In this Picture, the Center of view is taken fomewhat to the left hand, at C, the Diftance is CE². The height of the Eye fuppofes the Perfon fitting.

The meafure of its length, in front, may be fet off, on the fame Ground Line, GH, in proportion to the Chair; or on another, at MN, being fuppofed to ftand farther from the Picture; on which account, its geometrical Figure, in front, is by a lefs Scale, as MABN.

AB, the front Line of the Top, being determined, let it be bifected, and, AD and BF made equal to half the meafure, in any pofition required; from all which Points, draw to C; make A*b* equal to the breadth of the Top, and draw *b*E², cuting AC at b; draw bf parallel to AD. From b it is transfered, by a doted Line, to the other fide. The hollowing and thicknefs of the Covers, in front, are determined geometrically, by perpendiculars to AD, or BF, at f and D, or F.

Make A*a* equal to the diftance of the neft of Drawers, which rife, by a Spring, out of the Frame, and draw *a*E², cuting AC, at a; draw ac parallel to AB, cuting BC; obferving, in this operation, that there is a margin, of above half an Inch allowed on each fide, lefs than AB. Draw the perpendiculars ae and cd, and proportion them to the length ac; or fet up the full meafure on a Perpendicular, the breadth of the margin within, at B; on which, the feveral Drawers may be proportioned, at 1, 2, 3, geometrical. The front of the Drawers, being parallel to the Picture, they are geometrically divided into as many Drawers and in what order you pleafe.

The dreffing Drawer, being drawn out, it confequently projects on this fide the Picture. Having drawn the fronts of the two Drawers, over it, draw CG, and produce it, indefinite; produce HG, and make GI equal to the meafure you intend the Drawer to project, and draw E²I, till it cuts CG, produced, at K; draw KL parallel to GH, cuting CH produced, at L; KGHL reprefents the upper face of the Drawer, projected towards the Eye. HO (*the ring*) being made equal to its depth, draw CO, till it cuts a perpendicular from L, at J, which determines the front.

The divifions, for the Glafs, in the middle, the Boxes, &c. are determined, by their meafures on GH, or by a larger Scale, on KL, as K*c* being made equal to the firft divifion, *e*E² determines its apparent breadth, at d. After the fame manner the reft are determined.

The

The recefs, below the Drawers, being an elliptic curve, is thus determined.

The height being determined, draw ag, the tranfverfe Diameter of the Ellipfes, defcribe the geometrical curve a 3 5 g; ag being bifected at d, d 4 perpendicular to ag, is half the conjugate Diameter. Take as many Points, 1, 2, 3, &c. on each fide, as are neceffary, from which, draw Ordinates to the Tranfverfe at a, b, c, &c. obferving to make g, f, and e, refpectively, the fame diftances from that extreme, as a, b, and c from the other; from all which, draw Lines to C, and determine the length of each perfpectively, and refpectively, by means of the Point of Diftance E²; making b*b* reprefent b 2, and, c*c* reprefent c 3, &c. parallel Lines, *bg*, &c. will determine thofe on the other fide, giving the Points a, *b*, *c*, &c. through which the perfpective curve may be defcribed, as in the Figure.

What remains is fufficiently explicit by infpection.

Figure 133 reprefents a Lady's Secretary and Library; of which there needs no particular defcription. It is viewed directly central, fo that, the Vertical Line cuts it into two fimilar Figures. Fig. 133.

Being parallel to the Picture the whole front, of Drawers, &c. is geometrically drawn; the middle part, with the Defk Drawer, may be projected, as the laft Figure; or the Picture may be fuppofed to be wholly on this fide, fo that, only the Fall of the Defk is projected.

The Shelves, with the Books, recede from the front, it would be impertinent to fhew how it is managed here; the Books are difpofed at difcretion, and, the Door which is open is determined by Example 39; V is the Vanifhing Point.

For the reft, the Figure is more explicit than the moft elaborate defcription.

Figure 134 exhibits the reprefentation of a Bed with a Canopy-Teafter, viewed direct in front; one half fhews the naked Frame, the other half is furnifhed. Fig. 134.

The Front being parallel to the Picture, it is geometrically proportioned. C is the Center, and C V the Diftance of the Picture, by which the reft is proportioned.

The Column, on the left hand, being feen entire, is reprefented as in former Examples; the returning Lines, in the Bafe and Capital tend to the Center; alfo the infide of the Foot.

The length of the fide (GH) being determined, and the height of the Poft, at A, draw AC, cuting a perpendicular from H, at B, the apparent height of the head Poft. The geometrical curve of the Laths being determined, the perfpective curve, of the fide Lath, is determined by means of Ordinates, *a* 1, *b* 2, &c. the feveral heights being fet up from the corner, at A, and Lines drawn to the Center.

The Laths forming the Canopy, diagonal wife, are determined as groined Arches, by Ordinates from the Diagonals, D E and EF, E being the Center.

The hither one being determined from its known figure, dividing DE perfpectively, at a, b, c, &c. as the Bafe of the Original is divided by Ordinates, at difcretion; by which means, the feveral heights a*a*, b*b*, &c. are determined, and the Curve defcribed through the Points *a*, *b*, *c*, &c. Then, Lines drawn to the Center, from the Points a, b, c, &c. cut the other Diagonal (FE) at k, i, h, &c. at which Points, the Ordinates k*k*, i*i*, &c. being drawn, and from *a*, *b*, &c. Lines drawn to the Center, cuting them, at *k*, *i*, *h*, &c. through which, the curve of the other diagonal Lath is drawn.

In refpect of the Drapery, the Furniture of the other half, I fhall only obferve, that it is proportioned as the other; but as for the outline figure and ornament, I will not attempt to give any Rules; fuch Subjects are always beft drawn from a real Object, or they can only be done by a Perfon of experience and judgment. The figures of the Vallens are geometrical, in front, and no other are feen. The Canopy is but little feen, though it rifes confiderably; a Right Line, from *b*, fhews how much, fave the middle, which appears over it; on which a Vafe, or other ornament, at difcretion, may be drawn, crowning the whole.

Figure

Plate XXXV. Fig. 135. Figure 135 is a Defign for a large Library Bookcafe.

In order to give an Idea of the real proportion and figure of fuch an Object, to a Perfon who, perhaps, knows not what Perfpective means, and who has no other Idea of its proportion and form, than what the Figure really exhibits, it is moft proper to give it parallel, as it is here reprefented, but by no means to fhew an End. The Station is central, and confequently, the Point of View is in the middle. The receding parts, and breaking of the Mouldings around them, fufficiently indicate that it is perfpectively delineated; and, by reafon of its regular pofition, I am of opinion that it has a much more natural appearance, than when viewed oblique, from either end, which always occafions diftortion; as may be feen in Plate 23, Fig. 108, efpecially when the Object is long.

In refpect of the delineation, let it be obferved, that the Plinths of the middle part and the two Ends are in one Plane, which are geometrically proportioned, by the Scale. The parts which recede are determined as ufual.

At either Corner, as A, of the Plinth, draw the perpendicular AG; on which, fet up all the meafures of its height, as AB for the Dado part, with its Bafe and Sirbafe; BD the height of the upper Doors, &c. and DF, of the Cornice, &c.

To the Center, C, draw Lines from each divifion; make Bc equal to the receding of the upper part; or, becaufe the whole Diftance cannot be on the Picture, take CE half its Diftance, and B*a* half Bc, and draw *a*E, cuting BC at b.

Draw bd perpendicular, cuting the Lines drawn from B, D, &c. to C, which gives the height of the Doors, &c. perfpectively, according to their diftance from the Front. For their widths, make B*a*, equal to the projecture of the Mouldings, and *ab* to the width of the Door, and draw *a*C and *b*C cuting a Line drawn from b, parallel to the Bafe.

The other End and middle Doors are obtained by the fame means. Or, being all in the fame Plane, if either the height or width of one be obtained in its place, the whole is determined; for, being parallel to the Picture, the Doors are all reprefented fimilar to their Originals; that is, geometrical, in refpect of their outward figure, and alfo of the ornamental part, for Glafs, &c. or as thofe below.

For the Cornice; if CE, on each Side, be made equal to the Diftance of the Picture; they are the Vanifhing Points of the Diagonals, which are drawn through the Angles at the top, obtained from the Perpendicular at F, transfered to f. The general method, in Example 15, for Mouldings parallel to the Picture, is applicable here; X, being the Profile, and *E*, the Point of View.

The figure of the outline of the Pediment is geometrical; which certainly gives the beft Idea of its true Figure; and fo is all the Ornament, every where.

Fig. 136. If it be required to fee an End of any Object of which a Defign is to be made, the Front fhould then be inclined to the Picture, as in Fig. 136; which exhibits a reprefentation of a Library Table, by a larger Scale than the foregoing Figure. For however prepoffefed many Artifts are, in the parallel pofition of the Picture, I am confident, that, were they practifed a little in inclined pofitions, and faw clearly into Perfpective, they would not retain their prejudice long; for we cannot poffibly fee two Faces of a right angled Object, and the horizontal Lines, in either, appear parallel. Why, then, are they fo reprefented? Whenever a fingle Object is the fubject of a Picture, being right angled, and feen oblique, it is, therefore, moft palpably abfurd to be placed parallel to either Face; or, in any common Cafe, to have the Center of View out of the middle of the Picture.

The delineating this Object has nothing in it particular. The Inclination of it to the Picture and the Diftance being determined, the Vanifhing Points are determined, as ufual, both which are, in this Piece, out of the Picture.

S is the Center, and d the Diftance Point, by which the Front is proportioned; the meafures being applied on the Ground Line, AB, as in former Leffons, at *a* and *b*, for the Front; which give a and b, on A*B*, the line of the Plinth, and on F*B*, of the front Plane, on the Floor, tending to their Vanifhing Point.

As there is not room on the Ground Line to set off the full measure of the End, take A D a third part, or A e half; and let f or S, be in proportion, to the Distance of the Vanishing Point of horizontal Lines in the End; then, D f, or e S, cuts AD in the perspective length required.

From each Angle, A, *B*, and *D*, Perpendiculars are drawn, and the real measures of the heights of the Drawers, &c. being applied on EF, where the Seat of the Plane they are in, on the Floor, cuts the Intersection, lines drawn to the Vanishing Point of the front Lines, give their perspective heights.

Perpendiculars from a and b determine the opening; and if, from b, a line be drawn to the Vanishing Point of the End, and another from *D*, to the other, of the Front, cuting the former, at c, the perspective width of the inside End is determined. The Mouldings are done as usual.

E X A M P L E XLVI.

Is the representation of a large Chamber-Organ, seen oblique. Fig. 137.

The method of proceeding, in obtaining the general proportions, which is subject to one general Rule, it is unnecessary to repeat; the measures, or their ratio to each other in proportion to the Distance used, being always applied on the Intersection of whatever Plane the Lines are in, or on a Line parallel to it; as on A B, for the Piers, &c. I shall, therefore, pass over that description, and only explain the Parts, where there is any apparent difficulty.

The Finger-Board (U) is usually made to draw out, the length of the Keys, about 6 Inches. Its place being acquired, at a, and V being the Vanishing Point of the End, draw V a, and produce it; draw a e parallel to the Ground Line, make a e represent 6 Inches, in proportion to the length, and draw C e, (from the Distance Point of V) till it cuts V a produced, at b; and, from b, draw b c to the other Vanishing Point, of the Front, by the Expedients, Problem 13.

Draw b d parallel, on which, set off the number of Keys, as at U; then (having made h c represent the length required) join d c and produce it to the Vanishing Line cuting it at D; to which Point, draw Lines from all the divisions 1, 2, &c. which give the Keys.

The process may be done, more commodiously, below, on the Ground Plane, by dividing *bc*, below, as b c above; from which, by Perpendiculars, it may be determined, above.

The Mouldings, breaking about the Piers, &c. have nothing singular in them; their heights, on the Pedestal, are set off on A D, the Intersection of the front Plane, with the Picture; and on *AF* (for the upper part) the Intersection of the Plane of the Pipes, &c. above, at *B*, *C*, &c.

At Z (No. 2) is a geometrical Plan of the Breaks.

The Column of Pipes, at each End, are Segments of Circles containing five Pipes each, which are unequal; let their perspective Plans be formed above, at X and Y; from their extreme edges, *a*, *b*, &c. draw perpendiculars, as shewn by the dotted Lines, giving the places, below, of the cylindrical Part.

The Mouth of each Pipe, at f, &c. is at about one third part of their height, from which it is conical, inverted. The Ornaments, at the tops and bottoms, are regular, from the middle Pipe, which it would be impossible to describe, being circular; their Figure being known, such minutias are drawn by the Eye.

The Pediment, and middle compartments of Pipes, being inclined, it may be necessary to shew how they are described perspectively.

Suppose the geometrical Figure of the Pediment the same as in the Book Case (Fig. 135) which, inscribe in a Rectangle, and draw Ordinates parallel to both Sides, from various points in the Curve, as in that Figure.

The Cornice, at the End, being described, as usual, and the Angle F determined, draw F I to the Vanishing Point of the Front; and make F I represent the distance of the two extreme Angles; at each of which, draw perpendiculars.

Plate XXXV. fg is the Interfection of the Plane of the front of the Pediment; from *AF* tranf-fer the height of the top of the Cornice, to f, and make fg equal to the known height, which, divide in the fame ratio as HI (Fig. 135) from all which divifions, draw lines to the Vanifhing Point of the Front; and having divided GH perfpectively, as IK (Fig. 135) perpendiculars drawn from each divifion, cuting the other, give the Points *a*, *b*, *c*, &c. through which the perfpective curve is defcribed.

The middle Column of three Pipes is projected as the other.

If the geometrical figure of the two Sides be alfo determined; then, on *ag*, take the divifions *c*, *d*, *f*, &c. geometrical, and draw lines to the Vanifhing Point; let the horizontal Line *gk* be divided perfpectively, at *h*, *i*, &c. from which, perpendiculars being drawn, cuting the other at *m*, *n*, &c. give feveral Points in the Figure. The reft may be drawn, accurately enough, by hand.

The proportion of the Pipes may be determined on any horizontal Line croffing them, as *gk*, either above or below, as is moft convenient; dividing it perfpectively, in the ratio of the Pipes (by Prob. 8) as they are all in the fame Plane.

The mitre Angle, of the ftreight Mouldings with the circular, muft be determined by a geometrical Plan, at W, which varies accarding to what portion of the Circle is taken; alfo, where it falls againft the Plane, the projectures may vary confiderably; in a Semicircle they have their true projecture.

It is unneceffary to give more examples of Furniture, feeing that, if the Rules of Perfpective, here ufed, be clearly underftood, the Examples, given, are fufficient for any Perfon, who is clear in the Principles on which they are founded; and may, by fuch, be applied in any Cafe, whatever, in right lined Objects. Neverthelefs, as Chairs, from the various directions of the Lines in them, are fomewhat difficult to delineate, I fhall give another fpecimen, how far the Rules of Perfpective are applicable to fuch Objects, any how fituated to the Picture.

EXAMPLE XLVII.

Fig. 138.

How to reprefent a Chair, perfpectively, cafually fituated.

The pofition, and inclination of its Front to the Picture being determined, and the reprefentation of the hither Angle, ab, of its Foot, given at difcretion, to the fcale of the Defign you intend.

Let V be the Vanifhing Point of horizontal Lines, in the front of the Chair. C is the Center of the Picture, and CE its Diftance.

At No. 1. is a half Plan of the Seat, fhewing the fplay of the back Feet, at X, with the inclination of the fide Rails to the Front.

Draw EV, and make the angle VEY equal to the inclination of the Side to the Front, giving Y for the Vanifhing Point of the Rails, in that Side; alfo, make the Angle YEX, equal to the inclination of one Side to the other; giving the Vanifhing Point of the other fide Rails, &c.

Then, draw aV, and bV, aY, and bY; and cd, through b, parallel to the Vanifhing Line.

Make bc, equal to the width of the Front, in proportion to the height (a b.)

Make VD equal VE, and draw cD, giving the place of the other front foot, at i; alfo, the breadth of each are fet off at b*a* and c*b*.

Let bd be equal to the fide of the Chair, at its Seat, and de to the fplay of the back foot; make YG equal YE, and draw dG and eG, cuting bY at f and g; g is the place of the foot on the Floor; and if a perpendicular be drawn, at f, it will cut aY in the true width at the Seat.

Make a*c* equal to the width of the feat Rails, and b*d* to the height and width of the low Rail; draw *c*V; and *c*Y, cuting the Perpendicular from f; alfo, draw *d*Y, for the low Rail, to the back foot, which fplays back, from the Seat.

From g and h draw Lines to V, and from i and k to X, cuting them at l and m, which give the place of the other back foot, and compleats the Seat.

Make

Make b*e*, equal to the recefs of the inner Rail from the Front; draw *e*G cuting bY at *f*; from which, draw a perpendicular, giving its place on the low fide Rail, at *g*; and from *g*, draw to V, cuting the other Side Rail, the meafure of which was transfered from *d* to the other foot, at *h*.

The height of the low back Rail is obtained on the front Foot; and transfered to the back foot, by means of the Vanifhing Point Y.

Thus much is practicable, with the greateft accuracy; but beyond this, little can be done, the whole Back being curved every way.

The height may be truly determined, by means of perpendiculars, from the floor. Having obtained the Line gt, tending to V, over which, a Line, r s, through the corners of the top Rail, is perpendicular, through the Point where it is cut by bY, draw D*e* to the Ground line; make *e* 1 equal to its projecture over the Seat, and 1 2 to its known meafure. Draw 1D and 2D, cuting gt at g and t, from which Points, draw perpendiculars. From any Point, as G, in the Horizontal Line, draw Ge, through g; draw eq perpendicular, make eq equal to the height of the Chair, and draw qG cuting gr in the true place of that corner of the top Rail; rV gives the other, at s.

The various proportions of the Banifter, may, in fome degree, be obtained, by dividing a line (no) drawn through its middle, into the feveral heights, 1, 2, 3, &c. through which, Lines drawn to the Vanifhing Point, V, determine the raking of the feveral parts, as in the Figure. For, whether the Back of the Chair be ftreight or curved, it deviates fo little from a Right Line, and from a Perpendicular, that the meafures may be applied, geometrically, without any fenfible error.

In refpect of the modern Chairs, now in ufe, there is no fuch thing as applying the rules of Perfpective to them, except to afcertain the place of the Feet, on the Floor; alfo, by infcribing the Seat in a Square, or other Rectangle, its Figure may be nearly determined in its true place. Fig. 139.

The height of the Back, and of the Elbows may alfo be had, but, as the whole Chair is compofed of irregular curved Lines, it is abfolutely impoffible to give its true figure by the Rules of Perfpective; fuch Objects, muft therefore depend on the Hand and Eye, for a reprefentation of them; in which, a Perfon who has judgment in Drawing can only fucceed; but, being well verfed in Perfpective, he will find great affiftance from it.

WHEEL CARRIAGES are the next Subject to be handled; on which, as they are frequently neceffary to introduce into a Picture, or for the fake of giving a Defign to a Gentleman, by thofe whofe profeffion it is to make them, I fhall make a few Obfervations; but, like many other Objects, not compofed of Planes and Right Lines, little can be done by the rules of Perfpective, further than to give the proportion of one part to another; alfo the Wheels may be drawn with tolerable accuracy. Neverthelefs, I am confcious, that few Perfons who have occafion to delineate them (except the makers) will take the neceffary pains to project them by rule; for fuch as will, I fhall lie down fome which are practical.

N. B. It muft have been obferved, that to delineate any Object by the Rules of Perfpective, its true geometrical form and proportion of one part to another muft be known, or determined on; confequently, the more complicated the Object is, being compofed of irregular curved Surfaces, the greater is the difficulty in applying the meafures on the Picture; which, as the feveral parts to be delineated are not Planes, or plane Figures, their places can only be obtained, on the Picture, from their Seats on fome Plane or other, to which they are contiguous. By which means, a fufficient number of Points, in any curved Line in the Object, being perfpectively found, and joined carefully by hand, the perfpective Figure is determined.

EXAMPLE XLVIII.

How to reprefent a Coach in perfpective.

Let ABCD be a geometrical Plan of the Carriage of a Coach; AB is the Axle of the fore Wheels, and CD of the hind Wheels, whofe Diameters are ab and dc. EFGH is the Plan of the Top of the Coach, all the other Parts are in proportion to them; as in the Figure. Fig. 140.

Plate XXXVI. S is the Station, from which it is intended to be delineated, and IK the true poſition of the Picture from that Station. Sv being drawn, parallel to the Axles and front Lines of the Coach, gives their Vaniſhing Point; Sc, perpendicular to IK, gives its Center; and, Viſual Rays from various parts of the Coach, being drawn to S (anſwering the ſame as the Eye, in the Horizontal Plane) give their places on the Picture.

The Vaniſhing Point, of the Line of its direction on the Ground (with which, the Axles, &c. are at right angles) is out of the Picture; its diſtance from c (the Center) is to cS, as cS to cv. (Prob. 12.)

Fig. 142. Theſe Preliminaries being ſettled, as uſual, let C be the Center of the Picture, and V, the Vaniſhing Point, by a Scale of 2 to 1; AJ is the Ground Line; and A, the nterſecting Point of the line of direction of the Wheels, on the Ground, which are, here, ſuppoſed to be in the ſame Plane.

The Plane of the Wheels not being vertical, but ſomewhat inclined, from the Carriage, let DG be the Interſection of the hither Wheels, with the Picture*.

Draw AD perpendicular, make AB equal to the diameter of the fore Wheel; and AD of the hind Wheel; draw BF and DG parallel to the Horizon, cuting AG at F and G. Draw Ae and Gg, alſo from F, tending to the Vaniſhing Point of the direction of the Coach.

To obtain the true place and repreſentation of the hind Wheel, biſect AG, and draw sH perpendicular. Draw He, to the Vaniſhing Point; make HJ equal to the diſtance of the Seat, on the Ground, of the center of the Wheel, from the interſecting Point H, and draw JE, cuting He at e, (or, if *S* be its Seat on the Ground Line, draw *S*C) at which Point, draw a Perpendicular, and from s (in AG) draw a Line to the Vaniſhing Point, cuting it at *s*, the repreſentation of the Center of the rim of the Wheel.

Draw *s*G perpendicular to the Interſection AG; from which Point, ſet off the true radius of the Wheel, on each ſide, and draw Lines to the center of the Vaniſhing Line of the Wheels, cuting a Line drawn through *s*, parallel to AD, at e and g; eg is a Diameter of the Wheel, in its true place; from which, the circumference of the Wheel, efgh, may be deſcribed (by Prob. 3, Sect. 8.)

The fore Wheel (abcd) may be obtained by the ſame means.

The end of the Nave is not in the plane of the Wheel, but ſomewhat beyond it, at i and k; alſo, the Spokes diſh inwardly, from the front of the Wheel. Find the circumference of the Naves, *abcd* and *efgh*, in their true places, perſpectively, at the part where the ſpokes are fixed in it, and the place of each; alſo let each Rim be divided, perſpectively, into equal parts, the number of Spokes, at 1, 2, 3, &c. (by inſcribing a Poligon) from which they are drawn to their correſponding parts in the Nave; or to their common Center, cuting the Nave, in their true places; for, as they are not in the plane of the Rim, their Vaniſhing Points are not eaſily determined, nor are they neceſſary.

The hither Wheels being compleated, the Body of the Coach is next to be determined; the Sides of which not being in one Plane each, their Interſections are not eaſily determined. Therefore, from the Seat, on the Ground, of the hither Angle of the Coach, perſpectively determined, at i (from the Plan) draw ik perpendicular, in which, by means of the Vaniſhing Point, V, of horizontal Lines in the Front, determine the Angle k; the real height being IK, in the Interſection of the Front. The other Angles, m and n, may alſo be determined from their Seats; or, by their known diſtances from k, as in other common Caſes.

If the corners of the Coach are perpendicular (as in ſome Coaches) the meaſures may be determined on ik, geometrical; but if they are inclined, the point l may be obtained as k, from its Seat; and, from l, lines are drawn to the Vaniſhing Points, and proportioned as uſual.

The Door is not in the ſame Plane, with the Line kn, but may be determined from the Seat of the hither Line, op, where it opens; and, on any perpendicular

* This Interſection may be truly obtained, by Prob. 6. Sect. 12.

5 line,

line, cuting the Ground Line, as IK, set up from I, its true measures, of height, at *a* and *b*; then, from I, draw a Line through its Seat (s) to the Vanishing Line, cuting it at L; draw *a*L, and *b*L, cuting the Perpendicular, from s, at o, and p; from which, draw Lines to the Vanishing Point of the Side.

The curve of the Bottom being geometrically determined, it may be done perspectively, by means of Ordinates, as in foregoing Examples. From the Vanishing Point V, draw a Tangent to the hither Curve, at r, which also touches the other.

The contour of the Top may be determined, by several vertical Sections described, perspectively (as *dep*, and *fgh* at the Door) then, a Curve described over them all would be the true Contour.

The Axles being parallel to the front of the Coach, their Vanishing Point is V; their thicknefs may be proportioned to the diameter of the Wheel, and their Shape by means of Ordinates, at 1, 2, 3, &c.

The geometrical Figure of the Crane Neck being described (Fig. 141.) then, by means of squares (the whole being inscribed in a Rectangle, ABCD) it may be accurately drawn in Perspective; with the other appendages of Springs, &c. by the same means. Also the Circle, at X, on which the fore Wheels turn, is determined, at Y, in its true place perspectively, as any other Circle, parallel to the Horizon.

The height and true place of the Seat may be easily determined from its Plan on the Ground Plane, perpendicular over the fore Axle. The foot-board, at M, will be best obtained by the same means; or, by its Intersection and Vanishing Line, being a plane Figure. The place of the Bar, to which the Traces are fixed, is easily determined, at E, which tends to the Vanishing Point V.

The other smaller Appendages, such as Iron Stays to the Seat and Springs, Rings, Straps and Buckles, &c. it would be trifling to describe.

The true Figure and proportion of the Carriage being obtained, the Ornaments of carving, &c. will depend on the Hand and Eye, wholly.

And, last of all, the off Wheels must be drawn, after the same manner as the hither ones, nearly; observing that they incline the other way, from the Picture. But, as they are so much hid, by the other parts of the Carriage, their Circumferences and the places of the Spokes therein (with the Point to which they tend) may be obtained from the hither Wheels, by means of the Vanishing Points.

Figure 143 represents a Phaeton, more foreshortened, and going from the Picture; by which the hind part is more seen, as the fore part in the former Figure. Fig. 143.

A description of the method of delineating it, it is obvious, would be, in a great measure, a repetition, consequently tedious. The Position to the Picture being fixed and its Proportions known, the method of proceeding is as in the foregoing Figure, in respect of the Wheels and Carriage.

AB is the Ground Line, on which the Picture, standing perpendicular, is supposed to touch the hither hind Wheel, of which, BD is the Intersection. B*d*, and Bb, tending to their respective Vanishing Points, are proportioned, at *a*, *b*, *c*, &c. as usual, for the places and diameters of the Wheels, &c. which are described as the foregoing; and are transfered to the other side, by the Vanishing Point of the Axles, the true figures of which, being in vertical Planes, are determined as other Figures, from their known geometrical form.

On ef is set off, perpendicularly, the height of the hinder Axle, and the horizontal movement of the fore Wheels, determined at *f* by a perpendicular from *e*; being cut by a Line drawn from f, to the Vanishing Point of the Line of direction of the Carriage, on the Ground. *eg*, on the Ground, is a diameter of that Circle, which being described, it may be determined in its proper place by Perpendiculars; after which, the rest of the Carriage may be drawn by a Person who has knowledge of the several parts.

Plate XXXVI. The corners of the Chair, *m*, *n*, *o*, and of the Foot-Board, at *l*, may be aſcertained by perpendiculars, from their Seats on the Ground, perſpectively determined at m, n, &c. and certain other Points which are in a right lined direction, as *p* and *q*. For the reſt, it would be to as much purpoſe to give Rules for determining the figure and poſition of the Man in the Chair, and of the Horſes; ſeeing that, the curve Lines which bound the Figure, ſave thoſe at the top, are ſo variable, that no poſitive determination of their apparent Figures can be deſcribed; but muſt ever depend on the Eye, and judgment in drawing.

EXAMPLE XLIX.

To repreſent a Machine, for driving Piles; which is in the Repoſitory of the Society, for the encouragement of Arts, Manufactures and Commerce, in the Adelphi, Strand.

Fig. 144. Objects of this kind, I have frequently obſerved, are uſually drawn with one Face parallel to the Picture; not, I am perſuaded, with a view to make it better underſtood, but to ſave trouble; or, as it is moſt probable, for another reaſon, a want of judgment or knowledge in the Art, to do it otherwiſe; for, I am opinion, no Perſon would think that this Object would convey a more adequate Idea of its ſtructure and parts, by being parallel, ſeeing that, all the ſame parts are or may be ſeen, in one as in the other, without any apparent diſtortion; and which, by being parallel, is unavoidable, being ſeen oblique. I therefore think it unneceſſary to give a Specimen, in that poſition to the Picture.

C is the Center of the Picture, whoſe Diſtance is about 8 Inches; the Vaniſhing Points are determined, as uſual, according to the Inclination given.

Let A D be the Interſection of the Picture, with the plane of the Frame at the Bottom; and A, the Interſecting Point of the neareſt Angle of one of the Timbers; alſo, let E and F be the Diſtance Points, for the Vaniſhing Point of each Side, reſpectively.

The Vaniſhing Points being aſcertained, draw A *D* and A *H*, tending to them.

Make A*b* equal to the length of that End beyond the other croſs Timber; and *bd* equal to the width between the Timbers, allowing the thickneſs of one, and draw *b*E and *d*E, giving the Points c and d; through which, the other Timbers are drawn.

Make A*c* equal to the diagonal of the Inclination, and draw *c*V, to the Vaniſhing Point; through c and d, draw Lines to the Vaniſhing Point of the Side, and produce ec, to the Interſection, A B, cuting it at J.

Make A b equal to their thickneſs, and compleat the Ends (v and x); from the two upper Angles of each, draw lines to the Vaniſhing Points, on the left hand.

How that Frame is finiſhed, the Figure deſcribes ſufficiently.

The places of the other Timbers are obtained by ſeting the Diſtances, &c. on the Interſection AB, at *e* and *f*, and thence transfered to A*D*; or, from J, to J *I*.

From J, draw a Perpendicular (J L) the Interſection of the long Timbers; on which, ſet up all the meaſures of the Frame for the Wheels, at *j*, *i*, *k*, and *l*, from each of which, draw lines to the Vaniſhing Point of the Side, cuting Perpendiculars from c and f, giving the height of the Platform, &c. the length of which, (*mn*) may be obtained from the Timber, below; and the breadth (*no*) by the Vaniſhing Point of a Diagonal (*mo*.) at e.

The feet of the Supporters (r and f) being got, and the Bearer, or Head piece (X) drawn to the Vaniſhing Point of the Front (giving what thickneſs is neceſſary, at *l*) draw the Supporters, as in the Figure; alſo, the Bearers (y) and Stays (z) of the Platform. Their Vaniſhing Points may eaſily be obtained, knowing their inclination, in the vertical Vaniſhing Line of the End; by Prob. 4.

The top of the croſs Braces being got, by a line from *k*, the Figure ſhews how they are drawn, ſo as to ſhew their thickneſs, properly; firſt drawing the Front.

Produce the inside of the long Timber to the Ground Line, and somewhat beyond it, at I, draw IK, perpendicular, the Intersection of the vertical toothed Wheel, which begins the motion. Fig. 142.

Make I*p* equal to the height of its Center, and, let *pq* be its Radius, and *qr* its Diameter. Draw from *p* to the Vanishing Point of the Side; and, having bisected the head of the Frame, at g, draw, from g, to the Vanishing Point of the End, which will pass through the Center of the Wheel, at *s*, through which, draw *tu* its Diameter, and compleat the Rim h i k l (by Prob. 3, Sect. 8.) Its Vanishing Line passes through the Vanishing Point of the side Timbers; perpendicular.

The cross Bars (hk and il) have their Vanishing Points in it; making right angles, perspectively, with each other, at discretion.

The Teeth, where they are seen, may be done by hand; for although the Rim may be divided into equal parts, yet, to divide it into so many would be unnecessary trouble; observe, as the Wheel is contracted so are the Teeth.

The other vertical Wheel (Z) of a larger Diameter, on the same Axle, is managed by the same means; having the same Vanishing Line, and Vanishing Points. Its use is to accelerate the motion, by its weight.

In such Objects, great regard should be had to describe the hither parts first; taking particular care that they do not cross each other, improperly.

The middle, upright piece, m n, and the Bearer, n C, being drawn, the place of the long Axle is opposite the middle of the vertical Wheel; its place may be found in a Line drawn, on the Ground, through the foot of the upright Supporter, at r.

The small horizontal Wheel (pq) being nearly on a level with the Eye, appears almost in Right Lines. Its Diameter is, in proportion to the vertical Wheel, nearly geometrical.

The height of the upper horizontal Wheel may be had from any perpendicular Line, cuting the Ground Line, as J L; drawing a Line through r, the Seat of its Center, on the Ground, to the Horizontal Line, cuting it at E.

Make I L, equal to its height, and draw L E, cuting a perpendicular from r at s, and gives s for the Center of the Wheel.

Draw L M parallel to the Ground Line, make LM equal to the radius of the Wheel, and draw ME, cuting a Line drawn through s parallel to LM, at t; st is the Radius, in its true place; by which, the Circumference, tuvw, may be described. The thickness of the Rim, and height of the Teeth, are set up from L; and the cross Bars tend to the Vanishing Points of the Timbers; or at discretion.

The place of the Stay at u is also determined on IL, at N.

The large upright Timbers may be determined, in height, by the same means.

Produce the bottom Line of the hither cross Timber (at its foot) to the Ground Line, at O; draw OP perpendicular, and equal to the height. Then, having obtained the place of the upright Timbers, in the middle of the Rails, below, at R and S, and the perpendicular Lines being drawn, a line drawn from P to the Vanishing Point, on the left-hand, determines the height of the hither one, at T; and, from T to the other Vanishing Point, they may be compleated.

The Center and Diameter of the Wheel V may also be obtained on OP, at *p* and *q*, and transfered to its place, giving *s* for its Center, and *rs* for its Radius; by which it may be compleated. (Prob. 3, Sect. 8.)

Observe, it is the circle of the back part of this Wheel that is first got.

The place of the Weight, or Ram, W, is at pleasure; its height is in proportion to the upright Timbers; also the Stays between them, above.

The Roller (Y) for winding up the Ram, is a Cylinder, on the Axis of the vertical Wheel; and it is so contrived, as to revolve on the Axis freely, while the Ram descends, without retrograding the motion of the Wheels. When the Ram is down, it catches, and when the Ram arrives at the Top, the Cylinder is set at liberty again; so that, the motion of the Wheels is continual, the same way.

Behind the Frame is another heavy vertical Wheel (X) on the Axis of the Cylinder, intended to accelerate the motion, whilst the Ram descends.

EXAMPLE

Plate XXXVI.

EXAMPLE L.

Is the representation of a Crane, from the same Repository.

Having been minutely particular in the description of the foregoing Machine, less will suffice in this. It is worked after the same manner, by means of Winches; the Model has three, here is but one; which, is sufficient for the purpose of describing the manner of delineating it.

A geometrical Plan, of the place of each upright piece of Timber, &c. in the Frame, being drawn, to any Scale, and the position of the Picture determined, let Perpendiculars be drawn, from each, to the Picture, giving their Seats. Or, being so prepared, let the Lines, of some of the principal, be produced to the Picture; which, where it can be conveniently done, facilitates the process.

Fig. 145. The Center of the Picture is at C, in the same Horizontal Line; the Distance is about 6 Inches and a half. V is the Vanishing Point of horizontal Lines at the Ends, determined at discretion; the other runs out, on the right-hand, distant from the Center almost 14 Inches (a third Proportional to C V and the Distance of the Picture, always, when the Lines are at right angles with each other) BK, on the Floor, is the Ground Line of the Picture, on which it stands.

From A, the intersecting Point of the end Line, at the foot of the hither Supporter of the Frame, draw A V; and in it, find a, the nearest Angle (Prob. 7) from which draw a Line to the other Vanishing Point, and produce it to the Ground Line, at B; and draw BF, perpendicular; on which, the several measures, of the heights, are set up, from B, and Lines drawn to the other Vanishing Point cuting Perpendiculars from a and b.

The Axle (RS) of the large Wheel between the Supporters, is determined by a line drawn from G, the Seat of S on the Ground Line, to the Center, giving its place, at S (in the central Line L S) its Diameter is got by the same means. Or thus.

The height of the Wheel from the Floor being known, draw its Intersection HI, parallel to the Ground Line; and from G, draw Gg, perpendicular, and gC, cuting the Axle in the Center of the Wheel. Make gH and gI each equal to the Radius of the Wheel, and draw HC and IC, cuting a line drawn through the Center (J) parallel to HI, at h and i; by which Diameter (hi) the Circumference may be compleated. (Prob. 3. Sect. 8.)

Then, if V be made the Vanishing Point of one of the Spokes (kl) the Vanishing Points of the other are found by making Angles at the Eye, with the Radial of that, equal to the Angles they make with each other; which, in this, are equilateral, having six Spokes, forming a regular Hexagon. (Prob. 24.)

The Winch Wheel (U) touches the Picture; its place being obtained, at K, from the Plan, draw its Intersection Kt, and set up the height of its Center Ks: make rs, and st, each equal to its Radius; and, on the Diameter rt, describe the Circumference, and finish the Wheel from its known Figure.

On the Intersection Kt, of the Face of the Wheel, the height of the Supporters, and also the place of the cross piece (T) may be determined.

The other vertical Wheel, at Y, is got after the same manner, on P Q. Its use is to stop the Crane from going backward, by means of the Catch, at z.

At Z is a Lever, which, by a Cord and Pully, raises the Catch.

The upper Gudgeon-piece (X) being drawn; find the foot of the Crane, at L, in the central Line; its dimensions being obtained, draw the upright Piece, LM, and set up the measures of the Brace and Headpiece, at *d*, *e*, and *f*.

On the Floor, describe a portion of the circumference of a Circle, perspectively, on the Center L (Radius, the projecture of the Crane) and, according to the direction of the head of the Crane, its Seat being somewhere in that Curve, as at N, draw NO perpendicular, and make NO represent its height from the Floor.

The Rollers, which direct the Cord, are Cylinders; their upper Bases are determined at W, from which, perpendicular Lines are drawn to the head Gudgeon-piece at X, which is level with the Eye, so that, its Plane is not seen.

The Cord, the Pullies, Teeth of the Wheels, the canting of the Timbers, and all such minutias, I shall pass over, as the Figure describes those parts better than the Pen. For those, who would be particular, in large Drawings, various Lessons may be found in the Work, for truly projecting every part.

Plate XXX

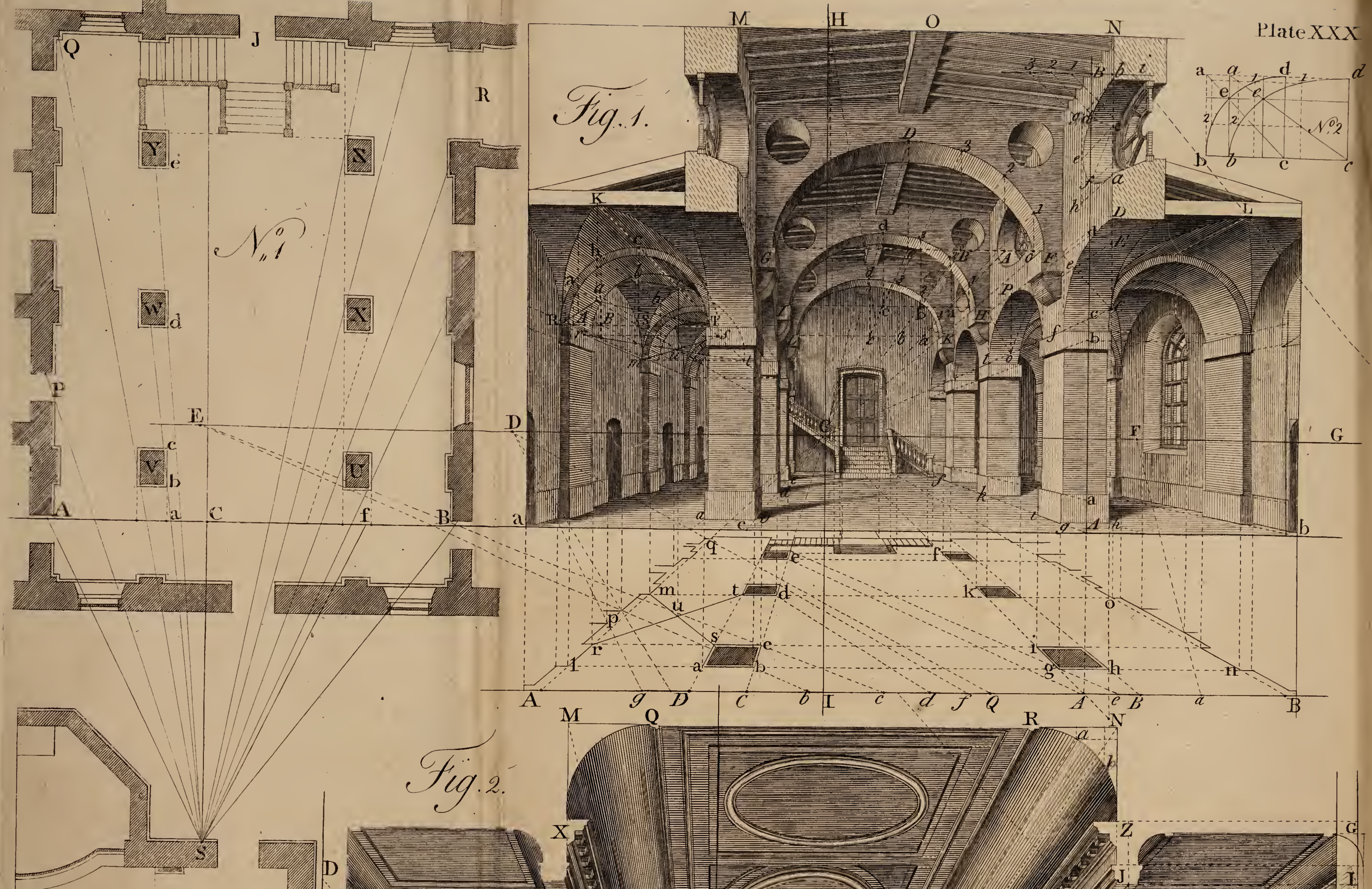


Fig. 1.

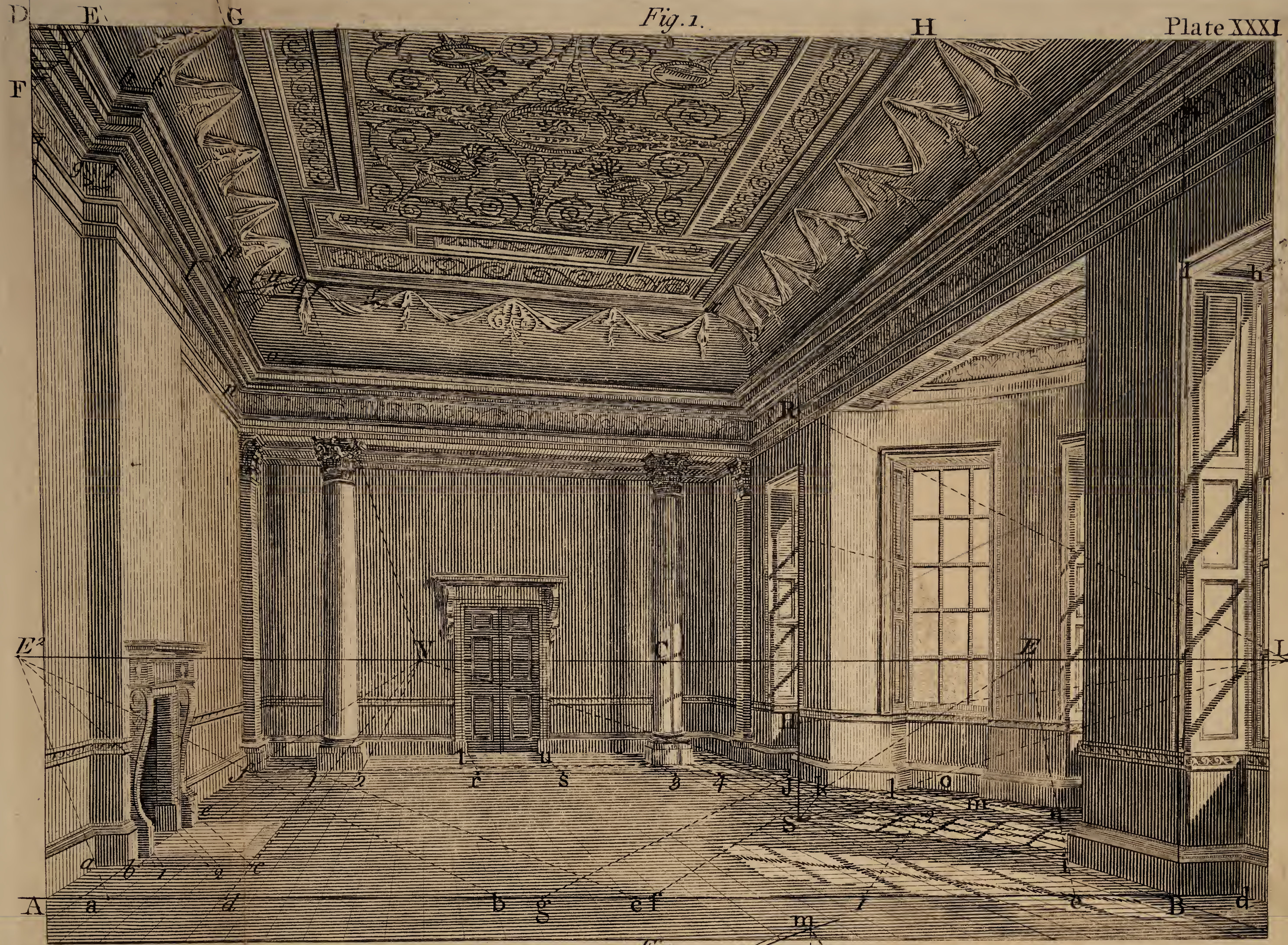


Fig. 2.

Publish'd by T. Malton March 16th 1775.

Plate XXXII.

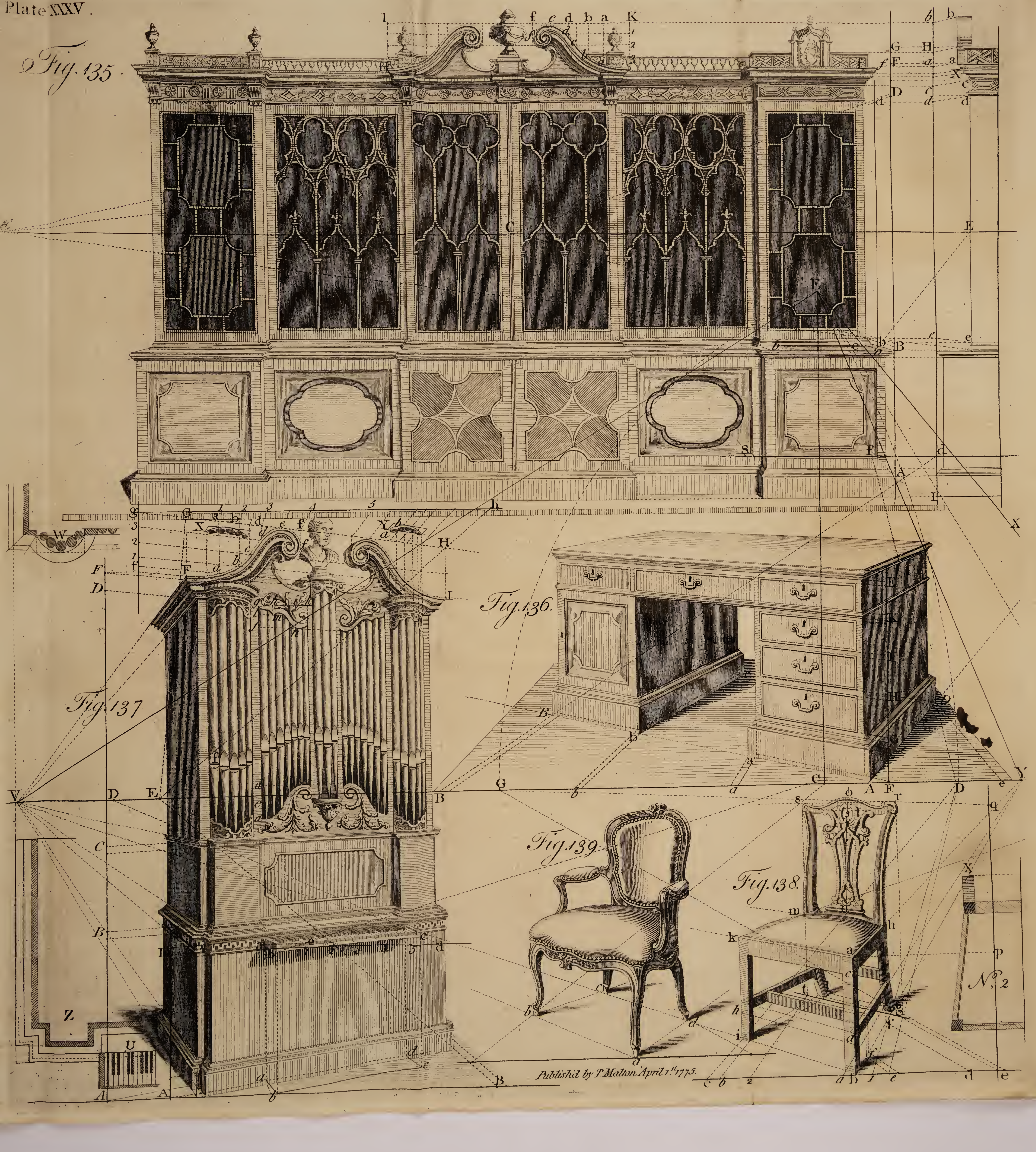
Publish'd by T. Malton March 1st 1775.

Pl: XXXIII

Publish'd by T. Malton March 1st 1776.

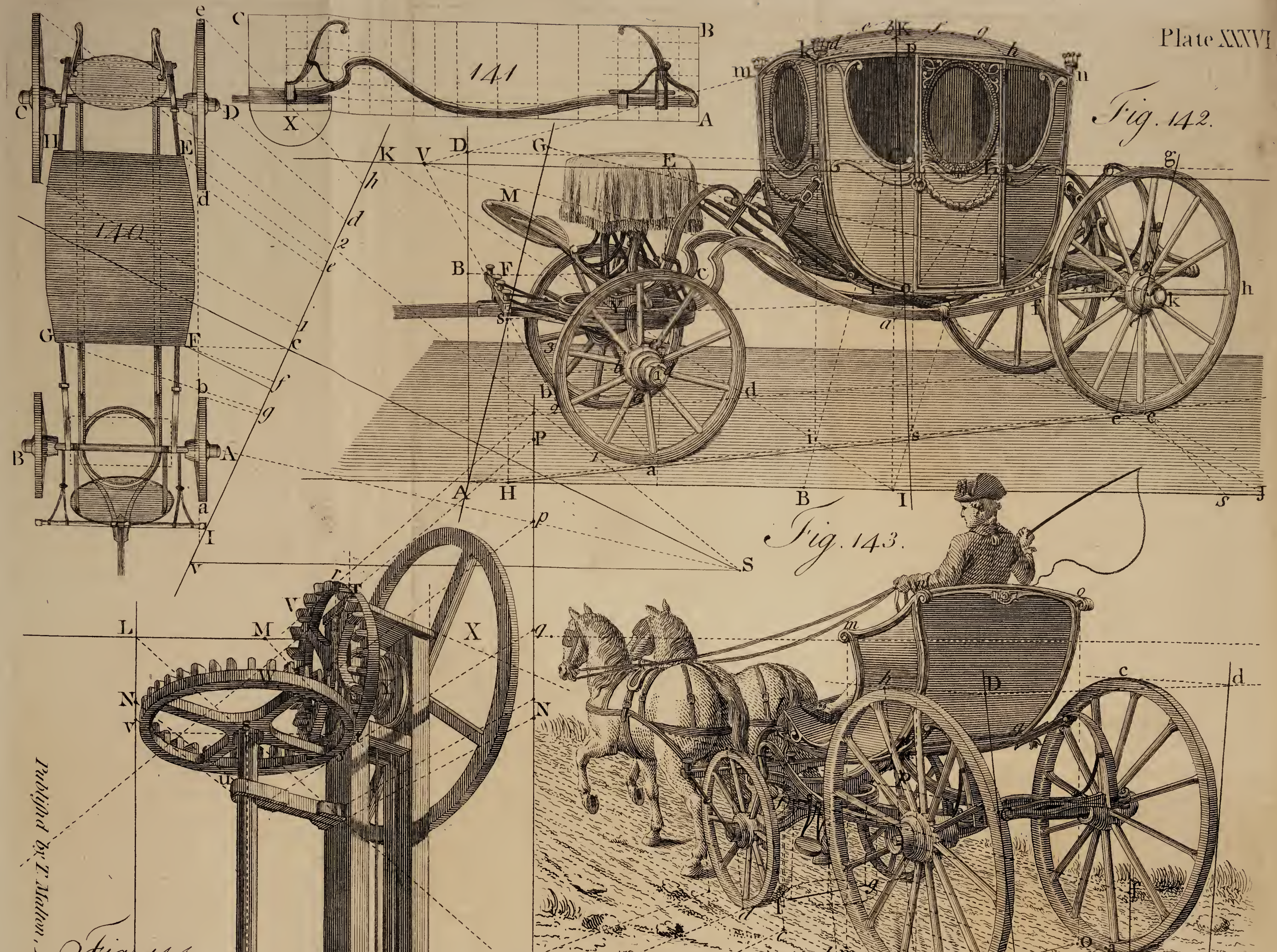

Plate XXXIV.
Fig. 129.
No 1.
No 2.
Fig. 130.
Fig. 131.
Fig. 132.
Fig. 133.
Fig. 134.

Plate XXXV.



Publish'd by T. Malton April 1st 1775.

Plate XXXVI.



Publish'd by T. Hinton, April 1st 1775.

SECTION XII.

On INCLINED PICTURES and PLANES, in general.

AS the Plan which I have hitherto purfued, has been folely confined to the ufeful ftudy of Perfpective, fo, this Section is adapted more for the curious Artift than the ufeful; yet, the inclined Picture may, in fome Cafes, be neceffary. However the univerfality of the Principles are, here, more fully evinced; it being demonftrated that the pofition of the Picture to the Horizon, is not of the leaft confequence, nor has any meaning in the Theory of Perfpective, as it has already been fhewn, in refpect of horizontal Pictures, and is, in this Section, made general.

For, whether the Picture be vertical or horizontal, and the Planes or Lines, in Objects reprefented on it, be inclined to the Horizon or to the Picture, or both, 'tis the fame thing if the Picture be inclined, and the Planes or Lines be either parallel or perpendicular to the Horizon; feeing they are, or may be, neverthelefs, inclined to the Picture. Wherefore, however either the one or the other be fituated to the Horizon, it matters not, if the inclination of one to the other be known, and the Interfecting Points of any two Lines in either, with the other, be determined; the fituation of the Eye being known, in refpect of either, as fhall be made manifeft, to conviction.

PROBLEM I.

The Interfecting Points of any two Lines in a Plane being given, and the Inclination of the Plane, to the Picture, known; to find the Vanifhing Line of that Plane, its Center and Diftance; the Center and Diftance of the Picture being given.

The Interfecting Points of two Lines, in any Plane with another, being given, the common Interfection of thofe Planes is determined. (Theo. 10.) Therefore, if A and B are the interfecting Points given, or found, A B is the Interfection of the Plane they are in, with the Picture. (Cor. 3.) Plate XXXVII. Fig. 146.

Let A and B be the Interfecting Points of two Lines, in fome Plane; and C the Center of the Picture, whofe Diftance is known.

Draw A B, and CE parallel to A B, equal to the Diftance of the Eye; through C, draw DF perpendicular to AB; make the Angle CEF equal to the complement of the inclination of the Plane to the Picture, cuting DC, produced, at F. Through F draw GH, parallel to AB, which is the Vanifhing Line required; its Center is F, and its Diftance is EF.

Turn up the Triangle CEF, on CF, till EC be perpendicular to the Picture; then, E, is the true Place of the Eye, and EC is the Direct Radial.

DEM. Now, becaufe DF is perpendicular to AB (the common Interfection) and CEF is a Plane, perpendicular to the Picture, paffing through DF, it is perpendicular to the other Plane †, for, it is perpendicular to their common Section; therefore, DF is the Vertical Line of that Plane, (Def. 11.) † 9. 7. El.

Then, becaufe CEF is the Complement of the angle of Inclination of the Plane to the Picture, EFD is the real Angle, (Cor. 3. 10. 1. El.) for, ECF is a Right Angle, by Conftruction.

Confequently, fince the Vanifhing Line of any Plane is parallel to the Interfection of that Plane, (Th. 2) if a Plane be fuppofed to pafs through the Eye, at E, parallel to the Original Plane, it is inclined to the Picture in the Angle, EFD; and the Line GH, in which it muft cut the Picture, is the Vanifhing Line of that Plane, (Def. 8.) for, the Sections of parallel Planes, by another Plane, are parallel between themfelves. — — — — — 8. 7. El.

But, EF is perpendicular to GH; therefore F is its Center (Def. 19.) and confequently, EF is its Diftance (Def. 20.) Alfo, CF is perpendicular to GH, (Th. 7.) for, it is a Line drawn in the Plane CEF, from the Point F, in which a perpendicular (GH) to that Plane, cuts it. 2. 7. El.

 COR.

Plate XXXVII. Cor. Hence the Corollary to the 11th Theorem is manifest; as the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture; so, the Center of every Vanishing Line is the Vanishing Point, of all Lines which are perpendicular to the Interfection of any Plane, of which it is the Vanishing Line, and of all Lines parallel to them.

Because, EF, producing the Center, (Def. 19.) is parallel to all such Lines.

It must be obvious that this Problem is univerfal, and cannot possibly regard any position either of the Picture, or of the Original Plane to the Horizon; for, let them be any how situated in that respect, every thing remains the same. But, nevertheless, in many Cases, the position they have to the Horizon is necessary, in practice, to be known; seeing that, all plane Objects, as Buildings, &c. are determined from the position of their Planes to the Horizon; as well as to the Picture; nor is it possible to determine any thing, in respect of their Projections on the Picture, without the previous determination of some Plane or other in respect of them and the Picture, and which is best done, by means of a Plane perpendicular to the Picture, either horizontal or vertical.

PROBLEM II.

The Center and Distance of the Picture being given, and the Vanishing Line of some Plane which is inclined to the Picture; to determine the Inclination of the Plane, and to find the Vanishing Point of Lines perpendicular to that Plane.

Fig. 147. Let AB be the Vanishing Line given, and C the Center of the Picture.

Through C, draw DF perpendicular to AB, indefinite; draw CE perpendicular to DF, i e. parallel to AB; make CE equal to the Distance of the Picture, and draw ED; also, EF, perpendicular to ED.

EDC is the angle of the Inclination of the Original Plane, to the Picture; and F is the Vanishing Point, of Lines perpendicular to the Plane.

Turn up the Triangle DEF perpendicular, also turn over the Plane X on AB.

Dem. Because CE is perpendicular to the Picture, and equal to its Distance, EC is the direct Radial, and E is the Point of View (Def. 15 and 16) and, because the Plane AEB passes through the Eye, and the Vanishing Line (AB) it is parallel to the Original Plane, (Def. 8.) and its Inclination to the Picture is the same, equal EDC.

But, the Plane DEF is also perpendicular to AEB, and consequently to the original Plane; therefore, it is the Vertical Plane, (Def. 5.) and DF is the Vertical Line, (Def. 11.) which makes equal Angles with them both; and, because it is the Interfection of a Plane cuting two parallel Planes, perpendicularly; therefore, the Angle EDF is equal to the inclination of the Original Planes.

But, EF is in the Vertical Plane, and it is perpendicular to ED (by construction) consequently, EF is perpendicular to the Plane AEB, and consequently it is parallel to all Lines perpendicular to the Original Plane; and therefore, F is their Vanishing Point. (Def. 22.) Q. E. D.

PROBLEM III.

The Center and Distance of the Picture being given, and the Vanishing Point of Lines, which are perpendicular to some Plane, to find the Vanishing Line of that Plane.

Fig. 147. C is the Center of the Picture, and F is the Vanishing Point given.

Join FC, and produce it; draw CE perpendicular to DF, and equal to the Distance given; draw EF, and ED perpendicular to EF, cuting FC, produced, at D; and through D, draw AB perpendicular to DF.

AB is the Vanishing Line required; which is manifest, from the foregoing.

PROBLEM IV.

The Vanishing Line of a Plane being given, and the Vanishing Point of the Interfection, of that Plane, with another Plane, perpendicular to each other; with the Center and Distance of the Picture; to find the Vanishing Line of the other Plane.

Fig. 147. Every thing remaining as in the last Figure; let AB be the Vanishing Line given, and A the given Vanishing Point.

Find the Vanishing Point, F, of Lines perpendicular to any Plane, whose Vanishing Line is AB (by Prob. 2.) and draw AF the Vanishing Line required.

Dem.

DEM. For, since the Plane, whose Vanifhing Line is required, is perpendicular to that Plane whofe Vanifhing Line is AB, F will be the Vanifhing Point of Lines perpendicular to the other Plane; and A is the Vanifhing Point of fome Line in it, by Suppofition; confequently, AF is the Vanifhing Line fought. — — — — Th. 10.

N. B. If the Vanifhing Line given was of a Plane perpendicular to the Picture, feeing it would pafs through the Center of the Picture. (Theo. 4.) wherefore, Lines perpendicular to it, would have no Vanifhing Point; for EG, perpendicular to EC, being parallel to DF (a Line in the Picture) can never cut the Picture and produce a Vanifhing Point; confequently, AF, the Vanifhing Line (in that Cafe) would be perpendicular to AB.

PROBLEM V.

The Vanifhing Line of a Plane being given, and the Vanifhing Point of the common Interfection of another Plane with that Plane, whofe Inclination to the former is known; to determine the Vanifhing Line of the other Plane; the Center and Diftance of the Picture being given.

Let AF be the given Vanifhing Line; and F the given Vanifhing Point. Fig. 148.

Find AB, the Vanifhing Line of a Plane, to which, Lines whofe Vanifhing Point is F are perpendicular (by the third.)

Make DG equal to DE (the Diftance of AB) and draw AG; alfo draw GB, making the Angle AGB equal to the inclination of the two Planes, cuting AB at B, and draw BF, the Vanifhing Line required.

DEM. If the Triangle DEF be turned up, as before, and the Plane AGB be alfo turned over on AB, DG will coincide with DE. Then, if the Plane AHF be turned over, on AF, AH will coincide with AG, and AHF will be parallel to the Original Plane, whofe Vanifhing Line is AF (Def. 8.) feeing it paffes through the Eye, E, and the Vanifhing Line.

But, AGB is perpendicular to AHF (9. 7. El.) for EF is perpendicular to AGB; confequently the Angle of inclination of any other Plane, with that Plane, may be determined in the Plane AGB†.

But, AGB is the Angle given, of the Inclination of the Planes; wherefore, if BIF be turned over on BF, till BI coincides with BG; then BIF, paffing through EF, is alfo perpendicular to AGB; confequently, it inclines to AHF in the Angle AGB.

† See Art. 4. Page 42.

But, Planes which produce the Vanifhing Lines of two Original Planes are inclined to each other as the Originals; and, their common Interfection paffes through the Eye, parallel to the common Interfection of the Original Planes. - - - - - Theo. 6.

Wherefore, EF is parallel to the common Interfection of the Original Planes; confequently, F is its Vanifhing Point, (Cor. 2.) for, it is the Interfection of the Vanifhing Lines; and BF, the Line in which BIF cuts the Picture, is the Vanifhing Line fought; (Def. 8.) for, BIF is parallel to the Original Plane, and paffes through the Eye, at E. Q. E. D.

N. B. If AGB was a Right Angle; then, AF, AB, and BF, would be the Vanifhing Lines of a folid Right Angle; each Plane, of which, being inclined to the Picture, refpectively, as the Planes AGB, AHF, and BIF; all which pafs through the Eye, at E.

The Center of each Vanifhing Line is where a Perpendicular from C cuts it; as D, K, and L, (Theo. 7.) and DG, KH, and IL are their Diftances refpectively; G, H and I, are confidered as the Eye for each Vanifhing Line, in the application of them to practice; each Diftance being the Hypothenufe of a right angled Triangle on CE; with CD, CK, and CL, refpectively. (Def. 20.)

Note. When the given Vanifhing Line paffes through the Center of the Picture, it is the Vanifhing Line of a Plane perpendicular to the Picture, which alfo paffes through F, as DF; in which Cafe, A and D coincide; and, the Angle is made with DG, as DGA or DGB; according to which fide of that Plane the other Plane inclines.

PROBLEM VI.

The Interfection of any Plane, which is perpendicular to the Picture, being given, and the Interfecting Point of the common Interfection of that Plane with another Plane, whofe Inclination to it is known, and, the Angle which their common Interfection makes with the Picture; to determine its Interfection and Inclination to the Picture.

Turn over the Plane Y out of the way, it being of no ufe in this Problem.

Let AB be the Interfection given, and A the Interfecting Point. Fig. 149.

Make

Plate XXXVII. Make BAE equal the Angle which the common Interfection of the two Planes makes with the Picture; and, at any point (E) in AE, make a right Angle, AEB.

Make BED equal to the angle of Inclination of the two Planes, and draw BD perpendicular to BE, cuting ED.

Draw BG perpendicular to AB; make BG equal to BD, and draw AG.

AG is the Interfection of the inclined Plane, with the Picture.

Turn up the Picture (X) on AB, perpendicular, and the Triangle BDE, on BE, alfo perpendicular; then, BD will coincide with BG; alfo, turn over the Triangle AIE; they will form a folid Angle, at the Point E.

Dem. BAE is the given Angle, which the common Interfection, AE, makes with the Picture; and, the inclination of the Plane AIE to AEB (which is perpendicular to the Picture, cuting it in AB) is BED; confequently, AG is its Interfection with the Picture.

For, if the Plane AEI be fuppofed continued; beyond the Picture, it will cut the perpendicular Plane in a continuation of EA, making equal Angles on the other Side; wherefore, A is the Interfecting Point of their common Interfection, and the Interfection AG remains the fame.

Secondly. From any Point (E) in AE, draw EC perpendicular to AB, and CH perpendicular to AG, cuting it in H; make CF equal CH and draw EF. CFE is the Angle of inclination of the inclined Plane to the Picture.

Dem. For, if the Picture be turned up, and the Plane AIE, meeting it in AG; then EH is the Hypothenufe of a right angled Triangle CHE, congruous with CFE; confequently, thofe Angles are equal.

But, CH and HE are both perpendicular to the common Interfection of the Plane AIE with the Picture, therefore, CHE (equal CFE) is the Angle of its Inclination to the Picture.

This Problem, is general; for, the Interfection, AB, given, is not, neceffarily, either horizontal or vertical; nor is the Picture neceffarily either, all that's required is, that the Plane be perpendicular to the Picture, whofe Interfection is given.

But if it was inclined to the Picture, that inclination being known, and which Side it inclined on, the reft is determinable; whereas, the given Interfection being of a Plane perpendicular to the Picture, it cannot poffibly be mifunderftood and applied.

In Prob. 5. Sect. 3. is fhewn how to find the Vanifhing Line of a Plane which is inclined to the Horizon, and to the Picture, when its Interfection with the Horizon is alfo inclined to the Picture, as in the laft Problem. The horizontal Plane, being confidered, fimply, as a Plane perpendicular to the Picture, the Problem becomes general, and univerfally applicable; however that Plane or the Picture be fituated in refpect of the Horizon.

I fhall here, as it is there propofed, give a brief Demonftration of that Problem, which will now, I prefume, be better underftood, the foregoing being previoufly neceffary; and the affiftance of moveable Planes will render it far more intelligible.

Let the Picture be turned up perpendicular, and the Plane Y perpendicular to it; alfo, turn over the Plane V, till *FB* coincides with *EB*; and W being turned over, *A*E will coincide with *AE*, &c.

The Planes being thus conftructed, let the former be alfo placed as was directed.

Then, if the Original Plane, Z, be horizontal, Y, i.e. *AEB* being parallel to it, is the Horizontal Plane; or, Z being confidered, fimply, as a Plane perpendicular to the Picture, Y is its Vanifhing Plane; for *AB* the Vanifhing Line, produced by it, paffes through C, the Center of the Picture (Th. 4.)

The Plane W paffes through the Eye, parallel to AGE, and therefore produces its Vanifhing Line, *AG*; and, *EB* being parallel to EB, alfo, the Angle *BEG* being equal to BEG, *EG* is parallel to EG; and confequently, *G* is the Vanifhing Point of EG (Def. 22.) alfo, *EA* being parallel to EA, *A* is the Vanifhing Point of EA. Wherefore, fince *A* and *G* are the Vanifhing Points of two Lines in the Plane, EA, and EG, confequently, *AG* is the Vanifhing Line of the Plane thofe Lines are in. (Theo. 10.)

Alfo, becaufe *E* is the Eye, and *EA*, *EG*, are parallel to EA and EG, refpectively; therefore, the Plane *AEG* is parallel to AEG (7. 7. El.) and it paffes through the Eye, at *E*; therefore, *AG* is the Vanifhing Line of the Plane. (Def. 8.)

Now, if the Plane V, be turned on *BG* into the Plane of the Picture, on either Side, *BF* being equal to *BE*, and the Angle *BFG* equal BEG; confequently, the Point *G* is the fame, however the Plane V be fituated to the Picture, in its revolution on *BG*; therefore, the Vanifhing Point *G*, is truly afcertained; and confequently, the Vanifhing Line *AG*; being parallel to the Interfection AG of the Original Plane, alfo *BG* to BG. (Theo. 2.)

For, the Original Planes, being fituated any where on the other fide of the Picture, being parallel, refpectively, to their prefent Situation, their Vanifhing Lines are the fame. (Theo. 3.)

PROBLEM VII.

The Inclination of a Line to the Picture being given, and the Angle of inclination of any Plane that Line is in, to the Picture; to determine the inclination of the Line, to the Interfection of the Plane it is in, with the Picture.

Let AB be the given Line (in the Plane AEB) inclined to the Picture in a given Angle. Fig. 150.

Make ABD equal to the Angle known, and, from any Point in AB, draw AC perpendicular to BD; make the Angle CAD equal to the Complement of the inclination of the Plane to the Picture, and ADC, is the real Angle.

On AB defcribe a Semicircle; and, with the Radius AD, defcribe the Ark DE, cuting the other at E; i. e. make AE equal AD; draw EB, and ABE is the Angle required.

Draw AE and produce it; make EF equal to CD, and draw BF.

Dem. Turn up the Triangle ADC, on AC, and ACB, on AB, till AD coincides with AE; then, turn up the Plane X, till BC coincides with BF.

Now, X being confidered as the Plane of the Picture, and ACB as a Plane paffing through the Line AB perpendicular to the Picture, ABC is the angle of its inclination to the Picture, given. But, ADC is the inclination of the Plane it is in, to the Picture, and EB is the Interfection of that Plane with the Picture; confequently, ABE is the inclination of the given Line, AB, to the Interfection, EB.

From thefe Problems it muft be obvious, that the pofition of the Picture, to the Horizon, is of no confequence in the Theory of Perfpective; but is very much fo in common Practice; becaufe, all perpendicular Lines, in Objects, are parallel to the Picture, being vertical; and horizontal Lines are eafily determined, whether perpendicular or inclined, their inclination being known. To an inclined Picture, their inclination is the Angle they make with their Seats, on the Picture.

N. B. The Vanifhing Line or Interfection of fome Plane in the Object, muft be given, to determine others, if neceffary; for which end, the Center and Diftance of the Picture are abfolutely neceffary. The Vanifhing Line of horizontal Planes, being perpendicular to the Picture, is therefore firft determined, and the Interfection of the Ground Plane; which of all other are fitteft. Without them, there would be great difficulty in proceeding. For want of that confideration, the work of Dr. Brook Taylor is almoft ufelefs to a Practitioner; he not giving, properly, one fpecimen, how to find the reprefentation of a Line, from its known length, fituation, and place, in refpect of the Picture; but only, by means of the Interfecting Points of other Lines, and drawing Vifual Rays, from the Eye to the Original Points, in their true places, which is not practical in many cafes; or, to determine the Figure from fome Line given in it, on the Picture; fo that, the Student knows not how or where to begin the procefs, from the geometrical proportions of the Object, and its known or determined place and pofition to the Picture, and to the Horizon.

EXAMPLE I.

To find the reprefentation of a right angled Parallelopiped, whofe Sides are known, fituated at fome diftance from the Picture; whofe Faces are all inclined to the Picture, the inclination of one being determined, and the inclination of one Side in that Face, to the Picture. The Center and Diftance of the Picture, with the Seat, and diftance of the neareft Angle of the Object, to the Picture, being given.

Let C be the Center of the Picture, and S the given Seat of the hither Angle. Plate XXXVIII. Fig. 151.

Find a, the reprefentation of that Angle. (Prob. 6. Sect. 4.) Through C, draw DF, at pleafure, and CE, perpendicular to DF; make CE equal to the Diftance of the Picture, and the angle CED equal to the Complement of the given Angle of inclination of a Face to the Picture.

Through D, draw AB perpendicular to DF (the Vanifhing Line of that Face, Prob. 1.) and, perpendicular to DE draw EF; F is the Vanifhing Point of Lines perpendicular to that Plane. (Prob. 2.)

Find the reprefentation of that Face whofe Vanifhing Line is AB, D is its Center, and DE its Diftance. (Prob. 21.) as follows.

Produce FD; make DG equal to DE, and draw GH parallel to AB.

Plate XXXVIII. Make the Angle HGA equal to the inclination of the given Line to the Interſection of that Face, whoſe Vaniſhing Line is AB (Prob. 7.) and, make AGB a right Angle; A and B are the Vaniſhing Points of its Sides; and, F being the Vaniſhing Point of Lines perpendicular to that Face, AF, and BF are the Vaniſhing Lines of the other Faces; as it is manifeſt by Problem 4.

Draw aA, aB, and aF, the indefinite Repreſentations of three Sides, forming the hither Angle; how to proportion them, I ſhall ſhew, as follows.

The meaſures of thoſe Sides are known, and the diſtance of the Angle a, from the Picture; which was found by its given Seat, S, and its Diſtance S*a*.

Now, A, B, and F, are the Vaniſhing Points of the Sides of the Object, any one of which may be in the ſame Plane with SC (Ax. 6.) wherefore, AC*S* is the Vaniſhing Line of ſuch a Plane. (Theo. 10.)

Make ab to repreſent a Line in proportion to that which Sa repreſents, as the Original of ab is to the diſtance of the Angle a (Prob. 10.) thus.

Make C*E* equal to CE, and perpendicular to AC; draw A*E*, and produce it; make *E*M, to *E*N, in the ratio of the Side, to the diſtance of the Angle a; join MN, draw *E*J parallel to MN, and draw SJ, cuting aA at b; ſo ſhall ab repreſent a Side of the Object, whoſe length and inclination to the Picture was given.

Draw bB and bF; and, by means of the Radials AG and GB, make ad, or bc, to repreſent a Line, in proportion to the Original of ab, as one Side of that Face is to the other (Prob. 10.) *viz.* as GI to GK; and draw Ad (through c) and dF.

The Face abcd, whoſe Inclination was given, being compleated; find e or g, ſo, that de, or bg, ſhall repreſent the proportion of the other given Side, by means of ab, or ad, as before; the Center and Diſtance of either Vaniſhing Line being determined, (by Prob. 1.) as *S*, the Center of BF, by a Perpendicular from C (Th. 4.) and *S*O the Diſtance, in A*S* produced; making BO equal BG, and joining OF, BO and OF are the Radials of the Sides in that Face, forming a right Angle BOF, at the Eye.

Make OP to OQ in the ratio of the Originals of ad to de; draw PQ and OR parallel to PQ. Draw aR, which will cut dF at e; and, through e, draw Bf, cuting aF at f; and laſtly, draw fA, cuting bF at g; which compleats the Parallelopiped, abdf, required.

For, becauſe of the Vaniſhing Points A, B, and F, the Sides ab, ad, and af, form a ſolid Right Angle, at a; AGB, ATF, and BOF being Right Angles, which bad, baf, and daf repreſent, reſpectively; and the other Sides vaniſh in thoſe Points, reſpectively; as dc and de, &c. which repreſent Parallels to ab, af, &c. (Cor. to Th. 3.) Therefore bdf repreſents a right angled Parallelopiped, whoſe proportion was known, and poſition to the Picture determined.

ac, ae, and ag, repreſent Diagonals, in each Face, reſpectively, which are in proportion to the Sides, as IK, PQ and TV, reſpectively, to GI and GK, OP and OQ, TX and TY, reſpectively.

The Parallelopiped, bdf, is truly determined, according to its poſition given, in reſpect of the Picture, its place in reſpect of the Eye, and its proportion in reſpect of its Diſtance; no regard being had to its poſition reſpecting the Horizon. Wherefore it is obvious, that, to determine its Poſition, in that reſpect, the horizontal Vaniſhing Line is eſſentially neceſſary, and the Vaniſhing Lines of its Faces are determined in reſpect of their poſition to the Horizon, as well as to the Picture, as ſhall be exemplified in the next.

EXAMPLE II.

To repreſent an Octaedron, perſpectively, ſituated on a Plane inclined to the Horizon, and to the Picture; the angle of Inclination to the Horizon, being given, and the Angle which its Interſection with horizontal Planes, makes with the Picture; together with the Seat of the Object on the inclined Plane; and, its ſituation in reſpect of the Picture.*

Fig. 152. No. 2. Let ABC (No. 2.) be the Seat of that Face on which the Object reſts, on the Plane Z, which is inclined to the Horizon in the Angle X; DF is its Interſection with a horizontal Plane, and DFG the Angle that Interſection makes with the Picture; FG is the Interſection of the Picture with the horizontal Plane. The Station Point is S, the Diſtance is SG, and SE the Height of the Eye.

* An Octaedron is a regular Solid, one of the five Platonic Bodies, having eight Faces, which are equal, equilateral Triangles; about which Solid, if a Sphere was circumſcribed, every Angle of the Solid would be in the Surface of the Sphere.

No. 1. Its geometrical Conſtruction is neceſſary to be underſtood, before it be poſſible to deſcribe it perſpectively. Its geometrical Plan, on the Plane on which it reſts, is a regular Hexagon, AEBFCD (No. 1.) ABC is the Face, on which it reſts. DEF is the upper Face, EAD, EBF, and FCD are inclined Faces, above; and AEB, ADC, and BFC, below, out of ſight; IGHC is its geometrical Elevation, or Section through EC, ſhewing the Inclination of its Faces, to each other; viz. the Angle GIK equal IGH.

These preliminaries being determined, let AB be the Intersection of the horizontal Plane, or Ground Line; and, at the height of the Eye (SE, No. 2.) draw the Horizontal Line, DF, parallel to AB; let C be the Center of the Picture.

Draw CE perpendicular, equal to the Distance of the Picture, and find the Vanishing Point, D, of the inclination given. (Prob. 2. Sect. 3.) Make DEF a right Angle, and through F, draw FG perpendicular to DF. FG is the Vanishing Line of a Plane, to which, Lines vanishing in D are perpendicular. (Prob. 3.) Fig. 142.

Make F*E* equal EF, and the Angle F*E*G equal X, (No. 2.) and draw DG, the Vanishing Line of the inclined Plane, (as by Prob. 5. Sect. 3.) indefinite.

Produce EC, cuting AB at S; make SB equal GF (No. 2.) draw BD, the indefinite Representation of DF, (No. 2.) and find the Points *a* and *b*, representing a and b, in which, BA and CA (No. 2.) cut DF. (Prob. 8.)

Find the Center (*C*) and Distance of the Vanishing Line DK (Prob. 1.) make *C*E[2] (in C*C* produced) equal to its Distance, and draw DE[2].

Make the Angle DEH equal BaD (No. 2.) H is the Vanishing Point of AB (No. 2.)

Make the Angles HE[2]I and IE[2]K each of 60 Degrees; H, I, and K are the Vanishing Points, of the Sides of the two Faces ABC and DEF (No. 1.) i. e. of AB, AC, and BC, (No. 2.) and consequently of the Sides of the opposite Face.

Find the representation of that Face, a b c (Prob. 18) having obtained the Points, *a*, *b*, and *c* (representing a, b, and D, No. 2.) by means of the Intersection AB; making D*E*[2], equal DE; or, by the Intersection BJ, of the Face, a b c.

Find the Vanishing Line HM, of the contiguous Face, whose common Intersection, with that found is a b, and its Vanishing Point H. (by Prob. 5.) Its Inclination is the acute Angle GIK (No. 1.) determined by a perpendicular from E, and IG being made equal IC; which Angle is 70 Degrees.

Find the Center (O) and Distance (OE[4]) of the Vanishing Line HM, (Prob. 1.)

Draw E[4]H, and find the Vanishing Points P and Q; as I and K, above; or make equilateral Triangles, X and Y, at the Eye (E[4]) and produce their Sides to the Vanishing Line HM, cuting it at P and Q.

Through a and b, draw Qa, and Pb, cuting at d; giving a b d for that Face.

The Point d is in the upper Face, and the Sides of opposite Faces are parallel, respectively, two and two; therefore, draw dI, and dK; and Pc, cuting dK at e; draw eH and join ae, which compleats the Figure.

N. B. This Figure having eight Faces, two and two of which are parallel, there are consequently (if none are parallel to the Picture) four Vanishing Lines; but here are only two required, DL, and HM; a third will pass through I and P, and the fourth through K and Q, which, being produced, would meet in the Vanishing Point of ae.

It may be observed, that the three Sides of each Face, vanish in its respective Vanishing Line, (Theo. 10.) and the Vanishing Point of the common Interfection, of any two Faces, is the intersection of the Vanishing Lines of those Faces, (Cor. 2. Theo. 6.) as H, of ab and ef, the intersection of the Vanishing Lines DL and HM; and K, of DK and QK, the Vanishing Point of de, and bc.

The next Figure exhibits two of the same Objects, on a level Plane (or any Plane perpendicular to the Picture) situate alike to the Picture, but on different sides of the Station Line; the Sides, in both, have consequently the same Vanishing Points, and, their Faces have the same Vanishing Lines, respectively. Fig. 143.

AB is the Vanishing Line of the Faces which are perpendicular to the Picture, and C its Center; a g is the Intersection of one of those Faces. The Angle a, of one Object, touches the Picture, the other is at some distance beyond it.

FG being the Vanishing Line of a Plane perpendicular to the common Intersection a b, and F*E* (equal EF) its Distance; the Angle F*E*G is made equal to that of a diagonal Plane with the Faces, (equal HIC, No. 1.) *viz.* 55 Degrees. AG is the Vanishing Line of the diagonal Plane a b c d, which is a Square.

The Vanishing Points, A, H, and B, of the Sides a b, a f, and b f, or e d, being found (as above) and *E*K drawn, perpendicular to *E*G, cuting GF, produced, at K, the whole is determined. BK and GH intersect at I, the Vanishing Point of e b and df; and BG, KH, and IA, being produced, would meet in the Vanishing Point of a e and f c.

Plate XXXIX. AB, AI, BG, and GI, are the Vaniſhing Lines of the Faces; and AG, BI, and KL of the three Diagonal Planes. The reſt the Figure deſcribes.

The Vaniſhing Lines of the Faces produce, by their Interſections, the Vaniſhing Points of every Side of the Figure, *viz.* ſix, A, B, G, H, I, and, if BG and IA were produced, they will meet in the Vaniſhing Point of a e and f c.

Theſe Examples are ſufficient, for finding the Vaniſhing Lines, and applying them to uſe, in projecting Figures in inclined Planes, in ſuch Objects. I do not intend to lead the Reader through all the Mazes, attending the projection of the Dodecaedron and Icoſaedron, becauſe I know no end it can anſwer to any Perſon; ſuch Objects never coming in the courſe of their ſeveral Studies. But, if any Perſon be inclined to amuſe himſelf with them, he may find Rules in this Section and the 5th. for his purpoſe, in every poſition he can deviſe. As they require a geometrical conſtruction, previous to the projecting them perſpectively, I recommend the Reader, who has curioſity, to the Work of Mr. Hamilton; or, to the ingenious Mr. Highmore, they being foreign to my Plan.

EXAMPLE III.

How to repreſent a piece of regular Fortification, in Perſpective.

The foregoing Examples are calculated more for Leſſons than real uſe; in this, the knowledge and application of inclined Planes will be found neceſſary.

In Fortification or military Architecture, as the Walls of the Poligons and other Outworks are all inclined to the Horizon, and moſtly to each other, it ſeems to be the fiteſt Subject of any I know. Indeed, the vertical Planes of other Buildings are, frequently, as much inclined to the Picture, but, their poſition to the Horizon familiarizes them, on a vertical Picture; and yet, in reality, there is no difference, in Theory, and very little in Practice.

Fig. 154. The figure, and poſition of the Poligon (No. 1.) to the Picture being determined, find the perſpective Plan of its Seat on the Ground Plane (Sect. 5.) Then, BC being the Ground Line, and A the interſecting Point of one Line in the Face X, find the Interſection (AD) of that Plane (Prob. 6.) (V is a ſection of the Wall, ſhewing its inclination to the Horizon) alſo, F being the Vaniſhing Point of a b, draw FG parallel to AD; FG is the Vaniſhing Line of that Face (Th. 2.) S is its Center, *E*S its Diſtance, and *E*SC the inclination of the Plane X, to the Picture.

Now, a b is determined, and if the inclination of the Sides a c and b d, to the horizontal Lines, a b and c d, which are parallel, be known, their Vaniſhing Points are determinable (Prob. 4. Sect. 3.)

Produce SC, and make SE^1 equal S*E*. Draw FE^1, and make the Angles FE^1G and FE^1I, equal reſpectively, to the known Angles, producing the Vaniſhing Point G, of a c; and, if E^1I be produced till it meets FG, it will give the Vaniſhing Point of b d; which does not fall within the compaſs of the Picture, yet, by means of E^1I, b d may be drawn, by Prob. 13.

Then, J being the diſtance of the Vaniſhing Point G, draw J a to the Interſection AD, cuting it at *a*; make *a b* equal to the length of the Side a e, and draw *b* J, which determines the Angle e; and e F gives the Angle d, b d being drawn indefinite. Or, if the Interſecting Points, D and *b* be obtained, by making AK equal to the perpendicular height of a e, (as at V) and drawing KD parallel to AB, the reſt is unneceſſary.

By the ſame means, any other Face (as Y) may be obtained, and continued around. If the inclination of the Faces, X and Y, to each other be known, the Vaniſhing Line GH is determined, by Problem 5; or, H, the Vaniſhing Point of ah, being found, draw GH; by which, the Face Y is deſcribed; as X, by means of the Vaniſhing Line FG.

The method uſed by the old Authors, for delineating ſuch an Object, is, by forming a perſpective Plan of the whole on the Ground, and drawing perpendiculars from the Seats of the interior Angles, as c d, &c.

Of

Of the INCLINED PICTURE.

I presume, the Reader will, ere now, be fully convinced of the universality of the Principles on which the Theory of Perspective is founded; and, that it is the same thing, whether the original Plane or the Picture be inclined, or both, in respect of the Horizon; seeing, it is the position of the original Planes and Lines to the Picture, only, that is considered, in projecting them.

In Example 1st. if the Parallelopiped, being right angled, had any of its Faces parallel to the Picture; or, if they were parallel to the Horizon, and the Picture vertical; or, which is the same thing, if any of its Faces are perpendicular to the Picture; then, the Sides which are perpendicular to them, are parallel to the Picture, and consequently, have parallel representations (Theo. 9.) But, the Picture being inclined to the Horizon, whilst some Face of the Object is parallel to it, the Sides may then be all inclined to the Picture; and, the perpendicular Lines, when the Picture is vertical, are parallel to it, and have no Vanishing Point; but, when the Picture is inclined to the Horizon, they are, consequently, inclined to the Picture, and vanish either upwards or downwards, according as the Picture is inclined, towards the top or bottom of the Object; of which I shall give some Examples.

EXAMPLE IV.

How to represent, an upright Object, on an inclined Picture.

Let A B be an object perpendicular to the Horizon, and let D F be a Section of the Picture, inclined towards the top of the Object; E is the Eye of a Spectator, E C, perpendicular to the Picture, produces its Center and measures its Distance; E D, parallel to the Horizon, determines the Horizontal Line, and E F, being perpendicular, that is parallel to A B, determines the Vanishing Point (F) of Lines perpendicular to the Horizon (Prob. 2.) Fig. 155. No. 1.

Let the Picture be prepared accordingly, drawing the Horizontal Line as usual, and the Vertical Line, at right Angles, cuting it at D. No. 2.

Take DC and CF in proportion to CE the Distance taken (as in No. 1.)

In this Case, it is obvious, that the Center of the Picture, is below the Horizontal Line; as, if the Picture inclined towards B, it would be above it, at S (No. 1.) but, seeing that the Vertical Plane is perpendicular to the Picture, its Center is always in the Vertical Line (D F.)

At No. 3, is shewn the position which the Object has to the Picture (ab) laterally; which Inclination being known, find the Vanishing Points, H, and I, of horizontal Lines in the Object (Prob. 2. and 4. Sect. 3.) D *E* (equal DE) is the Distance of the Vanishing Line; proceed, in every respect, as in Example, 1st.

F, being the Vanishing Point of Lines perpendicular to the Horizon, and, H and I, the Vanishing Points, of horizontal Lines in the Object, at right angles with each other; the Object being right angled; consequently, H F and I F, are the Vanishing Lines of the upright Planes in the Object (Prob. 4.)

The Seat (s) of any Angle (the nearest to the Picture is the most convenient) being determined; or, the intersecting Point (A) of *AB* (No. 3.) draw AB, the Intersection of any Plane that Line is in †. Find *a*, the representation of the Angle *A*, its distance from the intersecting Point (a*A*) being known. (Pr. 6, or 7. Sect. 4.) or by its distance, sd, (equal *AS*) from its Seat, i. e. from the Picture. † Theo. 2.

By means of the Intersection, A B, of the upper Plane of the cross Arms, is got *a*, *b*, &c. the real measures being applied on A B, as usual; or, on A L, the Intersection of the upright Plane (parallel to I F, Th. 2.) in which is the same line, ab; on which Intersection is also applied the measures of height (Ab, ac, &c.) and projected by means of the Point Q, the Distance of the Vanishing Point of those Lines; or, the intersecting Points, K, J, and L, may be found, thus.

Plate XXXIX. Fig. 155. A being determined (by its height above the Eye, the Seat of *A*, (No. 3.) and diſtance from its Seat, with the inclination of the Line it is in, to the Picture (dac) or to its Seat) draw Ab through A, perpendicular to HI; make Aa, Ac, and cb, in the ratio of Ab, bd, and dB; as CE, to CE, No. 1; draw aK, cJ, and bL, parallel to HI, cuting the Interſection AL, at K, J, and L; from which, draw lines to the Vaniſhing Point I, and through *a*, *b*, &c. to F, cuting them at *f*, *d*, *h*, &c. From *f*, and through *b*, *d*, &c. draw to H.

Find the Center and Diſtance of the Vaniſhing Line HF, and the place of the Eye at O (Prob. 1.) make OM, and ON, in the ratio *A*d to *A*e (No. 3.) draw OP parallel to NM; and, through *d*, draw P*e*, and compleat the End *ek*, by means of the Vaniſhing Point R (in IF) of a diagonal of a Square.

The reſt is obvious, from inſpection of the Figure.

No. 4. is the ſame Object on a vertical Picture, whoſe Diſtance is ED. (No. 1.)

Here, C is the Center of the Picture, anſwering to D (No. 1.) AB is the Interſection of the upper Plane of the Croſs; and, AD and BE of the upright Faces.

The Interſecting Point of every horizontal Line is geometrically determined; the Interſections of the Planes cuting each other in the Interſecting Points, A, B, D, and E, and the reſt are ſet off geometrically from them; or the Lines are determined, and proportioned, otherwiſe, as uſual.

No. 5. exhibits the ſame Object, when the Picture is inclined towards its bottom, (as GH, No. 1.) its Center is at S, above the Object, as in No. 2, it is below.

SE is the Diſtance of the Picture; and, if the Angle SED be made equal to SED (No. 1.) the Horizontal Line *DI* is determined; alſo, EF, perpendicular to DE, cuts the Vertical Line, DF, at F, the Vaniſhing Point of perpendicular Lines, in the Object.

In this Example, the Ground Line (GH) may be uſed, conveniently; whoſe diſtance from the Horizontal, is DB, not EK (No. 1.) i. e. it is in proportion to the Diſtance of the Picture, ES.

AB is the Interſection of the Plane of the croſs Arms, in which the Interſecting Points, A and B, &c. may be determined, as in No. 2. and, BF is the Interſection of a vertical Plane of the Object. The reſt it is needleſs to particularize; the Vaniſhing Points of horizontal Lines being determined as uſual; obſerving, that the Diſtance of the Horizontal Line is not the Diſtance of the Picture, as in a vertical Picture; ſeeing that, horizontal Planes are not perpendicular to the Picture, the Picture being inclined to the Horizon. Wherefore ED, in both Pictures, is the Diſtance of the horizontal Vaniſhing Line; being conſidered, in every reſpect, as an inclined Plane, in reſpect of a vertical Picture.

In theſe Examples may be ſeen the univerſality, and the ſuperiority of Brook Taylor's Principles to all other; by which is ſhewn (in this laſt Section) that any Plane Object, whatever, may be projected, from the known poſition of one Plane to another, in the Object, and the proportion of the Lines in thoſe Planes, together with their poſition to each other; without regarding their poſition to any other Plane, whatever, except the Picture. Whereas, by the Old Authors, it was almoſt impoſſible to project them at all; or, with the utmoſt difficulty, by the Seat of each Angle on the Ground Plane, and its height above it, a troubleſome and laborious proceſs; by which was obtained the ſeveral Angles, and then joined by Right Lines; without Vaniſhing Points, of which they had not the leaſt conception, in any other Lines but horizontal; and thoſe they called accidental Points, for they had no certain method of producing them. Having found the two extremes of an inclined Line, then, drawing the Line, and producing it to the Horizon, they found its Vaniſhing Point (properly called accidental) which is now fixed with abſolute certainty, in all Poſitions of Lines to the Picture, and directs the certain poſition and place of each Line on the Picture, indefinite. And, by means of Viſual Rays, drawn on the Picture, certain portions are cut off (truly mathematical) which repreſent the Originals, as they appear to the Eye, in the true Point of View.

BOOK

BOOK IV.

Of SHADOWS, in general; of LIGHT and SHADE, REFLECTION, KEEPING, &c.

SECTION I.

An introductory PREFACE, concerning LIGHT and SHADE.

IN the foregoing Work is contained the full knowledge and practice of linear Perſpective; the laſt Book containing Rules, neceſſary for the projection of any regular Object, almoſt, whatever; with Examples, varying the application of the Rules, frequently, as occaſion requires. The linear part being projected, ſtrictly by the Rules there preſcribed, will convey a juſt Idea of the figure of the Object, and the proportion of its parts to each other (the Eye being in the true Point of View.) Nevertheleſs, there requires ſomewhat more to be done, in order to give an appearance of ſolidity, and receding of one part behind another; which indeed is effected, in ſome meaſure, by their perſpective proportions; and perfected, by a juſt gradation of Light and Shade, properly diſtributed to each part of the Object; and which, with the projection of Shadows, &c. is the Subject of this fourth Book,

It muſt not be expected that I ſhould define what Light is; having already given my Opinion on that Subject (Sect. 1. Book 1.) yet, it may be proper to give ſome general Idea of what is underſtood and meant by Light, as it is uſed by Artiſts in general, in the application of it to a Picture.

LIGHT, in that reſpect, means nothing more than the bright parts of Objects; which differs greatly in degree, according as their Surfaces are ſituated in reſpect of ſome luminous Body; which is effected either, directly, from the Luminary, or reflected, from ſome illumined Object.

SHADE is a privation of Light, the dark parts of Objects; occaſioned either by the Object itſelf, on thoſe parts which are not towards the luminous Body, or, by ſome other opake Subſtance, interpoſed between the luminous Body and the Object; depriving it, either wholly or partly, of Light.

To give or preſcribe Rules, abſolutely, for perfecting a Picture, in reſpect of Light and Shade, is as impoſſible as in reſpect of Colour; yet, by adhering to Reaſon, and carefully obſerving Nature, we may arrive at a tolerable degree of perfection. In the firſt place, it is neceſſary to conſider how the Object is ſuppoſed to be ſituated to the luminous Body, from which it is to receive the Effect.

It has been almoſt a general Rule, amongſt Artiſts, to ſuppoſe Light to flow from the left hand to the right; but that is entirely arbitrary, and can have no foundation in the nature of things, but merely an habitual cuſtom; it is, however, proper to imagine it to flow from one hand or the other; for, to ſuppoſe the Luminary, or luminous Body, directly oppoſed, either on this or on the other ſide of the Object, can never produce a pleaſing effect; ſeeing that, in the firſt caſe, it is almoſt wholly illumined, in the other, it is wholly deprived of Light; an agreeable mixture of both, judiciouſly diſpoſed, is what contraſts one Object, or part of an Object from the other, and renders the whole agreeable to the Eye, as a pleaſing imitation of Nature.

Plate XL. There is, moreover, an inconveniency arifing from the cuftom of fhading Pictures, on the left hand, generally, feeing they cannot fuit both fides of a Room; for, being placed, in that refpect, properly, adds greatly to the effect of the Piece.

Let it be carefully obferved, that, that part of an Object, to which any luminous Body is moft directly oppofed, will be the brighteft; but, when, from the fituation of the Eye, it is much contracted, or forefhorted, it may appear as dark, or perhaps darker than other parts, on which, the Light is not fo direct.

Fig. 1. For Example. Let Z be the Plan or Bafe of fome prifmatic Object, and AB, &c. Rays of Light, falling thereon. Becaufe thofe Rays fall more perpendicularly on the Face BC, than on BD, that Face is, confequently, more illumined than BD, and BD than DF, whilft FG is wholly deprived of Light; and GH, being farther removed from the Light, will be darker than FG, from the effect of Light fimply, or direct. Now, if a Spectator be fo fituated, at E, that the Face BC, which is moft illumined, is much contracted, it may in fome fituations of the Object to the Light, appear darker than BD, which is more oppofite to the Eye; whereas, if the Station be at *E*, where both appear equally contracted, the Face BC will appear the brighteft; but, if the Light was in the direction of *E*B they would be, and alfo appear equally bright, without diftinction.

Any direction to the Rays of Light may be given, at difcretion, as AB; and, by drawing others, parallel to AB, it is eafy to know which Face fhould be the brighteft, and which the next, &c. but to determine in what degree, pofitively, is not poffible; a careful obfervation of Nature is the beft criterion by which to judge of that; and, even in that cafe, it is not eafy to determine, without experience and found judgment.

After the fame manner, the inclination of the Rays of Light to the Horizon, being taken into confideration, may be determined, whether horizontal Faces of Objects, below the Eye, or inclined Faces, are more illumined than the vertical. As Light flows from above, the whole Hemifphere being illumined (when the Luminary itfelf is not much elevated) the horizontal faces of Objects, or fuch as are much inclined, are, generally, the brighteft; provided they are oppofite to that quarter from which the principal Light flows. Notwithftanding it is ufual to fhade the Roofs of Buildings, Pediments, &c. more than the vertical Planes, it can only be from a fuppofition of their being of darker materials, as Lead, Slate, &c. for, if they were compofed of the fame, that is, if they are of one uniform Colour (and otherwife, no pofitive determination can be made, in refpect of Light, from obfervation) inclined Planes, will be brighter than either vertical or horizontal, being, generally, more oppofed to the Light.

In refpect of curved Surfaces, from the effect of Light, fimply or directly, being convex, thofe parts which are moft towards the luminous Body, are the brighteft; and confequently, thofe which are fartheft removed from the Light will (without reflection) be the darkeft. On concave Surfaces, the effect is reverfe.

Fig. 2. On cylindrical Surfaces, as Columns, ftreight Mouldings, &c. the Light is always parallel to the fides, or edges of the Mouldings; the greateft Light being on that part which is moft perpendicularly oppofed to the luminous Body (as AB) from which it gradates regularly, on each fide; confequently, that fide which is fartheft from the Light will be the darkeft, fuppofing the Luminary to be fituated on this fide of the Object, on either hand. But, if it be fuppofed fituate on the other fide, the Edge, AB, towards the luminous Body, will be the brighteft; and it will be gradated from that Edge, not to the other, as in No. 2. (which has not the appearance of an entire Column or Cylinder, but of a Segment, cut off parallel to its Axe) but, it will be darkeft fomewhat from the other Edge (as No. 3.) not owing to reflection from other Objects, but from the luminous Body, being on the other Side; which, as it is more or lefs direct, will occafion the darkeft part to be more or lefs removed from the middle.

Convex fpherical Surfaces are brighteft on that part to which the luminous Body is perpendicular, from which it gradates every way equally; infenfibly varying, to the extremes, provided no other Object interfered, to reflect Light. If a Hemifphere, or leffer Segment, be properly fhaded, it will reprefent equally as well a concave as a convex Surface, by fuppofing the Light on one fide or on the other; that is, when the Eye is fo fituated as to fee the whole circumference of its Bafe, or nearly; as Fig. 3. Fig. 3.

Refpecting ftreight mouldings, which are compofed wholly of Planes and cylindrical Surfaces, their different fituations, towards the flow of Light, occafion variety of Shades. As for example; the Cima-recta (compofed of a convex and a concave cylindrical Surface) in its proper pofition, in a Cornice, &c. has two ftrong Shades and one Light, in the middle; whereas the very fame Moulding, reverfed, as for a Bafe, has two Lights, and one Shade, in the middle, with a faint one at each Edge. The Cima-reverfa, in its proper pofition, has two Lights and one ftrong Shade, in the middle; the fame Moulding reverfed, has two faint Shades and one Light, in the middle.

Figure 4 is the Profile of a Cornice; fuppofe the Light to flow in the direction AB, at difcretion. It is eafy to determine, what parts of the Cornice will be light, and which fhaded. For, drawing feveral Lines, touching the projectures and prominances, parallel to AB, it is obvious that the edge a, obftructing the Light, muft neceffarily occafion a ftrong Shade, below; which gradually dies away into the Light, at b, in the middle, where the Rays fall oblique on it; and where, the prominance or fwell of the convex part occafions another Shade, at c; begining faint, and gradually ftrengthening, reverfely. Fig. 4.

The fame reafoning accounts for the Shade in the middle of the Cima-reverfa.

The great projecture of the Corona, (X) throws all below in Shade, which gives great expreffion and force to the whole, making the upper part appear to ftand off, from the Canvas or Paper. The whole, below, being immerfed in Shade, would be totally loft to fight, was it not, in fome degree, illumined by reflection; on which I fhall fpeak in its place; let it fuffice, here, to obferve, that the effects are almoft wholly reverfed, and but faintly expreffed.

Let No. 2. reprefent the upper Mouldings reverfed, for a Bafe Moulding. No. 2.

Here, it is obvious, that the Rays, falling on it in the fame direction, illumines the whole; infomuch that, no part can be faid, with propriety, to be in Shade; neverthelefs, the parts *a*, *b*, &c. which are moft directly oppofed, will be brighter than the other, which are faintly fhaded, as the Surface falls off.

I fhall juft make an obfervation on the prevalent Cuftom of fhading Mouldings, in architectural Defigns, as ftrongly illumined by the Sun; which entirely deftroys their proper effect.

Suppofe AB (No. 3.) to reprefent a Ray from the Sun. It is obvious, that all the part, from B to *b*, will be in Shade; and, being a Shadow, projected by the upper edge (BC) ftrongly defined by a Right Line (*bc*) has not a very agreeable effect. The Shade, below, is alfo more fudden and hard. No. 3.

Now, if by means of Light and Shade, it is intended to give an Idea of Mouldings, I would afk, ferioufly, which has the moft natural appearance and effect? Suppofe the Profile cut off, or covered; would any Perfon conceive what this Moulding is intended to reprefent? But, where is the neceffity to fuppofe the Sun to fhine on them? as there are ftrong Shades when it does not; nor are they intended as Pictures, but Defigns, which ought to exhibit what is intended, in the moft expreffive manner poffible.

If my opinion might be allowed to have any weight, I fhould fuppofe that a Section fhewing the infide of a Building, geometrical, would be more expreffive, by means of penumbral, inftead of ftrong edgy Shades, which are by no means natural, in fuch Cafes. Nay, fo fond are fome Architects of forced Effects, that we frequently fee a ftrong right line of Light introduced into a Section of a Dome, through

Plate XL. Fig. 5. through an opening at the Top (AB) as ſtrongly defined on the part at C, which is nearly horizontal (where it is abſolutely impoſſible for Light to come) as elſewhere, at D; where, an elliptical figure of the opening would, only, be deſcribed; the concave Cylinder, below, being ſhaded without any edge at all.

Similar to this are the forced and unnatural Effects, which may frequently be ſeen, in Prints and Paintings, of a ſtrong Light being introduced, ſtriking in a Right Line, from the upper edge of the Cornice, acroſs the Side of a Room, as if it was open at the top. Alſo, on exterior Objects, a ſtrong diagonal Shade frequently croſſes a Building, without the leaſt apparent Cauſe for ſuch an Effect.

There is (to me) another great impropriety; which, though it does not come immediately within my cognizance, I ſhall beg leave juſt to mention; which is, the introducing Landſcape, and ſome appearance of Perſpective, in a geometrical Deſign. I am well aſſured that it need but be pointed out, to convince any ſenſible Perſon of its inconſiſtency, as a Picture; exhibiting a geometrical Elevation of a Building, placed upright in a Garden, &c. where every thing elſe appears real.

When I ſee a number of Drawings, geometrical Elevations of Buildings, at an Exhibition, vying with each other, which ſhall make the beſt figure, as a Picture, more than as a Deſign (for which, the Architect is obliged to ſome Landſcape Painter, frequently) I am apt to imagine myſelf in Vauxhall Gardens, where, the luxuries of Art eclipſe the beauties of Nature; and, inſtead of a Garden, we, in reality, find ourſelves in ſome ſpacious Building; the ſeveral Walks and Avenues of which, reſembling Galleries of communication to various detached Appartments.

SECTION II.

The Theory of SHADOWS, projected by the Sun.

IN treating on Shadows, it is neceſſary, firſt, to conſider the Situation, Diſtance, and Magnitude of the luminous Body, by which the Shadows are projected. Secondly, the Situation of the Object to the Luminary, and, of the Lines bounding Planes, &c. whoſe Shadows are to be projected; their Poſition in reſpect of ſome other Plane, on which the Shadow is to be projected; and laſtly, the Poſition of the Luminary, in reſpect of the Picture.

The Sun, which is the grand fountain of Light, being placed in the wide Expanſe, diſpenſes its Light, equally, all around, and illumines a concave Sphere, as far as its influence extends; which differs, in degree of Splendor, according to the Diſtance from the luminous Body. If an opake Body be interpoſed, that Body is illumined on its Surface towards the Sun; and conſequently, muſt deprive ſome part of the Expanſe of Light; and, if another opake Body intervene, the Shade, or Shadow of one, is thrown upon, or projected on the other.

A SHADOW, therefore, is the outline projection of one Object on another, Plane or other Superficies, in Right Lines, from ſome luminous Body to the Plane of Projection; i.e. the Plane on which the Shadow is projected.

By Rays of Light is to be underſtood Right Lines, from the Luminary, paſſing by the extremes of an Object, and projecting its Shadow.

Fig. 6. To illuſtrate what has been advanced, ſuppoſe the Body, a Globe, at S, to repreſent the Sun, and CD a portion of a concave Sphere, illumined by its Light; let E be ſuppoſed an opake Body, illumined on the Surface towards the Sun.

7 Now,

Now, if this opake Body be also a Globe, representing the Earth or any other Planet; the Sun being, in proportion to it, immensely great, the Space deprived of Light, in the Expanse, will form a Cone; as ACB.

If a Plane, or any other opake Surface, be placed at FG, within the extent of the Cone, it will receive, thereon, the Shadow of the opake Body, at H; which Projection, if the Plane be direct to the Sun, will be a Circle; but, if the Plane be inclined, as DL, it will form an Ellipsis, at I. These Projections, or Shadows, are less than the Object; because the luminous Body is greater. But, if the luminous Body be equal to the Object, or at an infinite Distance; the Shadow, on the direct Plane, will be equal to the Object, in diameter, and may be considered as its orthographic Projection.

Hence it is evident, that, from the magnitude of the Sun in proportion to the Earth, the Rays of Light, from its extremes to the Earth, are converging; but, when its immense Distance is considered; they are, to all sense, parallel. Then, seeing that the largest Object, on the Earth, can scarce be said to bear any proportion to the whole, we may consider them as being perfectly parallel.

Next, the Altitude of the Luminary is to be considered, so as to give the best and most pleasing effect to the Picture; this is generally at the discretion of the Artist. It may, to some, be necessary to explain what is meant by its Altitude.

Let the Semicircle AFB be supposed an Arch in the Heavens, in which is the Sun's *apparent* diurnal motion; and let E be the Earth, *supposed* at rest. Fig. 7.

If the Sun be at C, the Ark CB is the measure of its Altitude; and, the Angle CEB is the Angle of Elevation; EB, being considered as the Horizon, and CE the direction of the Rays of Light. If it be at D, then, DEB is the Angle of the Inclination of the Rays of Light to the Horizon, and the ark DB the measure of its Altitude; but, when its place is at F, in the middle of the Arch (90 Degrees, each way, from A and B) its Rays have, then, no Inclination to the Horizon, AB; and consequently, a perpendicular Line, GE, will project no Shadow.

This can only happen to those parts which lie between the Tropics; for although, at the Equinoxes, the Sun appears to describe a Semicircle, at all parts of the Earth; yet, according to the distance from the Equator, the Plane of its apparent motion is more or less inclined to the Horizon of the place; consequently, its Altitude, when in the Meridian of any place, is more or less elevated above the Horizon, from the Equator to the Poles, where it describes an entire Circle, around the Horizon, having no Elevation, or Meridian.

Lastly; the Angle of Inclination which a vertical Plane, passing through the center of the Luminary, makes with the Picture, is necessary to be known, when it is not in the Plane of the Picture.

Now, although the Rays of Light, proceeding from the Sun, are parallel amongst themselves, in respect of the Earth, yet, their inclination to its Surface varies infinitely.

Suppose a Globe illumined by the Sun, at S, whose Center only is considered*. Fig. 8.

* In projecting Shadows by the Sun, in order to describe the true Contour, or outline of the Shadow, it is necessary to consider it as a luminous Point, only, at an infinite Distance; for, if its magnitude be taken into consideration (since every part of its Surface emits Light) the Shadow of a Point, would always be, in proportion to the Sun, as the distance of the same Point, from the Plane of Projection, is to its distance from the Sun. Let S be the Sun, and A, a Point whose Shadow is projected, on the Plane X. Fig. 10.

Now, the whole extent of the Shadow of the Point A is a Circle, whose Diameter is *ab*; for, since it is obvious, that every part of the Sun must emit Light; consequently, a Ray of Light emited from a, and passing through the Point A, will project its Shadow to *a*; and a Ray from b will project the same Point to *b*, &c. while the Center (S) only, being considered, will project it to B, which is the center of the Shade, and its real Shadow. For (supposing the Point to have substance) every other part of the Shadow, is more languid, the farther it is from the Center, B; consequently, at its extremes, it cannot be distinguished from the surrounding Light; seeing that, every Point, in the Sun's surface, emits light. Wherefore, since the Triangles, aAb, *aAb*, are similar, it will consequently be, as, *Aa* is to Aa, so is *ab* to ab, that is, as AB to AS. (6. 6. El.)

Hence, it is easy to account for the Penumbra of Shadows; which, at a distance, appear distinctly defined, but on approaching near, we find it otherwise; insomuch that, except where the Lines, in any Object occasioning the Shade, cut the Surface on which the Shadow is projected, we cannot trace a line at all; and, the farther the Shadow is from that Point, the more penumbral it becomes; that is, the less distinctly defined; till, at a considerable distance, it becomes insensibly mixed with the Light.

Let

Plate XL. Fig. 8. Let AB be the Horizon of the part at D, to which, the Sun is directly oppofite. The Ray SC is perpendicular to the Horizon, AB; and confequently, a perpendicular Line, as CD, to the Horizon of that place, can have no Shadow, but will be projected towards the Center. But, if any other Tangent, as EF, be drawn; the direct Ray SC, to the Globe, is inclined to it, in the Angle SDE; and therefore, Lines perpendicular to EF will project Shadows. As CE to D.

Fig. 9. To illuftrate this more clearly; fuppofe the Plane AB, horizontal, a portion of the Earth's Surface, to which the Rays are inclined, and SE the direction of a Ray of Light. The Angle of Inclination, to the Plane AB, is SED; and, the length of the Shadow of the Perpendicular CD, is DE.

Hence it is manifeft, that the Shadow of a Right Line, on a Plane, is always a Right Line (1. 7. El.) for it is projected by a *Plane of Shade*, occafioned by the Line; as CED, which cuts another Plane, whofe common Section is the Shadow.

And, it is evident, that, the greater the angle of Elevation is, the fhorter is the Shadow; for, FC projects the fame Point, C, to G; confequently, being elevated perpendicularly over C, the Line CD will project no Shadow.

In projecting Shadows by the Sun, there are three Cases to be confidered, or fituations of the Luminary; *viz.* it muft be either on this fide, or on the other fide, or in the Plane of the Picture.

In order to determine the Shadows of Objects, delineated on the Picture; having firft confidered and determined on the fituation and altitude of the Luminary, the next thing requifite, if the luminous Body, or Point (for it is always confidered as fuch) be not in the Picture, is to find the Vanifhing Point of the Rays of Light; but, when it is in the Plane of the Picture, the Rays are parallel on the Picture, and confequently they have no Vanifhing Point; the Angle of Elevation is, then, only neceffary to be confidered.

Fig. 11. Let FGHI be the Picture, and EC the Diftance of the Picture. Alfo, let SE be fuppofed a Ray of Light.

Imagine a Plane (ESB) to pafs through the Luminary, and through the Eye, at E, cuting the Picture in FG; in which Line, whether the Sun be on this, or on the other fide of the Picture, as at S, its apparent, or tranfprojected place muft neceffarily be; as at F or G.

For, being on the other fide, at S (fuppofed at an immenfe Diftance) and being in the Plane ESB, confequently ES is in that Plane, and will cut the Picture in their common Interfection (FG) at F. Wherefore, F reprefents the Sun on the Picture, and is, confequently, the Vanifhing Point of the Rays of Light.

When the Sun is on this fide, at S^1, or S^2, being ftill in the fame Plane, produced, confequently S^1, or S^2E, being produced, will alfo cut the Picture in FG, as at G; and, becaufe all the Rays are parallel amongft themfelves, and all Lines which are parallel have the fame Vanifhing Point, confequently, G is their Vanifhing Point. (Def. 22.) for S^1.G is a Ray of Light, to which all other Rays are parallel; or, it is a Right Line, paffing through the Eye, parallel to them.

N. B. In the former Cafe (when the Sun is on the other fide of the Picture) it is obvious, that its place, in the Picture, muft neceffarily be above the Horizontal Line, as it cannot be feen till it is above the Horizon; fo, in this Cafe (feeing it is tranfprojected) being above the Horizon, its tranfprojected place muft neceffarily be below the Horizontal Line; from which (in either Cafe) it is farther removed, as the Luminary is more elevated above the Horizon. Alfo, whether its real place be on the right hand, or on the left, its apparent place is the fame, in the former Cafe, but reverfed in the other. (See Transprojection, in the Introduction; Page 52.)

Hitherto I have proceeded introductorily, which, I have called the Theory of Shadows; feeing that, all which has been faid is theoretic. I fhall now proceed to practice, and lie down fuch Rules as are neceffary, for projecting the Shadows of all regular Objects, particularly fuch as are right lined; which may be defcribed on plane Surfaces, with facility and certainty, by adhering to the following Rules.

Plate XXXVII

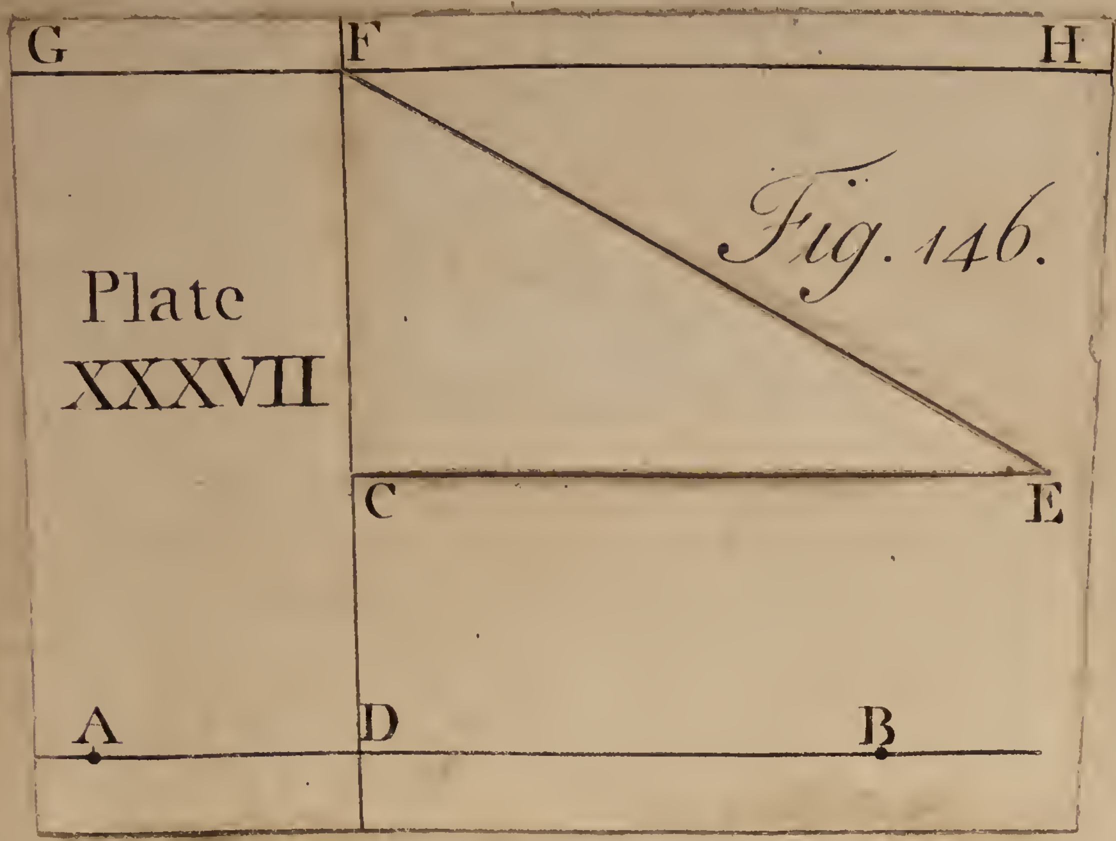


Fig. 146.

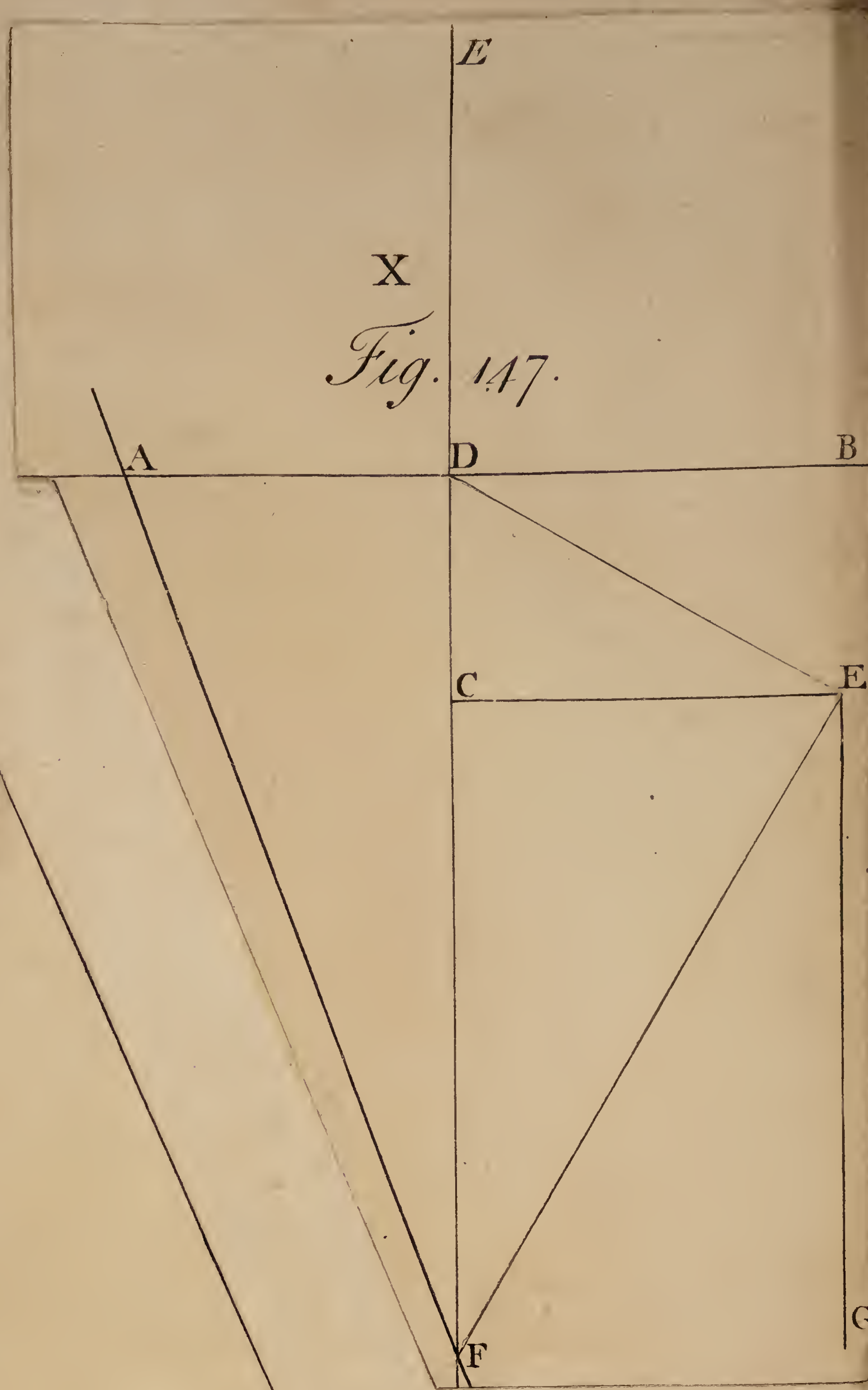


Fig. 147.

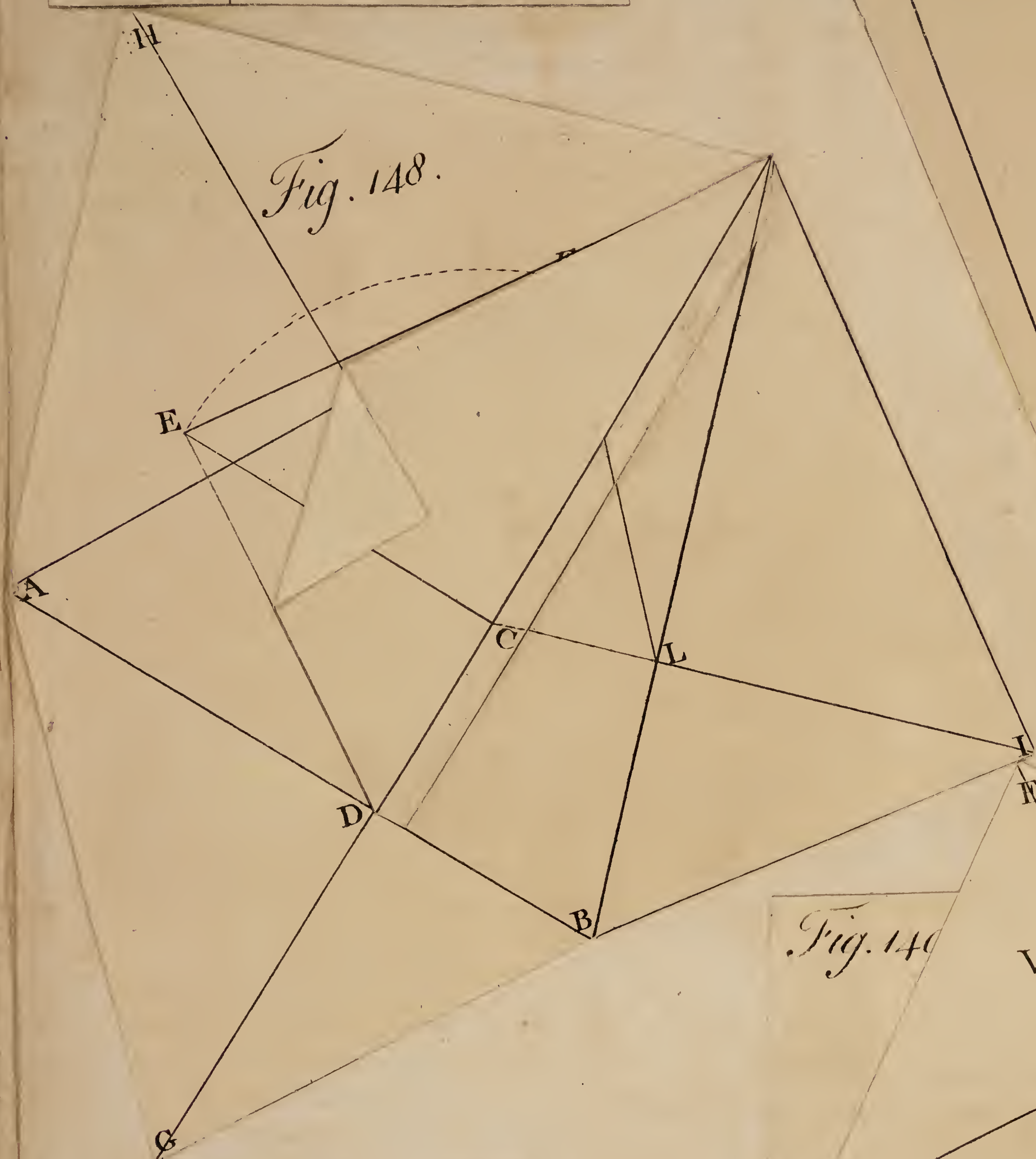


Fig. 148.

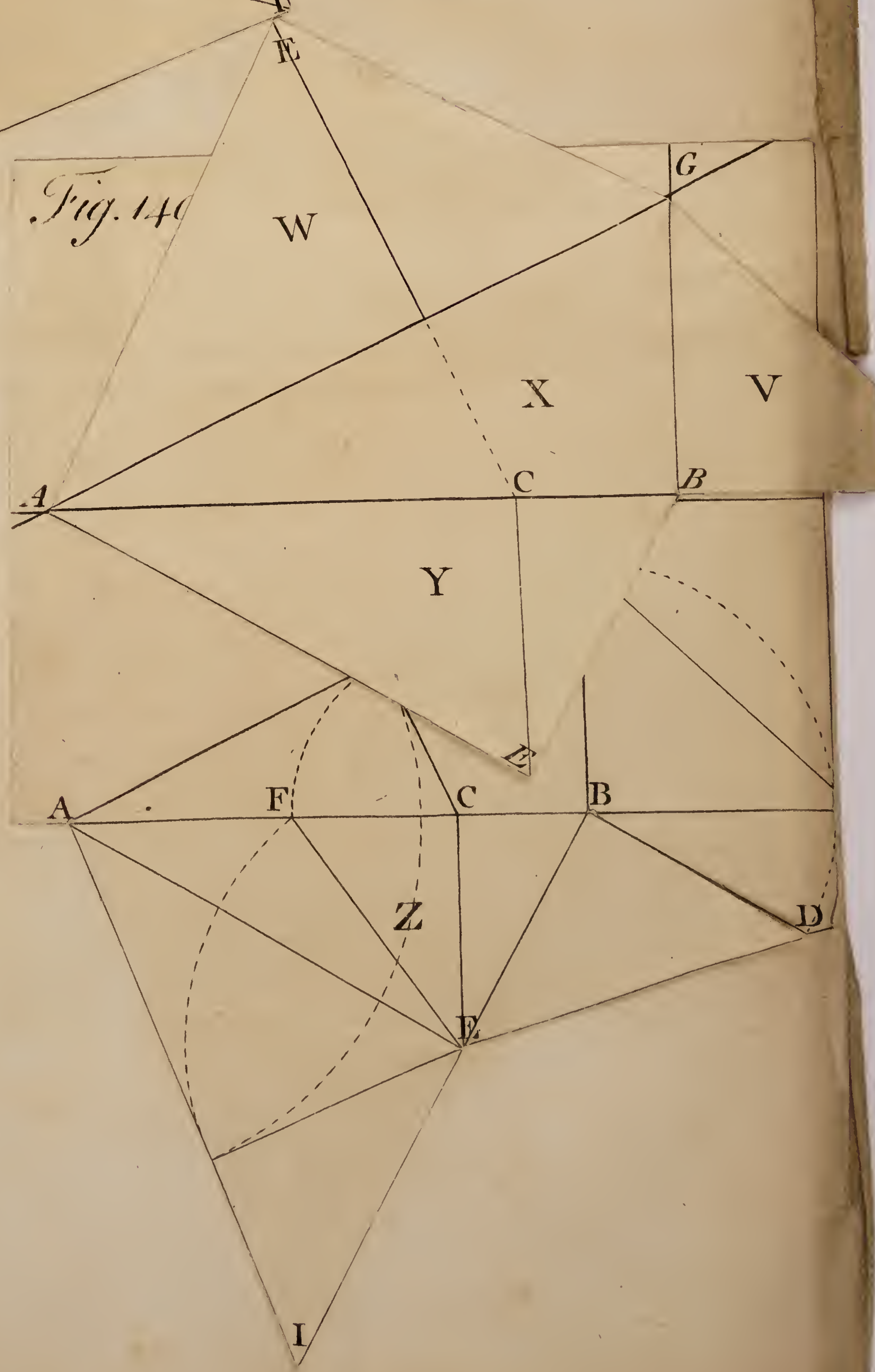


Fig. 149

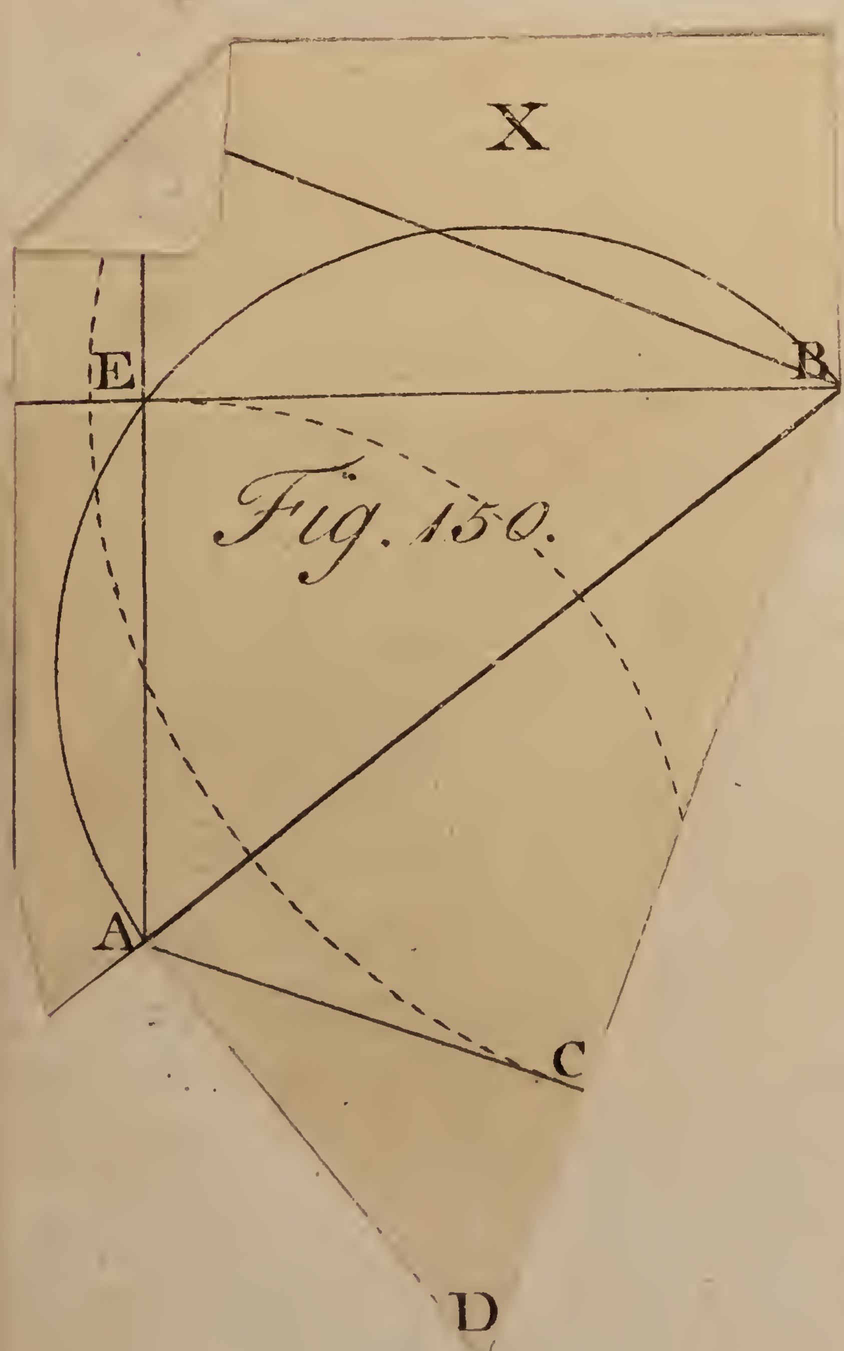
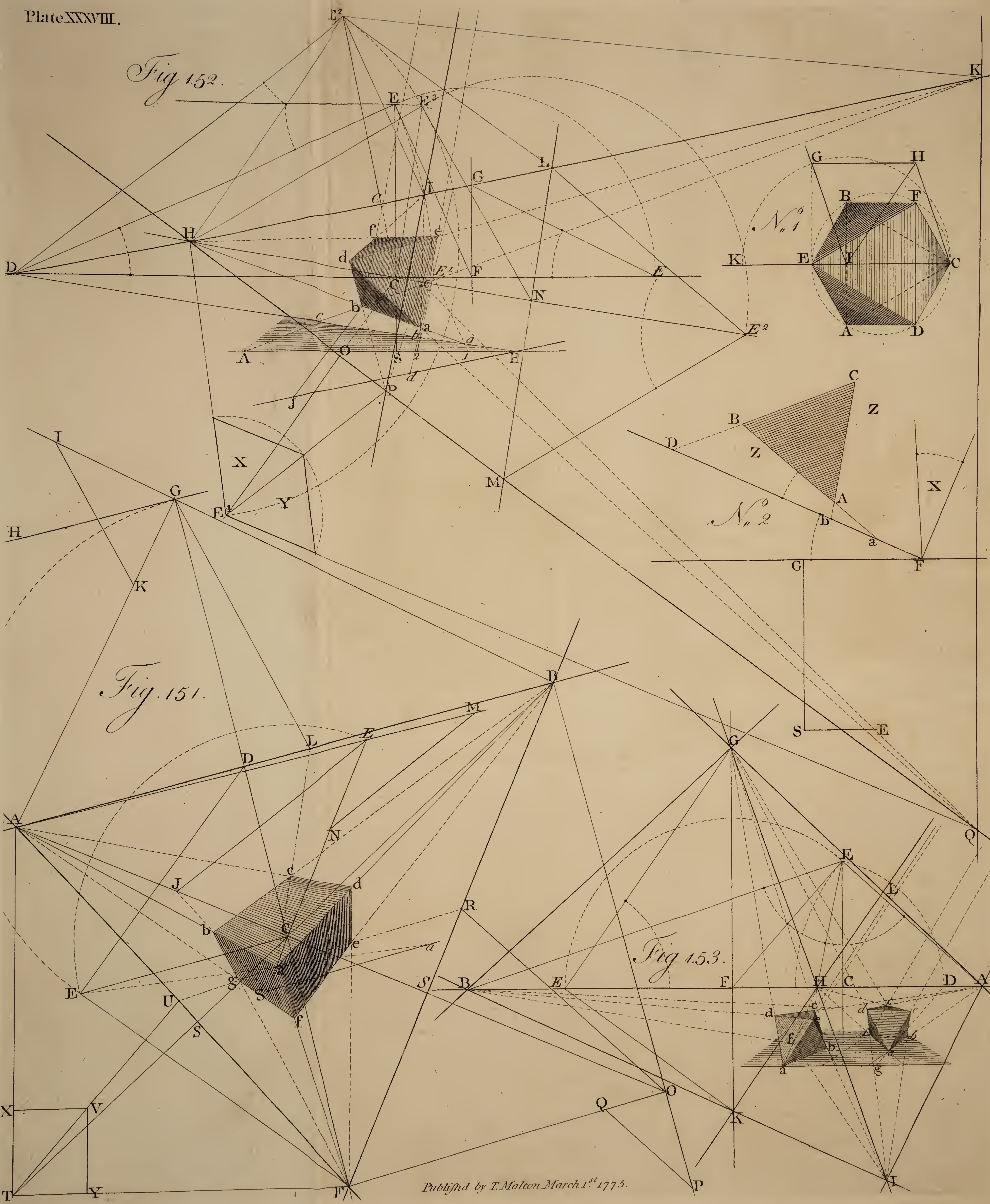


Fig. 150.

Plate XXXVIII.



Publish'd by T. Malton March 1st 1775.

Plate XXXIX

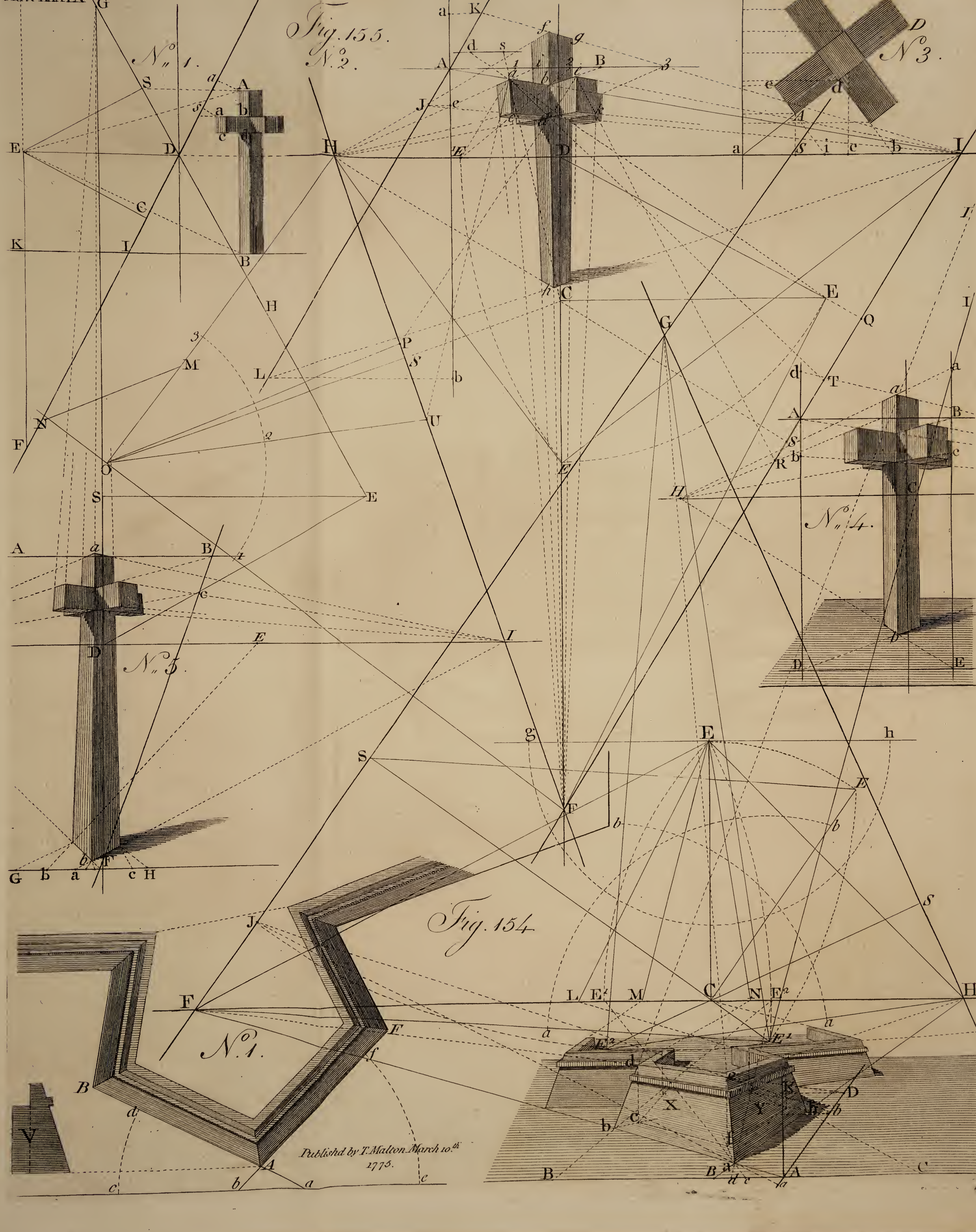
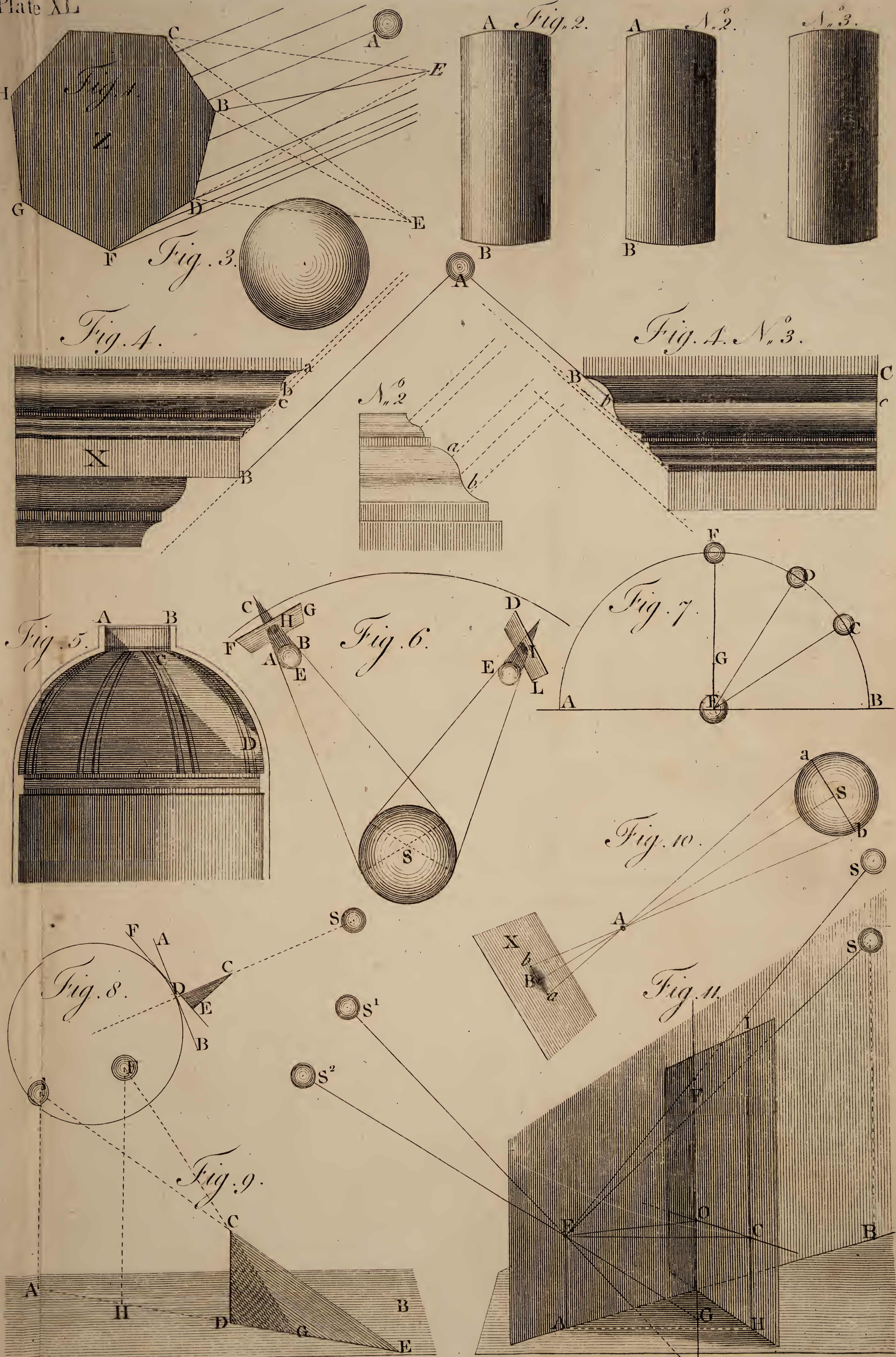


Fig. 155.
No. 1.
No. 2.
No. 3.
No. 4.
No. 5.
Fig. 154
No. 1.
Publish'd by T. Malton March 10th 1775.

Plate XL.



Publish'd by T. Malton, May 21.st 1775.

FIRST. The indefinite projection of the Shadow of a Right Line, on any Surface whatever, is the Interfection of that Surface by a Plane, paffing through the luminous Point and the Line; which, for diftinction fake, I fhall call the PLANE OF SHADE. This is obvious in itfelf.

SECOND. The Vanifhing Line of the *Plane of Shade*, projecting the Shadow of any Right Line, is a Right Line drawn through the Vanifhing Point of the Rays of Light, and the Vanifhing Point of the Line whofe Shadow is required (Th. 10.)

Becaufe, the Vanifhing Point of the Line, whofe Shadow is projected, and the Vanifhing Point of the Rays; projecting the Shadow, are in the Plane of Shade.

THIRD. The Vanifhing Point of the Shadow of any Right Line, on a Plane, is the interfecting Point of the Vanifhing Line of that Plane, and the Vanifhing Line of the PLANE OF SHADE. (Cor. 2. Theo. 6.)

Becaufe, the common Interfection of thofe Planes is the Shadow required.

N. B. When the Sun is in the Plane of the Picture, there being no Vanifhing Point of its Rays, a Right Line, drawn through the Vanifhing Point of any Line whofe Shadow is required, parallel to the given Ray, cuts the Vanifhing Line of the Plane of projection, in the Vanifhing Point of the Shadow.

Alfo, if the Original Line be parallel to the Picture, and confequently has no Vanifhing Point; (the Sun being on either fide of the Picture) then, a Right Line drawn through the Vanifhing Point of the Rays, parallel to the Line whofe Shadow is required, cuts the Vanifhing Line of the Plane of projection, in the Vanifhing Point of the Shadow.

But, when the Luminary is in the Plane of the Picture, and the Line, whofe Shadow is required, parallel to it, the Shadow has no Vanifhing Point; for it is, in fuch Cafe, neceffarily parallel to the Picture; therefore, parallel to the Vanifhing Line of the Plane of projection.

SECTION III.

Of the PROJECTION of rectilinear SHADOWS.

PROBLEM I.

Having the Angle of the Sun's Elevation (X) *and the Inclination, to the Picture, of a vertical Plane paffing through its Center* (*equal* Z) *with the Center and Diftance of the Picture given; to find the Sun's place on the Picture, or the Vanifhing Point of the Rays of Light.*

Let CE be the Horizontal Vanifhing Line, and C the Center of the Picture. Fig. 12.

Make CB equal to the Diftance, perpendicular to CE, and draw AB parallel to CE. Draw BD, making the Angle ABD equal to Z, cuting CE, at D; and through D, draw FG, perpendicular to CE. FG is the Interfection of a vertical Plane paffing through the Sun, with the Picture, whofe inclination to it is equal Z.

Make DE equal to DB, and the Angle DEF, or DEG, equal to X, the Angle of ths Sun's Altitude; F or G is the Vanifhing Point fought.

DEM. Turn up the Triangle CBD, perpendicular, and turn over the Plane FEG, or FG, till DE coincides with DB.

Then, if the Sun be on the other fide of the Picture, EF is parallel to the Sun's Rays; becaufe the Luminary is in the Plane EFG produced, and, the Angle DEF is equal to its Elevation; confequently, EF produced would pafs through the Center of the Luminary, wherefore, F is the place of the Sun on the Picture; i. e. F reprefents the Sun; confequently all the Rays center there, and therefore it is their Vanifhing Point. For its Diftance is infinite, to all fenfe.

CASE 2d. When the Sun is on this fide the Picture, being behind the Spectator, it cannot appear in the Picture, but its place is tranfprojected to G; for, DEG is the Angle of its Elevation, and it is in a continuation of the Plane FEG; wherefore, EG is parallel to the Rays of Light; confequently, G, the tranfprojected place of the Luminary, is their Vanifhing Point.

Or, as Brook Taylor, very pertinently, calls it, the Shadow of the Spectator's Eye, on the Picture; for E is the Eye, wherefore GE, being produced, would pafs through the Sun's Center.

Plate XLI.

PROBLEM II.

The Vanishing Point of the Sun's Rays being given, and the representation of a Line perpendicular to some Plane, whose Vanishing Line is given; together with the Center and Distance of the Picture; to find the representation of the Shadow of that Line, on the Plane, and its Vanishing Point.

Fig. 13. Let S be the Vanishing Point of the Rays, VL the Vanishing Line of the Plane, and AB the given representation of a Line perpendicular to that Plane. C is the Center of the Picture, whose Distance is known.

Find F, the Vanishing Point of Lines perpendicular to the Plane, whose Vanishing Line is VL (Prob. 2. Sect. 12. B. 3.) consequently, AB vanishes in F; draw SF, cuting VL at V. V is the Vanishing Point of the Shadow.

Draw VB and SA, intersecting at D; BD is the Shadow of AB, required.

Dem. Whether S be considered as the Image of the Sun, on the Picture, or *S* its transprojected place, SF is the Vanishing Line of the Plane of Shade, for all Lines perpendicular to that Plane; seeing that, F is the Vanishing Point of all such Lines; wherefore, V, its intersection with VL, is the Vanishing Point of the Shadow, of all Lines perpendicular to the Plane whose Vanishing Line is VL.

For, SV and AB represent parallel Lines (Cor. to Theo. 3.) wherefore, a Plane may pass through both Lines (Ax. 5.) and consequently, SA, VB, AF, and SF are all in that Plane.

But, F is the Vanishing Point of Lines perpendicular to the Plane; and, because S represents a Point at an infinite Distance in that Plane, SF is the Vanishing Line of a Plane passing through AB.

But, S represents the Sun, the Vanishing Point of the Rays of Light, and V is its Seat on the Plane (whose Distance is supposed infinite) consequently, V is the Vanishing Point of the Shadow, BD; for, the Plane of Shade, SAF, projecting its Shadow, is parallel to a Plane passing through the Eye and the Points S and F, seeing they are at an infinite distance; and the Shadow, BD, is the intersection of that Plane with the Plane of Projection; therefore its Vanishing Point is V, the intersection of their Vanishing Lines, VL, and SF. (Cor. 2. Theo. 6.)

This Problem is universal; I shall, next, apply it to Planes which are perpendicular to the Picture.

No. 2. Secondly. Let AB be a Line perpendicular to the Horizon, whose Shadow is required, and S the representation of the Sun, on the Picture; or *S* its transprojected Image, the Sun being supposed on either Side.

Draw SV perpendicular to the Vanishing Line; then, V is the Seat of the Luminary, its Distance being supposed infinite.

Draw VB and SA, as before, intersecting at D; BD is the Shadow required.

For, because AB, whose Shadow is to be projected, is perpendicular to the Horizon, and the Picture is supposed vertical; it is parallel to the Picture, consequently SV, parallel to AB, is the Vanishing Line of the Plane of Shade; wherefore, V is the Seat of the Luminary, the Vanishing Point of the Shadow; and, SD is a Ray of Light, projecting the Shadow of the Point A, which determines its length, BD.

This is universally applicable to all Planes which are perpendicular to the Picture, whether they be horizontal, vertical, or inclined to the Horizon.

Thirdly. When the Luminary is in the Plane of the Picture.

In this Case, the Rays having no Vanishing Point, the inclination of the Rays to the Plane of projection, being determined, they are all parallel on the Picture; seeing, the distance of the Vanishing Point is infinite.

No. 3. Draw BD parallel to the Vanishing Line, and SA, cuting BD at D, making the Angle SDB equal to the Angle of the Sun's Elevation; BD is the Shadow required.

For, because the Luminary is supposed in the Plane of the Picture, it is consequently in every Plane parallel to the Picture; wherefore, the Plane of Shade, SDB, is parallel to the Picture, seeing the Line AB is parallel; and consequently, BD, its Shadow, is parallel to the Vanishing Line of the Plane of Projection, seeing it is parallel to the Picture.

When

When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this side.

Let AB represent a Line perpendicular to the Plane X; and, let S be the transprojected Image of the Sun, on the Picture; C is its Center. Join CS. Draw BD, parallel to CS, and AS, cuting BD, at D; BD is the Shadow required No. 4.

Because the Plane (X) is parallel to the Picture, it has no Vanishing Line; wherefore, seeing the distance of the Luminary is supposed infinite, its Seat, on that Plane, is also at an infinite Distance.

But CS, produced, is the Seat of a Ray of Light, projecting its Vanishing Point, S; for it passes through the Eye, which is perpendicularly opposite to C; wherefore, the Seat of the Luminary is at an infinite distance, in the Line SC, produced. Consequently, the Shadows BD, *BD*, &c. are all parallel to CS; and S is the Vanishing Point of the Rays, which determine the length of the Shadow.

PROBLEM III.

To project the Shadows of Right Lines, on a Plane to which they are parallel; and, in all positions to the Picture.

First, when the Lines are parallel to the Picture, and the Luminary in the Plane of the Picture.

The Shadows of Lines, on a Plane to which they are parallel, cannot be determined, conveniently, without having their Seats on that Plane; and is the same as finding the Shadows of Lines perpendicular to the Plane; or, the Shadow of one extreme of the Line being found, the other is easily determined.

Let AB be a Line parallel to the Picture, and parallel to the Ground Plane, on which the Shadow is to be projected. Its Seat is *ab*. Fig. 14.

If the Sun be in the Plane of the Picture, and if it was in the Zenith of that place, the Shadow of AB is *ab*. But, if the altitude of the Sun be the Angle SCD; then, drawing AD, parallel to SC, the Shadow of AB is CD; for, the Shadow of a Perpendicular, A*a*, is *a*D; and CD is equal AB, seeing, ABCD, the Plane of the Shadow, is a Parallelogram.

After the same manner, the Shadow of the inclined Line, EF (being parallel to the Picture) is projected; by producing the Line till it cuts the Plane, at G; then, drawing GI, parallel to the Vanishing Line, and SH, SI, through F and E, parallel to SC, cuting GI, at H and I; HI is its Shadow. No. 2.

Secondly. If the Line AB be either perpendicular or inclined to the Picture, and parallel to the Horizon, its Seat, at least of one extreme (B) must be determined; and as the Point B may be in some vertical Plane, (as X) its Seat on the Ground is in the Intersection of that Plane, at b; and supposing the Plane X parallel to the Picture, the Luminary is in that Plane; consequently, the Shadow of the extreme B will be somewhere in the Intersection of the Plane. No. 3.

Draw SD, through B, making the Angle SDb equal to the altitude of the Sun, giving the Point D, in the Intersection of the Plane X, for the Shadow of B.

Then, because AB is perpendicular to the Picture, C, is its Vanishing Point; and, because the Shadow is projected on a Plane, to which AB is parallel, the Shadow is necessarily parallel to AB, and consequently it has the same Vanishing Point, C.

Wherefore, draw CD and produce it; draw AE parallel to SD, (the given Ray of Light) cuting CD, produced, at E, the Shadow of A.

DE is the Shadow of AB on the Ground; to which it is parallel and equal.

For the Plane of Shade ABDE is a right angled Parallelogram in Perspective.

FG is its Shadow on an inclined Plane, to which AB is parallel; projected by the same Rays of Light, or Plane of Shade.

SCHOL. Because the Sun is in the Plane X, it cannot be said to be illumined; consequently, no Shadow can be projected on it; otherwise, if the Luminary was ever so little on this side, the Shadow of AB will be first projected on it, from B to D, and then projected to E; in which Case, BD would not be a Ray of Light, but a Shadow, and consequently, not parallel to AE.

Plate XLI.

Case the Second and Third.

Fig. 15. When the Luminary is on this ſide, or on the other ſide of the Picture.

First. Let AB be parallel to the Picture, and to the Plane, on which its Shadow is to be projected. Let S be the Image of the Sun, on the Picture; or ⊙ its transprojected Image. Let VL be the Vaniſhing Line of the Plane of Projection.

Now, the Line AB being parallel to the Picture, has no Vaniſhing Point; conſequently, its Shadow, being on a Plane to which it is parallel, is parallel to the Line. Let D be the Seat of A, on the Plane; C is the Center of the Picture.

Becauſe the Plane, on which the Shadow is to be projected, is inclined to the Picture, through C, draw its Vertical Line EF; and becauſe D is the Seat of A, on the Plane, draw AD and produce it, cuting the Vertical Line at F, the Vaniſhing Point of Lines perpendicular to the Plane.

For the Seat of a Point on any Plane is produced by a perpendicular to that Plane.

Draw SF, or ⊙ F, cuting the Vaniſhing Line at V, or L, the Seat of the Luminary on the Plane, at an infinite Diſtance.

Draw VD, or LD; and SA, or ⊙A, cuting VD or LD, at a or *a*, the Shadow of the extreme A, on the Plane (Prob. 2.) Draw ab, or *ab*, parallel to the Van. Line, that is to AB; and, through B, draw SB, or ⊙B, cuting ab, or *ab*, at b, or *b*.

ab is the Shadow of AB, by means of S, the Image of the Sun on the other ſide of the Picture; or, *ab* is its Shadow, projected by the Vaniſhing Point ⊙, the transprojected Image of the Luminary, on this Side; both which, are Vaniſhing Points of the Rays of Light. aD, or *a*D, is the Shadow of the Perpendicular AD.

No. 2. Secondly. When the Line AB is perpendicular to the Picture, and parallel to the Plane of Projection; C is the Center.

Becauſe the Line whoſe Shadow is to be projected is perpendicular to the Picture, and parallel to the Plane of Projection, that Plane is conſequently perpendicular to the Picture, and its Vaniſhing Line VC paſſes through the Center. (Theo. 4.)

Let D be the Seat of the extreme A, on the Plane, and let S, or ⊙, be the Image of the Luminary; projected or transprojected.

Draw SV, or ⊙ V, perpendicular to the Vaniſhing Line; and, through D, draw VD indefinite. Draw SA, or ⊙ A, till it cuts VD, at a, or *a*, the Shadow of the extreme A, on the Plane, in either Caſe.

For, AD is perpendicular to the Plane, and Da or D*a*, is its Shadow. (Pr. 2.)

Draw Ca, or C*a*, indefinite, and SB, or ⊙B, cuting it at b, or *b*.

ab is the Shadow of AB, the Sun being beyond the Picture; or, *ab* is the Shadow of AB, the Sun being on this ſide, by means of its transprojected Image, ⊙.

For, AB being parallel to the Plane of Projection, its Shadow is parallel to AB; conſequently, becauſe AB is perpendicular to the Picture, the Center, C, is their common Vaniſhing Point. And, becauſe the Rays of Light are parallel amongſt themſelves, they have the ſame Vaniſhing Point, S or ⊙.

N. B. When the Line, AB, is inclined to the Picture, and the Plane of Projection is perpendicular to it, there is no difference, in the proceſs, but only in its Vaniſhing Point.

No. 3. Thirdly. When the Line AB is inclined to the Picture, and the Plane of Projection alſo inclined to the Picture, let VL be its Vaniſhing Line.

Find the Vaniſhing Point (F) of Lines perpendicular to the Plane, (Pr. 2. S. 12.) and draw SF, or F⊙, cuting the Vaniſhing Line at E, the Seat of the Luminary.

Thro' D, the Seat of A, draw ED indefinite, and SA, or ⊙A cuting ED, at a or *a*.

Then, V being the Vaniſhing Point of the Line AB, and becauſe the Shadow is parallel to it, draw Va, or V*a*, indefinite, and SB, or ⊙B, cuting it, at b, or *b*; ab, or *ab*, is the Shadow of AB, projected on the Plane whoſe Vaniſhing Line is VL.

PROBLEM

PROBLEM IV.

To determine the Shadows of Lines inclined to the Plane of Projection and to the Picture; in any Angle, whatever.

Let AB be inclined to the Plane Z, on which the Shadow is to be projected; and, let ST be a given Ray of Light, the Luminary being supposed in the Picture. Fig. 16.

Through V, the Vanishing Point of AB, draw VC parallel to the given Ray, cuting the Vanishing Line of the Plane Z, at C, the Van. Point of the Shadow.

For, the Luminary being in the Plane of the Picture, its Rays have no Vanishing Point; consequently, a Right Line drawn through V, the Vanishing Point of any Line, parallel to the Rays, is the Vanishing Line of the Plane of Shade, projecting the Shadow of that Line.

Therefore, draw AC, the indefinite Shadow of AB; and BD parallel to the given Ray; that is, parallel to VC, the Vanishing Line of the Plane of Shade. AD is the Shadow of AB on the Plane Z.

If the Line EF, whose Shadow is required, does not cut the Plane, on which it is to be projected, let it be produced till it cuts the Plane, at G, and draw GC. Or, if the Seat of either extreme be given, or found, as *a* of the Point E, or *b* of the extreme F; then, having obtained the Shadow of either, e or f (Prob. 2.) draw Ce, or Cf; and Rays, through the extremes, E and F, parallel to the given Ray, ST; by which means the Shadow, ef, of EF is projected.

CASE the Second and Third.

When the Luminary is beyond, or on this side of the Plane of the Picture.

Let AB be a Line, whose Shadow is to be projected. V is its Vanishing Point. Let S and ⊙ be the projected and transprojected Images of the Sun. No. 2.

If the Line does not cut the Plane of Projection, let it be produced, to C.

Draw SV or ⊙V, cuting the Vanishing Line EF, at F, the Van. Point of the Shadow. Draw CF; and SA, SB, or ⊙A, ⊙B, cuting it at a and b, or *a* and *b*. ab is the Shadow of AB when the Luminary is beyond the Picture, and *ab*, when it is on this Side; according to its determined place. Or the Shadow may be found by means of the Seat of either extreme, given or found, on the Plane of Projection; as in the first Case.

SCHOL. Any Line, whose Shadow is to be projected, cuting the Plane of projection (whose Vanishing Line is given or found) the process is the same, whether the Plane of projection be perpendicular or inclined to the Picture; for, if the Vanishing Point of that Line be obtained, the Vanishing Line of the Plane of Shade is also; whose Intersection with the Vanishing Line of the Plane of projection is the Vanishing Point of the Shadow; which, it must be obvious, always passes through that Point in which the Line cuts or would cut the Plane.

2. When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this side; in which Case, whether the Line be perpendicular or inclined to the Plane, it is the same to the Picture, and, its Vanishing Point being determined, the process is the same. In this Case, the Plane of projection having no Vanishing Line, the Shadow, of any Line thereon, is parallel to the Vanishing Line of the Plane of Shade.

3. All Lines which are parallel to such Planes are also parallel to the Picture; and their Shadows, being parallel to the Lines, are always so represented; which is not the Case on other Planes, but only, when the Lines are parallel to the Picture, as well as to the Plane of projection.

4. The Seats of all such Lines, on the Plane, must be had, or the Shadow cannot be determined; in which Case, it is but finding the Shadows of two equal Lines, perpendicular to the Plane (as in Fig. 14.) and, joining their extremes, the Shadow of the parallel Line is obtained, as in the second Problem.

Having, in these Problems, given every Rule which I conceive necessary for the projection of right lined Shadows, I shall, next, give some Examples of their utility, in the application of them to the Shadows of Objects.

Plate XLII.

As every Object is compofed either of plane or curved Surfaces, or both, fo likewife, thofe Surfaces are bounded by right Lines or curved; and, as the outline of the Shadow only is required, we fhould carefully obferve on what part of the Object the Light falls, that we do not give the Shadows of fuch Lines as can (from their fituation to the Light) have no Shadow; for, according to the altitude of the Luminary, or its fituation to the Object, different Lines in the Object define its Shadow. Wherefore, fince it is from the linear perfpective reprefentation of the Object, that the Shadow is projected, it is fometimes difficult to determine the Line which cafts the Shadow, becaufe they are frequently out of fight; wherefore, they ought to be determined with caution, before we begin the procefs.

The confideration of the Luminary being beyond, or on this fide, or in the Plane of the Picture, is alfo a circumftance which fhould be maturely confidered. The beft effects, are (in my opinion) produced, by confidering the Sun to be on this fide; becaufe it is obvious, that, when it is beyond the Picture, though ever fo little, that is, though ever fo much inclined, it is alfo beyond every Object reprefented on the Picture; but, if the Sun itfelf be reprefented, that is, if it be fo little elevated and inclined to the Picture, that the Sun's apparent place is within its limits, it muft then be entirely behind the Objects; confequently, the whole of every Object, on that fide, is immerfed in Shade; and the Shadow, of each, is projected towards the Eye, greatly diftorted, and larger than the Object. Whereas, the Sun being on this fide, towards either hand, as is moft fuitable to the Objects (according to what Faces we would have illumined) the Shadows are projected forwards, from the Eye, diminifhed, and great part of it hid by the Object; which, with the various effects of Light and Shade, on the Objects, cannot fail rendering the whole agreeable, and moft pleafing to the Eye.

EXAMPLE I.

To reprefent the Shadow of a Plane Figure, on a Plane.

Fig. 17. FIRST. Let the Plane ABCDE be perpendicular to the Ground Plane, on which the Shadow is to be projected. Let S and ⊙ be the Image and tranfprojected Image of the Sun; VL is the Vanifhing Line of the Plane ABCDE, and FG of the Plane of projection.

Draw SF, or ⊙F, perpendicular to the Horizontal Line, cuting it at F; the Vanifhing Point of Lines perpendicular to the Horizon. (Prob. 2.)

Through A, draw Fb, indefinite; and, if the Shadow be projected by the Image, S, through B, draw Sb, cuting the former, at b. Or, draw B⊙, cuting it at *b*.

Then, for the Shadow of BC (on this fide) join SV; if it be parallel to the Horizon, draw bc alfo parallel; if it be not parallel, then, SV produced will give its Vanifhing Point. For the fame Line on the other Side, draw V⊙, cuting the Horizontal Line at H, and draw *b*H. SC, and C⊙, cuts them at c and *c*, refpectively.

CD is parallel to the Plane of projection, and G its Vanifhing Point; therefore, draw cG, or *c*G, and SDd, or D⊙, giving d, or *d*, for the Shadow of D.

Laftly; for the Line DE (on this fide) draw SL, cuting the Horizontal Line at I, and draw Id. For the other Side, draw ⊙L; which being produced to the Horizontal Line, will give the Vanifhing Point of *d*E, and compleats that Shadow. Or, join dE, or *d*E, only.

AbcdE is the Shadow of ABCDE projected on the Ground Plane, by the Image (S) of the Sun, on the other fide of the Picture; and, A*bcd*E is its Shadow, by the tranfprojected Image, ⊙, being confidered on this Side.

Fig. 18. SECONDLY. Let the Plane ABCDE be either perpendicular or inclined to a vertical Plane, whofe Vanifhing Line is VL, on which the Shadow is projected.

Whether the Plane, whose Shadow is required, be perpendicular or inclined to the Picture, the process is the same. HF is the Van. Line of the Plane ABCDE.

First. Let S be the Image of the Sun, on the other side of the Picture.

The Line, AB being parallel to the Picture, draw SV parallel to it; V is the Vanishing Point, of its Shadow; wherefore, through A draw Vb, indefinite, and, through B, draw Sb, cuting it at b, which determines its length, Ab.

H being the Vanishing Point of the contiguous Line BC, draw SH, and produce it to I, the Vanishing Point of its Shadow.

Draw bI, and Sc, through C, cuting it at c. bc is the Shadow of BC.

Then, becauseCD is parallel to the Plane of projection, and G is its Vanishing Point, draw cG, and SD, cuting it at d, the Shadow of D.

Lastly, draw SF, giving the Vanishing Point J if it was necessary; but it is obvious, that, dE, being joined, compleats the Shadow, and tends to J.

If the Shadow was obstructed by another Plane, cuting it at MN, the Vanishing Line of which is FN; SV, being produced, would cut that Vanishing Line in the Vanishing Point of the Shadow of AB, on that Plane; from which, draw M*b*. SH produced, cuts the Vanishing Line at P; draw *b*P, cuting the Ray SCc at *e*; and, SG produced cuts it at O; draw *e*O; or join *ef* only, which compleats it.

Second. Let ⊙ be the transprojected Image of the Sun, on this side.

AB being parallel to the Picture, draw ⊙K parallel to it, cuting the Vanishing Line, VK, at K, the Vanishing Point of its Shadow.

Draw AK, and B⊙, cuting it at *b*, the Shadow of B. ⊙H cuts the Vanishing Line at L; draw *b*L, and C⊙, cuting it at *c*. Then, draw *c*G, and D⊙, cuting it at *d*. *cd* is the Shadow of CD, being parallel to the Plane of projection.

If ⊙F be parallel to the Vanishing Line, VK; then, *d*E is also parallel to it. Or, it will tend to that Point, in which ⊙F, being produced, would cut VK.

SCHOL. If the Plane of projection be much inclined to the Picture, there can be no Shadow cast on it, the Sun being beyond the Picture; or they will be drag'd out to an immoderate length. For, if the Image of the Sun appear in the Picture, it will be behind the Plane of projection, being much inclined.

By observing the Rules, given above, the Shadow of any plane Object may be projected on any Plane, or on various Planes, of which I shall give some Examples.

EXAMPLE II.

To project the Shadows of right angled Parallelopipeds, on the Ground Plane.

First. When a Face of the Parallelopiped is parallel to the Picture, and, the Luminary in the Plane of the Picture.

X is the Object whose Shadow is to be projected. Fig. 19.

In this Case, the Luminary being in the Picture, the Shadow of the whole Solid is no more than of the Plane ABDE. For, the Plane X being parallel to the Picture, cannot be illumined; seeing that the Sun is in the Picture, it is, consequently, in every other Plane parallel to the Picture. The whole Shadow of that Plane is, therefore, but a Line, generated by producing the Plane.

Draw Ab, indefinite, parallel to the Vanishing Line, HL, and, if SB be a given Ray of Light, produce it, till it cuts Ab, at b, the Shadow of B.

Then because BD is perpendicular to the Picture, C, the Center, is its Vanishing Point; and, being parallel to the Plane of projection, its Shadow consequently parallel, and has, therefore, the same Vanishing Point.

Draw bC; and through the angle D, draw Dd parallel to Sb, cuting bC at d; or, draw Ed parallel to Ab, which compleats the Shadow, AbdE, required.

SCHOL. If a Line (FG) projected from the Plane X, its whole Shadow will be projected on the Ground. For, since the Luminary is in the Plane X, no part of its Shadow can be projected on that Plane; wherefore, fg, its Shadow, is wholly on the Ground Plane.

Plate XLII. Fig. 20. SECONDLY. When its Planes are inclined to the Picture.

Let AD be a right angled Parallelopiped, the Vaniſhing Points of its Sides, are H and L; and let ST be a given Ray of Light.

Draw Ab parallel to the Horizon; and through B, draw Bb parallel to ST.

Then, L being the Vaniſhing Point of BC, draw bL; and through C, draw Cc, parallel to ST, cuting it at c. Laſtly, draw cH; and Dd parallel to ST; and, through d parallel to Ab. Abcd is the Shadow required.

For the Rays, projecting the Shadow, are all parallel amongſt themſelves; and the Perpendicular AB, being parallel to the Picture, has its Shadow (Ab) parallel to the Vaniſhing Line; alſo, the Shadows of BC and CD have the ſame Vaniſhing Points, reſpectively; becauſe, BC and CD are parallel to the Plane of projection.

CASE 2d. When the Luminary is on the other ſide of the Picture.

Fig. 21. Firſt; let the Plane AD be parallel to the Picture, and, let S be the Image of the Sun on the Picture, being ſituated beyond it.

Draw SV, perpendicular to the Horizon, V is the Vaniſhing Point of the Shadows of Perpendiculars, on the Ground Plane. (Prob. 1.) Draw VA and SB, and produce them, cuting at b, the Shadow of B. Then, draw bd parallel to BD; and, through D, draw Sd, cuting it at d; bd is the Shadow of BD.

Laſtly; becauſe DE is perpendicular to the Picture, and C is its Center, draw dC; and, through F, draw VF, or SE, cuting it at e, and compleats the Shadow, AbdeF; which, as it is projected back, towards the Eye, is larger than the Object.

SCHOL. When the Object is inclined to the Picture, as in No. 2, the proceſs is the ſame in every reſpect, having regard to the Vaniſhing Points of the Lines BC and CD; to which their Shadows, bc and cd, are reſpectively parallel, and conſequently have the ſame Vaniſhing Points, G and F.

No. 2. CASE 3d. When the Luminary is ſituated on this ſide the Picture.

Let AD be a Parallelopiped, right angled, as before; and let ⊙ be the tranſprojected place of the Luminary.

Draw ⊙V perpendicular, and, draw AV and B⊙, cuting at *b*, the Shadow of B. Then, draw *b*G and C⊙, cuting at *c*; and laſtly, draw *c*F, which compleats the Shadow A*bcd*. The Shadow of the Angle D is not ſeen.

E X A M P L E III.

Plate XLIII. *To project the Shadow of a plain Building, on the Ground.*

Fig. 22. Let ADF repreſent a Building, obliquely ſituated to the Picture; and let the Sun be in the Plane of the Picture. The direction of its Rays is SC.

Draw Ab parallel to the Vaniſhing Line, and Bb to the given Ray, cuting Ab, and determining the Shadow of AB.

Through V, the Vaniſhing Point of BC, draw VG parallel to the given Ray of Light; and draw bG; SC produced, cuts it at c, the Shadow of C.

Becauſe the Vaniſhing Point of CD is below the Horizontal Line, equal VH, the Vaniſhing Point (I) of its Shadow, will be on the other ſide VH, equal GH.

Make HI equal GH, and draw cI; and Dd parallel to SC, cuting it at d.

Becauſe of the Sun's altitude, and the ſituation of the Building, the other ſide of the Roof is illumined; and conſequently, the Shadow of CF (the Ridge) is not projected, which is evident; becauſe, being parallel to the Plane of projection, its Shadow is parallel to itſelf, and conſequently has the ſame Vaniſhing Point, L. Wherefore, if from c, the Shadow of C, a Line be drawn to L, it will fall on this Side cd, the Shadow of CD; conſequently it has no Shadow, here.

Therefore, draw dL, cuting DE; de is the Shadow of a Line from D, parallel to CF (the Eaves of the Roof) on the other ſide of the Building.

SECONDLY.

SECONDLY. Let the Sun be ſuppoſed beyond the Picture, and let S be its Image.

Draw ST, perpendicular; and, through A, draw T b indefinite; through B, draw Sb, giving Ab, for the Shadow of AB. Fig. 23.

Then, BC being parallel to the Horizon, and L its Vaniſhing Point, through b, draw Lc, and through C draw Sc, giving bc, the Shadow of BC.

V being the Vaniſhing Point of CD, draw SV. If SV, be parallel to the Horizon, draw cd, alſo parallel; if SV, be not parallel, produce it, till it cuts the Horizontal Line, on one ſide or the other, and draw cd, tending to that Point; alſo, through D, draw Sd, cuting it at d, the Shadow of D.

Again. Y being the Vaniſhing Point of DE, draw SY, cuting the Horizontal Line at I, the Vaniſhing Point of the Shadow of DE.

Draw dI, and SE till it cuts dI, at e; and laſtly, join eF, which will tend to its Vaniſhing Point T, and compleats the Shadow, AbcdeF.

In this Caſe, it is obvious that the whole Building, ſave the Plane DB, is in Shade.

CASE 3d. When the Luminary is on this ſide of the Picture.

Let ⊙ be its tranſprojected Image. ⊙K being drawn, perpendicular, K is the Vaniſhing Point of the Shadows of perpendicular Lines. (Prob. 2.)

Draw AK and B⊙, interſecting at *b*, the Shadow of B.

Then draw ⊙V, cuting the Van. Line at I, the Van. Point, of the Shadow of BG.

Draw *b*I, and G⊙, cuting it at *g*, the Shadow of G.

Join ⊙Y; and if it be not parallel, produce it to the Vaniſhing Line.

Through *g*, draw parallel, or tending to the Point in which ⊙Y, cuts the Horizontal Line; which compleats the Shadow, as much as can be ſeen.

I preſume, it is obvious, from theſe Examples, that, the moſt judicious choice is to ſuppoſe the Luminary on this ſide of the Picture, as in this laſt Caſe. Firſt, becauſe the object is more advantageouſly ſituated to the Luminary, and conſequently, will have a more agreeable effect of Light and Shade. Alſo, becauſe the Shadow is projected from the Eye, ſo that, it does not appear diſtorted and prepoſterous, as in the ſecond Caſe; and it is far from being pleaſing to have the Shadows of perpendicular Lines all parallel amongſt themſelves; as when the Luminary is in the Plane of the Picture. Therefore, as I have given Rules and Examples in each, I ſhall, hereafter, conſider the Luminary only on this ſide of the Picture, unleſs it be otherwiſe determined.

EXAMPLE IV.

To project the Shadow of an upright priſmatic Object, whoſe upper Face is inclined to the Horizon, part on the Ground, and part on a vertical Plane, inclined to the Picture, whoſe Interſection with the Ground is given.

Let X be the Object, whoſe Shadow is to be projected; let ⊙ be the tranſprojected Image of the Sun, and JI the Interſection of a vertical Plane with the Ground; KL is its Vaniſhing Line. (Prob. 3. Sect. 3. Book III.) Fig. 24.

Draw ⊙H perpendicular to the Horizontal Line. Draw AH, and B⊙ cuting it at *b*, the Shadow of the Angle B on the Ground.

But, A*b* cuts the Interſection JI, at a; wherefore, becauſe the Plane Y is vertical, AB is parallel to the Plane; and, being alſo parallel to the Picture, its Shadow is conſequently parallel; draw ab, parallel to AB, till it cuts the Ray B⊙, at b, the Shadow of B on the vertical Plane; as *b* is, on the Ground.

The Vaniſhing Point of BC is F; which, being inclined to the Horizon is not in the Horizontal Line. Draw ⊙F, and produce it, till it cuts KL, the vertical Vaniſhing Line, at K, the Vaniſhing Point of its Shadow; then, draw bK, and C⊙ cuting it at c, the Shadow of C on the vertical Plane.

Plate XLIII. Then, G being the Vanifhing Point of CD, a Line on the other fide, parallel to BE, which projects a Shadow, draw ⊙G, cuting the vertical Vanifhing Line at L. Draw cL and D⊙ interfecting at d.

If DM be a perpendicular Line, draw de parallel to it, cuting the Interfection at e, and join eM, which tends to the Vanifhing Point, H, of Perpendiculars, and compleats the Shadow; Aaef on the Ground, and abcde on the vertical Plane, as it was propofed.

If the vertical Plane was out of the way, its Shadow on the Ground is A*bcdf*; in which, the Shadow of BC tends to N, and CD to O, in the Horizontal Line.

EXAMPLE V.

To project the Shadow of a Crofs, perpendicular to the Horizon, part on the Ground; alfo, on a Plane inclined to the Horizon, and to the Picture, whofe Vanifhing Line, and Interfection with the Ground Plane are given.

Fig. 25. Let AFBK be the Object, inclined cafually to the Picture; whofe Vanifhing Points are M and N, in the Horizontal Line. Let QR be the Interfection of the inclined Plane with the Ground, whofe Vanifhing Line is OP. ST is a given Ray of Light, the Luminary being in the Picture.

Becaufe the Object is inclined to the Picture, and the Luminary is in the Picture, the Side AB and its oppofite, IJ, will project Shadows.

Draw Aa, and Jb parallel to the Horizon, cuting the Interfection, QR, at a and b. Draw a*b*, and b*i*, parallel to the Vanifhing Line OP, indefinite; and draw B*b*, parallel to ST, cuting a*b*, at *b*, the Shadow of B.

For, the Line AB being parallel to the Picture, the Plane of Shade occafioned by that Line is, in this Cafe, parallel to the Picture; confequently, its fection with every Surface, on which the Shadow of AB falls, is parallel to the Picture; therefore, Aa is parallel to the Vanifhing Line of the Ground Plane (MN) and *ab* to OP, of the inclined Plane.

Draw NO and MP, parallel to the Ray of Light, ST; O is the Vanifhing Point of the Shadows, of all Lines parallel to CD; and P, of all that are parallel to EF, on the inclined Plane.

Draw Cc parallel to ST, cuting the Shadow of AB at c. Draw Oc indefinite, and D*d* parallel to the Rays, cuting it at *d*, the Shadow of D.

DE being parallel to the Picture, draw *de*, its Shadow, parallel to the Vanifhing Line, OP, meeting the Ray of Light from E, at *e*.

P being the Vanifhing Point of the Shadow of EF, draw *e*P, meeting the Ray from F, at *f*; and draw *f*O, till it cuts the Interfection QR, at *j*; from which it tends to N, the Vanifhing Point of FG; being parallel to the Ground.

*d*O cuts the Interfection at *a*, from which Point it alfo tends to N, till it meets the Ray, from K, at *k*. Draw *k*M, cuting the Ray from L; and LG being parallel to the Picture, draw *lg*, parallel to the Horizon, which compleats the Shadow of that Arm, on the Ground.

The Shadow, *bbi*, of the Top, having the fame Vanifhing Points, refpectively, it would be unneceffary to defcribe, being obvious, from the Figure.

Secondly. *When the Luminary is on this fide of the Picture.*

In the foregoing Cafe (as in other preceding) the Vanifhing Point of the Shadow of every Right Line, on the inclined Plane, was produced, by drawing a Right Line, parallel to the given Ray, from the Vanifhing Point of the Line whofe Shadow was required, to the Vanifhing Line of the inclined Plane; becaufe the Luminary being fuppofed in the Plane of the Picture, all its Rays are confequently parallel amongft themfelves.

In this, the Rays of Light having a Vanifhing Point, a Right Line joining the Vanifhing Point of the Line, and the Vanifhing Point of the Rays, cuts the Vanifhing Line, or being produced, would (if it be not parallel) cut the Vanifhing Line of the Plane of projection, in the Vanifhing Point of the Shadow (Prob. 4.) or, the Rays, inftead of being parallel amongft themfelves (as in the former Cafe) converge in a Point, in which confifts the difference.

⊙ is the Vanifhing Point of the Rays, MN the Vanifhing Line of the Ground, and OP of the inclined Plane, of which, QR, is its Interfection with the Ground. M and N are the Van. Points of hor. Lines in the Object, as before. Fig. 26.

S being the Seat of the Luminary, on the Ground, draw AS, cuting the Interfection, QR, at a. Then, becaufe AB is parallel to the Picture, produce ⊙S, (being parallel to AB) till it cuts the Vanifhing Line of the inclined Plane, at T. Draw aT; B⊙, a Ray of Light, cuts aT at *b*, the Shadow of B; and C⊙ cuts it at *c*; through which, a Line drawn from O gives the Shadow of DK, on the inclined Plane; from the Interfection QR it tends to N. Rays from D and K, tending to ⊙, determine the extremes of the Shadow.

It would be ufelefs to defcribe the whole, the fame Letters of reference, as in the foregoing Cafe, fhew how the Shadow of every Line is produced, by adhering to the former Leffons, in this Cafe.

EXAMPLE VI.

To project the Shadow of a Right Line, inclined to the Horizon and to the Picture, on feveral Planes, varioufly inclined to both. Plate XLIV.

AC reprefents a Ladder, leaning againft a Building, on which its Shadow is to be projected. ⊙ is the tranfprojected place of the Luminary; H and I are the Vanifhing Points of horizontal Lines in the Building, and E of the Ladder. FI is the Vanifhing Line of the Roof of the Building; and GI of the adjoining Shed; on which, the Shadow is alfo projected. Fig. 27.

Draw E⊙, and produce it to the vertical Vanifhing Line (KI) of the Sides of the Building, cuting it at K; the horizontal Vanifhing Line is cut at L, and the inclined Vanifhing Lines, at M and N.

K, L, M, N, and E, are all Vanifhing Points of the Shadow. (Prob. 4.)

From the foot of the Ladder, draw Lines to the Vanifhing Point L, cuting the Interfection of the Plane V, with the Ground, at a and b; AabB, is its Shadow on the Ground.

K being the Vanifhing Point of the Shadow, on the vertical Planes, draw Lines from K, through a and b, till they cut the common Interfection of the Planes V and U, at c; from which, draw Lines to E, the Vanifhing Point of the Ladder; becaufe it is parallel to the Plane U.

M being the Vanifhing Point of the Shadow, on the Plane X, draw dM, cuting the Interfection of that Plane with the next, at e; from which Points draw Lines to K; for the Shadow on the Plane Y, being parallel to V, has the fame Van. Point.

From the Points *ff* (where the Ladder touches the Eaves of the Roof) draw to N; and, where they cut the Interfection of the Chimney with the Roof, draw to K, and E; from C and D the ends of the Ladder, draw to ⊙, cuting them, which compleats the Shadow of the fides of the Ladder.

If from each Step, on either Side, Lines are drawn to ⊙, cuting the Shadow of that Side, at *a*, *b*, *c*, &c. and from thofe Points, Lines are drawn to I, the Vanifhing Point of the Steps, their Shadows will be compleated, on the Planes V, X, Y, and Z, as well as on the Ground; becaufe the Steps are parallel to all thofe Planes; being parallel to the Horizon.

On the vertical Plane U, their Vanifhing Point is where a Right Line drawn from I, through ⊙, would cut the vertical Vanifhing Line EH, of that End.

The Chimney being perpendicular, ⊙S, drawn perpendicular, cuting the Vanifhing Line (GI) of the Roof, at R, gives its Vanifhing Point; J⊙ determines its length.

OPQ reprefents a Sign Poft, whofe Shadow, Oo, on the Ground, and opQ, on the Wall, is projected by the fame Point (⊙) whofe Seat is S; which, being fo near the Vanifhing Point I, confequently, the Luminary is nearly in the Plane OPQ; wherefore, the Shadow of both, on the Wall, are almoft in one Line.

In

Plate XLIV. In this situation of this Object, the Shadow is more pleasing supposing the Sun on the other side of the Picture; as O*o*, on the Ground, *op* of the Post, on the Wall, and *p*Q of the Bearer. The Sun being on the left hand, V (at a proper distance) may be the Vanishing Point, of the Shadow *p*Q; and, the Seat of the Luminary, is the Vanishing Point of O*o*, on the Ground (Prob. 2.) OP being perpendicular. The Rays of Light, P*p*, &c. indicate the rest.

E X A M P E VII.

To project the Shadow of a Building, and the several parts of it on the other; the Luminary being on this side of the Picture, having its Altitude and Inclination given.

Fig. 28. Let the Angle of the Sun's Elevation be equal X; and the Inclination to the Picture, of a vertical Plane passing through its Center, equal Z.

Find ⊙, the transprojected place of the Luminary; by Prob. I.

Draw ⊙S perpendicular, giving S, the Seat of the Luminary, on the Ground.

Then, draw AS, cuting the Intersection of the Plane of the Arches, on the Ground, at *a*; from which, draw *a*a, perpend. the Shadow of AB on that Plane.

Produce ⊙S, till it cuts VO, the Vanishing Line of the inclined Plane X, at O; the Vanishing Point of the Shadow of AB, on that Plane.

Draw aO indefinite; and, because of the projecture of the Cornice at C, casting a Shadow (JB) on the Plane W, draw B⊙, cuting aO at b, the Shadow of B; from which, draw bc, to the Vanishing Point V; because, *b*C, projecting that Shadow is parallel to the Plane X.

Then, M being the Vanishing Point of CD, draw ⊙M, and produce it, till it cuts the Vanishing Line of the Planes V and W; and, VO being also cut at P, draw cP, cuting the next plane at e. and draw eQ, to the Vanishing Point, in VE.

L being the Vanishing Point of DE, draw ⊙L, and produce it to the Vanishing Line VM, of the Roof, cuting it at N. Draw NE, and D⊙ cuting it at d, and draw Md, to f, which compleats the Shadow of that part. CD, projecting the Shadow df, is parallel to the Plane Y, and has, therefore, the same Vanishing Point, M.

As the Luminary is not much elevated, the Shadow of the angle of the Cornice falls on the Wall, at h. F being its Seat on the Ground, draw FS, cuting the intersection of the Wall with the Ground at *f*; from which draw *f*h, perpendicular, and H⊙, cuting it at h.

RT is the Vanishing Line of the Wall Z, and M being the Vanishing Point of HI, ⊙M, cuting RT gives the Vanishing Point T, of hi, the Shadow of HI, on the Plane Z. From i, the Shadow of IK tends to K, in RT; K⊙ cuts it at k; from which, the Shadow of K*E* (the Ridge, which is not seen) tends to R, till it cuts the Intersection of the Wall with the Ground, whence it tends to V. *E*⊙ cuts it at *e*; from which Point, *ed* is the Shadow of the Hip, at the farther corner.

To determine the Shadow of the projecture of the Cornice. Let mn be the seat of the first Truss, on the top of the Cornice, and m its utmost projecture, which projects the Shadow; or, take any other Point, in mY.

L being the Vanishing Point of the Truss, i. e. of mn, draw from ⊙ through L, till it cuts the Vanishing Line of the Planes W; from which Point, draw no, and m⊙, cuting it at o, the Shadow of m, on the Plane W, produced.

Draw oV, cuting both Planes W, in Gr, and BJ, the Shadow of the extreme projecture of the Cornice, on those Planes.

Having, by the same means, obtained the Shadow of the lower edge of the fronts of the Trusses, their places are obtained by drawing Lines, from each to ⊙. The first falls on the Wall, at *p*, and the first on the other side, at *q*, on the low Roof.

The Shadows of the Piers of the Arcade, against the Wall, within, are projected by means of the Vanishing Point S, till they meet the wall, where they are upright. Lines drawn to ⊙, determine their height. (See Sect. IV. for the Arches.)

Thus, the Shadow of the whole Building is compleated; in which are all the variety of Examples, necessary for projecting the Shadows of right lined Objects.

SECTION IV.

Of the Shadows of curved Lines and curve-lined Objects, on plane and curved Surfaces.

AS the Perfpective of curve-lined Objects is more difficult and liable to error than right-lined, on account of the continual bending, and varying in the direction of curve Lines; fo, their Shadows are with more difficulty projected. As there are no vanifhing Points of the Original Lines, fo neither can there be vanifhing Points of the Shadows; alfo, when the Shadows, either of right or curved lines, are projected on a curved Surface, there being no Vanifhing Line, the figure of the Shadow cannot, fo certainly, be defcribed thereon. Neverthelefs, Rules may be prefcribed, for projecting Shadows, in fuch cafes; which, if followed, will produce the true projection of as many Points, in the Curve, as are neceffary, for defcribing the contour of the Shadow, with tolerable accuracy. The principal Shadow being determined, the minutias, of Mouldings, &c. may, by a Perfon who has judgment, be done by Hand, and in many cafes they muft; for, I fairly own, that it is more than human patience can bear, nor indeed is it poffible, to project Shadows, in all Cafes, with mathematical exactnefs.

The Shadow of a Circle, projected upon any Plane, to which the Circle is parallel, is a Circle, of equal dimenfions; in all fituations of the Luminary, whatever.

For, whether its fituation be on this, or on the other fide, or in the Plane of the Picture; whether it be more or lefs elevated, 'tis ftill the fame; the projecting Rays, being parallel amongft themfelves, generate a Cylinder, for the moft part oblique; confequently, being cut by a Plane parallel to the Circle, its projection muft alfo be a Circle, feeing it is the oppofite Bafe of the Cylinder; but, being cut oblique it is an Ellipfis. So that, in Arches, &c. parallel to the Picture, the Luminary being on this fide, and projecting the Shadow on a vertical Plane, parallel to the Arches, the Center of each being determined, the reprefentation of the Shadow of each Arch, may be defcribed with Compaffes; which will be in proportion to the Arches, as the diftance of the Plane of the Arches, to the diftance of the Plane of projection. But, being projected on the Floor, or any other Plane, not parallel to the Arches, their Reprefentations are Ellipfes; except when the Cylinder, projecting the Shadow, is cut fubcontrary*; in which Cafe, the Shadow will be a Circle.

In all other pofitions of the Arches, to the Picture, and in all fituations of the Luminary, being projected on a parallel Plane, the Shadows being Circles, their Reprefentations are Ellipfes, defcribed as the Arches, themfelves; and being projected on any other Plane, the Shadows being Ellipfes, their reprefentations are alfo Ellipfes, more excentric, generally, though not always fo.

The Shadow of a Sphere or Globe (as well as its perfpective Projection) on a Plane, is an Ellipfis, in all Cafes, whatever; except when a Right Line paffing through the Center of the Luminary, and of the Sphere, is perpendicular to the Plane of Projection.

* An oblique Cylinder or Cone is cut fubcontrary, when the Plane of fection is fo fituated, that the Axe of the Cylinder or Cone has the fame inclination to it, contrarywife. As, if the Sun be elevated 45 Degrees, and be fituated fo, that a vertical Plane, paffing through its Center, is perpendicular to the Plane of the Arches, *i. e.* to a vertical Circle; then, if the fhadow be projected on the Floor, or any horizontal Plane, it will be a Circle; for the Rays cut the Floor, in the fame Angle, *viz.* 45 Degrees.

 EXAMPLE

Plate XLV.

EXAMPLE VIII.

To project the Shadows of Cylinders, on a horizontal Plane, lying along; another upright, so situated, that its Shadow crosses one of the former.

Fig. 29. The Bases of the Cylinder (X and Y) being inclined to the Picture, their representations are Ellipses, whose Shadows, may be thus projected.

No. 2. Let A B C D be the representation of a Square, circumscribing a Circle, whose sides are parallel and perpendicular to the Horizon. Draw the Diagonals A C and BD, also the two Diameters a b and c d, parallel to the sides of the Square; and, from the Points e and f, draw perpendiculars to the Base, A D.

Find the Shadow of the Square A B C D (by Prob. 2. and 3.) and draw the Diagonals, A*C* and D*B*, draw dS, also g S and hS, cuting the Diagonals; and through their Intersection, draw *ba*, to the Vanishing Point of A D, &c.

Then, if a Curve be described through the eight Points, *a*, *e*, d, *f*, *b*, *g*, *c*, and *d*, it will be the true representation of the Shadow of the Circle, a c b d.

Or, because there is but half its Shadow required, proceed thus,

Draw a b perpendicular, through its Center; also, c d and e f parallel to a b.

Find the Shadows of a, c, e, &c. (Prob. 2. Case 3.) and describe the Curve *ae*b.

Then, the Sides of the Cylinder being parallel to the Plane of projection, draw *c*V, a Tangent to the Curve; and, the Shadow of the other End being described by the same means, compleats the Shadow of that Cylinder.

Fig. 30. Second. The Shadow of the Cylinder Z, on account of its situation, is but little seen. Its Base is turned from the Light, in the other it was illumined, so that, 'tis the other edge of the Circle which projects the Shadow.

Third. The Sides of the upright Cylinder being perpendicular to the Horizon, draw AS, a Tangent to its Base, at A, cuting aV, the seat of the Side be, on the Ground, of the Cylinder Z, at *A*; from which Point, draw *Ab* perpendicular, cuting be (the greatest projecture of the Cylinder Z) at *b*.

But, on account of the inclination of the Rays of Light, the Shadow of the Side AE will be continued on the Ground, till it falls into that of the other, at *a*; and it is continued down the side, from *b* towards *a*, till it is lost, insensibly, in the shade of the other Cylinder.

Take as many Points, b, c, d, in the curve of the Base, of the Cylinder Z, as are necessary; and others, answering to them, that is, of equal height, perspectively, at B, C, D, on the Side, AE, of the upright Cylinder; from all which, draw Lines to S, cuting others, drawn from the corresponding Points b, c, d, to the Vanishing Point V, at *b*, *c*, *d*, &c. through which, a Curve being described, will be the Shadow of the Side AE, on the Cylinder Z.

For, the Side of the Cylinder (AE) projecting the Shadow, is perpendicular to the Plane of projection; wherefore, the Plane of Shade, occasioned by that Line, vanishes in ⊙ S; and, all horizontal Lines (in that Plane) from the Points A, B, C, D, &c. vanish at S; and, consequently, they must cut others, b*b*, &c. drawn in the Side of the Cylinder Z, being of equal height. Therefore, *abcd* is the section of the Plane of Shade, with the surface of the Cylinder Z, made by the Side, AE, of the upright Cylinder; and consequently, *abcd* is its Shadow on that Surface.

The upper Base (EF) of the upright Cylinder, is above the Eye, and, being parallel to the Plane of projection, its Shadow is a Circle; wherefore, having obtained the Shadow of any Diameter, as ef, of EF, by drawing E⊙ and F⊙; the former cuting AS, at e, and drawing ef to the Vanishing Point of EF; then, the Shadow being a Circle, and projected on a Plane, not parallel to the Picture, its representation is an Ellipsis (Theo. 2. Sect. 5. Book I.)

Describe the representation of a Circle, whose Diameter found is e f (Prob. 2. Sect. 8.) and from G, draw a Tangent, tending to S, the Shadow of the other Side of the upright Cylinder, which also cuts the other in the same manner, and compleats the Shadow, as much as can be seen. Let it be observed, that the Shadow, on the Cylinder Z, is darkest, where the Light is strongest.

EXAMPLE IX.

To project the Shadows of Right Lines, on a convex cylindrical Surface.

Let AK be a Right Line, any how situated, in respect of the Cylinder X; its Vanishing Point is V. Its Shadow on the Cylinder is required; ⊙ being the transprojected Image of the Luminary. Fig. 31.

Draw V ⊙, cuting the Horizontal Line at J, the Vanishing Point of the Shadow on the Ground. Draw A J, cuting the Base of the Cylinder at a; and, through a, draw ⊙ *A*, cuting AK at *A*.

Aa, on the Ground, is the Shadow of A*A*, part of the Line AK; beyond which, its Shadow is projected on the Cylinder.

Having obtained S, the Seat of the Luminary, (Prob. 2.) also, R, the Vanishing Point of the Seat of AK, on the Ground, draw AR; and, from various Points, B, C, &c. (in AK) at discretion, beyond *A*, draw B*B*, C*C*, &c. perpendicular; from all which Points, *B*, *C*, &c. so obtained, draw Lines to S, cuting the Base of the Cylinder, at b, c, &c. from which, draw perpendicular up the Cylinder; and from the several Points, B, C, &c. draw lines to ⊙, cuting the corresponding Perpendiculars, at *b*, *c*, *d*, &c. through all which, a Curve, *abcdef*, being described is the Shadow of *AF*, part of A K, on the cylindrical Surface.

The remaining part of its Shadow, falls on the Ground, behind the Cylinder.

SECOND. FH, on the top of the Cylinder, is a Square, which may represent the Abacus of the Tuscan Capital; to project its Shadow on the Cylinder and Floor.

Take as many Points f, g, h, &c. as are necessary, in the lower Lines, which cast their Shadows on the Cylinder; from all which, draw to S; also, from the Angle G, cuting the upper Base, or Curve, at 1, 2, 3, &c.

Draw perpendiculars from each Point, down the Cylinder, which are cut, by Rays from each Point, f, g, &c. to ⊙, at *f*, *g*, &c. the Angle G is projected to *G*. The Ray k ⊙, touches the Cylinder, at *k*, whence, it is projected to *K*, on the Floor.

Curve Lines, drawn through *Gfgh*, and *Gik*, are the Shadows of the Lines Gk and Gl on the Cylinder; kF is projected on the Floor.

Draw Sa, a Tangent at the Point a, cuting the Ray, from k at *K*; a*K* is the Shadow of the side of the Cylinder on the Floor.

Draw *KF* to the Vanishing Point of GF, and F ⊙, cuting it at *F*, the Shadow of the Angle F; draw *FL*, to the Vanishing Point of GH, and FI, the upper Line, on the other side, which casts the Shadow; I ⊙ cuts it at *I*; through which, draw *IM*, tending to the vanishing Point of HI, and compleats the Shadow on the Floor.

THIRD. NO is a Line projecting from the Cylinder, at pleasure; let P be its Vanishing Point; to project its Shadow, on the Cylinder.

Draw PQ, perpendicular, cuting the horizontal vanishing Line, in the vanishing Point of the Seat of NO, on any horizontal Plane, imagined at discretion (above or below the Vanishing Line) cuting the Cylinder. Let *N*u be a section of such a Plane with the Cylinder.

Draw Oo, perpendicular, cuting *N*u at o, and, through o, draw Qn, indefinite; from the extreme N, and from various other Points, as r, s, in NO, draw perpendicular, cuting Qn, at n, t, and u, from which, draw to S, cuting *N*u, at *N*, *t*, and *u*. Draw *Nn*, *tr*, and *us*, perpendicular, and N ⊙, &c. cuting them at *n*, *r*, and *s*; through which Points, the Shadow of NO is described. The rest is obvious, from the Figure.

EXAMPLE

Plate XLV. Fig. 32.

EXAMPLE X.

To project the Shadow of a Tuscan Base on the Floor, casually situated to the Picture.

⊙ is the transprojected Image, and S the Seat of the Luminary.

Draw AS and B⊙, cuting at *b*, and draw *b*V; C⊙, cuting *b*V, determines the Shadow of the edge of the Plinth, BC, provided nothing else interfered. But, the Torus, &c. and Shaft of the Column unite their Shadows with it.

For the Column, draw SB and SF tangents to its Base, on the Ground; from the points of contract, draw BD and FE, perpendicular; which determine where the total Shade, on the Shaft, begins. BD, and EF, are the Sides of the Column which project the Shadow, on the Ground.

Draw D⊙, and E⊙, cuting BS, and FS, at d and e, the Shadows of D and E.

DE, is a Diameter of the Column; and de being drawn, tending to the same Vanishing Point, may be considered as its Shadow; on which, as a Diameter found, describe the representation of a Circle (Prob 3. Sect. 8.) or, of a Semicircle, dfe, only, which terminates the Shadow. The Column being supposed cut, by a horizontal Plane, through DE, its Shadow is, consequently, a Circle; being projected on a Plane to which it is parallel.

The Shadow of the Fillet gh, may be projected after the same manner; but, as so little of it is seen, it would be unnecessary trouble.

Draw a Tangent to its Base, at b, from S, and draw bg perpendicular, and g⊙, cuting bS, at *g*, the Shadow of the Point g. By the same means the Shadows of other Points, may be obtained.

The Shadow of the Torus is not easy to describe; seeing its Surface is continually varying, it is scarce possible to determine what part of it projects the Shadow; except the Point *d*, of the greatest swell, determined, as the Fillet, by a perpendicular from c. No other Point in that Circle projects a Shadow, on this side; to determine, absolutely, what other Points do, is what I shall not attempt, being persuaded, my Readers will excuse me that trouble.

N.B. The Shadow of the Shaft falls, first, on the Cavetto, before it reaches the Ground; the Fillet on the Torus, and the Torus on the upper Plane of the Plinth; which the Figure describes.

The Shade on the Torus, and on the Column, &c. is more hard and edgy when the Sun shines on them, than otherwise; yet there is no edge defined; also, the Light is broader, and gradates more suddenly into the Shade; and the Reflections are much stronger; but that, I shall reserve to another Section.

EXAMPLE XI.

To project the Shadow of a Doric Capital, on a vertical Plane, whose Vanishing Line (VY) *and Intersection* (AB) *with the Ground Plane are given.*

Fig. 33.

CD cuts the Plane at a; find its Shadow, cd (Prob. 4.) ⊙[2] being the Vanishing Point of the Rays, and G, of the Line; which, being joined and produced, cuts the Vanishing Line, VY, at V, the Vanishing Point of the Shadow.

After the same manner, the Shadow de, of DE, &c. may be obtained; and the whole Shadow, cdef, of the square of the top of the Abacus, in which draw the Diagonals; and let the lesser Trapezium *abcd*, be the supposed Shadow of the square of the Column, parallel to the former, and found by the same means; in which, the Ellipsis, inscribed, may be the Shadow of the top of the Column, in the Plane of the Abacus, from which draw Perpendiculars; or, rather (because of the swelling of the Column) somewhat diverging, and gently curved.

The Shadows of the Angles, g and h, are obtained as the Square of the top; and, if great accuracy was requisite, the square of the Astragal, ik may be also projected, in its true place. But, as nothing more of its Shadow is defined than what projects beyond the Column, it may be dispensed with, the Points, *i* and *k*, being determined, are sufficient.

Rays of Light, passing by the Ovolo to ⊙ ², may be some guide for describing its Shadow; but, on account of the continual variation of its Surface, every way, 'tis not easy to determine at what Points they touch it.

Neither can the curve of the Shadow of fg, be described on the Ovolo, with certainty, as on a Cylinder, yet somewhat may be done towards it; for example; the Shadow of the point b is required.

In this process, it is necessary to have the perspective curve (*aef*) of the seat of the small Fillet, from which the Ovolo projects, on the under Plane (g h) of the Abacus.

Then, draw bS, cuting it at *a*; draw *ab*, perpendicular, and describe the curve *bc*, a section of the Ovolo, by the Plane of Shade; b ⊙ ² being drawn, cuting that Curve, determines the Shadow of b, on the Ovolo, at *c*; which, because it touches the Curve only, is the limit of the Shadow on the Ovolo.

The rest of the Shadow of that Line, from b to g, falls on the vertical Plane.

- After the same manner as many Points may be determined as the Student has patience for; and when done, 'tis a great chance if it be more exact than a judicious Person would describe by guess; nevertheless it is useful, to give an Idea how such Shadows may be projected, and what kind of curves they describe.

E X A M P L E XII.

A vertical, concave, cylindrical Surface being so situated, that the Shadow of one part is projected on the other; how to determine that Shadow; also, of Right Lines any how situated to the Surface, and to the Luminary.

Let ABX be a Wall, inclosing a circular Area. ⊙ ³ is the Vanishing Point, of the Rays of Light, and S, of the Shadows of Perpendiculars. Fig. 34.

Draw SC, a tangent to the Curve, which determines the point where the Shadow begins, whatever be the Altitude of the Luminary.

Draw AS, cuting the Curve on the Ground, at a; draw a b perpendicular, and B ⊙ ³, cuting it at b, the Shadow of B.

By the same means, the Shadows of as many Points (D, E, &c.) as are requisite, may be found; through which, the Curve, Cdeb, may be described; which is the true Shadow of the Curve CDEB, on the concave Surface.

Second. Let FG be a piece of Timber, projecting perpendicularly from the Wall, whose Shadow is required; its Van. Point is where GF would cut SR produced.

Draw Ef perpendicular, either to the top or bottom of the Wall; and, through f, draw a Line to the Vanishing Point of FG, cuting a perpendicular from G. fg is the Seat of F G, on the Plane of the top, being horizontal.

From several Points, h, i, &c. draw perpendicular, cuting fg, at k, l, &c. from g, k, and l, draw lines to S, cuting the Curve of the Top, at m, n, o, from which, draw Perpendiculars, indefinite; and from the Points g, h, i, &c. draw to ⊙ ³, cuting them, respectively, in the Points *g*, *h*, *i*, which determine the Shadows of those Points on the Wall; through which, a Curve, being described, is the Shadow of the lower edge of the Timber. The rest is evident.

HI is another piece, also perpendicular to the Wall, its Vanishing Point is Y. Its Shadow is described after the same manner; or by means of a section of the Wall (Hi) parallel to the Horizon. Draw IS, cuting it at h; from which, draw perpendicular, and I ⊙ ³ intersecting at K, which determines its length; the Shadow being so nearly perpendicular, it deviates but little from a Right Line.

If its Vanishing Point was at S, the Shadow would be perpendicular.

Third. LM is a piece of Timber, inclined to the Wall, its Vanishing Point is Q.

Draw Q ⊙, cuting the Horizontal Line at P, the Vanishing Point of the Shadow on the Ground. Draw LP, cuting the Curve at the bottom, at p; through p, draw ⊙O, cuting LM, at O. Lp is the Shadow of LO, on the Ground.

Plate XLVI. Draw MN, perpendicular; and LP, being drawn, LN is the Seat of LM, on the Ground. From feveral Points *a*, *b*, *c*, (between O and M) draw perpendicular to the Seat LN, at *d*, *e*, *f*; from which, draw to S, cuting the Curve at the bottom, at *g*, *h*, *i*, and from them draw perpendiculars up the Wall; alfo, from *a*, *b*, *c*, draw to ⊙, cuting them, refpectively, at a, b, c; through which, a Curve being defcribed is the Shadow of OM, on the concave Wall.

For the Shadow of each Perpendicular *ad*, &c. is projected firft, from *d* to *g*; and *g*a, &c. is parallel to *ad*, &c. *ad* being parallel to the Wall.

E X A M P L E XIII.

To project the Shadows of Arches; or, the Shadow of the circular outline, on the infide, cylindrical, and plane Surface.

This Example is the fame, in every refpect, as the firft part of the laft, except in the pofition of the Cylinder to the Picture; their pofition to the Horizon is of no fignification.

Fig. 35. First; the Arch (X) is parallel to the Picture; C is the Center.

The Luminary being on this fide, and ⊙ its Image; draw C⊙, indefinite.

Then, becaufe the plane of the Circle is parallel to the Picture, draw a Tangent to the Curve, at A, parallel to C$\odot^4$; alfo, from various Points B, D, &c. at difcretion, draw parallel Ordinates, cuting the oppofite fide of the Arch, at b, d, &c. from all which, draw to C; and laftly, draw B⊙, D⊙, &c. cuting bC, dC, &c. refpectively, at *b*, *d*, &c. through which, a Curve being defcribed, is the Shadow of all that part of the Circle, from A (where the Shadow begins) to G, where it falls into the perpendicular.

If from A, to *b*, there be too much fpace, for, defcribing the curve accurately, take another point (a) and project its Shadow to *a*, and more if neceffary.

SCHOL. In this, the affinity to the former Example is obvious. The Right Lines, B*B*, D*D*, &c. tending to C, (the Center of the Picture) reprefent perpendiculars to the Bafes of the Cylinder; as, in the former they are perpendicular, being parallel to the Picture; alfo, the Lines Bb, Dd, &c. which are, in this Cafe, parallel to C⊙, in the former they vanifh in S; becaufe they reprefent parallel Lines, in a Plane not parallel to the Picture; as, in this Cafe, they are parallel, being in a Plane parallel to the Picture. The Rays of Light, in both Cafes, tend to their Vanifhing Point.

2. The Shadow, from A to *e*, notwithftanding its apparent convexity towards the Front, is really concave, the fame as the other; the convexity arifes only from its pofition to the Eye, owing to the exterior Curve Abe; which, in the former, is concave, the Eye being between the two Bafes, nearly oppofite to that part of the Surface on which the Shadow falls; in which pofition of the Eye, the true curve of the Shadow is feen.

Fig. 36. Fig. 36. reprefents the fame thing, inclined to the Picture; in which, the perpendiculars B*B*, &c. vanifh in H, in the Horizontal Line HL, as before in the Center; confequently, b*b*, d*d*, &c. vanifh in the fame Point.

Alfo Bb, Dd, &c. vanifh at S, in the Vanifhing Line of the Arch, where H$\odot^5$ cuts it; H being the Vanifhing Point of Right Lines perpendicular to the Plane of the Arch (Prob. 2.) The Rays of Light vanifh at $\odot^5$; confequently, HS is the Vanifhing Line of the Plane of Shade, for Lines perpendicular to the Plane of the Arch, and S is its Seat on that Plane.

Fig. 37. Fig. 37. reprefents another Arch, with the Light flowing inwardly (as in Example 42. Book 3. of the Piazza.) In which Cafe, it is the exterior Curve, from A to F, which projects the Shadow, the Sun being on the other fide of the Picture (in this Cafe) at S^2; and, G is the Vanifhing Point of Perpendiculars, as before.

The Figure, having the fame Letters of reference, explains the reft.

Here, the curve of the Shadow is feen properly, that is, concave; and it is the fame in both the other, in the Object, though reprefented convex.

In Example 42, of the Piazza, the Luminary is on this fide of the Picture, although it may to fome appear abfurd; but, the direction of the Shadows indicate it. It is indeed very much inclined, nearly approaching to parallel; that is, the Luminary is nearly in the Plane of the Picture.

EXAMPLE XIV.

To project the Shadow of the circular outline of a Nich, on the interior, spherical, and cylindrical Surface.

FIRST; let the Front of the Nich be parallel to the Picture.

C being the Center, draw C⊙, indefinite, the Intersection of a Plane passing through the Luminary, perpendicular to the Picture. Fig. 38.

Draw a Tangent to the Curve at A, parallel to C⊙, and several parallel Ordinates, B*B*, D*D*, &c. at discretion. On the Diameters B*B*, D*D*, and E*E*, describe the representations of Semicircles, in Planes whose Vanishing Line is C⊙ (Prob. 2. Sect. 8.) (CI is its Distance) which are sections of the Head, parallel amongst themselves, and to their Plane of Shade.

As the Section, through F*F*, cuts the cylindrical part of the Nich, also, and that through G*G* entirely, those Sections are not Semicircles; the former, where it cuts the Head, is circular, but where it cuts the Cylinder (which only is wanted) it is elliptical; and that through G*G* is a semi-Ellipses, whose transverse Diameter is G*G*, and its Conjugate, the Diameter of the Nich.

Describe the representations of such Ellipses; which being done, from B, D, &c. draw Rays to ⊙, cuting the Curves, respectively, at b, d, &c. through which, describe the Curve A b d e f g, the Shadow of the exterior Curve (from A to G.)

Draw g h, perpendicular, the Shadow of the edge GH, to which it is parallel, and join H h; which compleats the whole Shadow, in that situation of the Sun.

If the Luminary be less inclined to the Plane of the Nich, but in the same Plane of which CI is the Vanishing Line, its place being $\odot^2$; A*abdefgh* is the Shadow, which has but little convexity, being farther removed from the Curve of the Head, A*BE*.

SCHOL. This process, though apparently troublesome, is the shortest of any that has yet been published, and much more simple than by vertical Sections, as exhibited in Fig. 39;* in which, the several Points, B, D, &c. in the Arch, by means of perpendiculars to the right Line IK, at the bottom, have corresponding Points, *B*, *D*, &c.

Then (S, being the Seat of the Luminary) lines are drawn from *B*, *D*, &c. to S, cuting the curve of the bottom of the Nich, at b, d, &c. from which, perpendiculars are drawn to the Curve FGH, where the spherical Surface begins; through which, the representations of parallel Sections are made (as in the Figure) a most difficult process, as, each Curve is a different portion of a Circle. Right Lines from each Point, B, C, &c. tending to $\odot^3$, (as in the former Figure) give the Shadow of each Point in its respective Section; through which, the contour of the Shadow is described. Fig. 39.

Sir William Chambers (if Fame reports true) was the first who took notice, in any publication in England, of this seeming paradox, *viz.* a convex Shadow of a concave Line; which, since, has been most servilely copied, by almost every Architect, or other Artist, who had occasion to represent a Nich, either in external or internal Designs. Yea, to such an extravagant excess it is carried, that, in geometrical Sections, and where it is impossible that the Sun should ever shine, almost every Nich is so shaded; which, I do maintain, is, in such Case, most unnatural; nor is the Shadow really convex, but only apparently so; yet it is frequently exaggerated, preposterously.

But, I presume that, in Sun-shine, it has not always the appearance of convexity; for, I have shewn (Fig. 38.) that, the less the Inclination of the Sun, to the Plane of the Front, the less convex is the Curve. There is, also, another circumstance which occasions a different appearance. Those Authors, who have favoured the World with the method of describing it, have always given it direct, with the Center of the Picture in the middle of the Nich, as in Fig. 38. But, certainly, in the Front of a Building, perspectively delineated, the Eye cannot be

* In this Example, Fournier is most egregiously mistaken; for, he makes all these vertical Sections pass through B, the Vertex of the Nich, when it is evident, that they are parallel amongst themselves, and to the Plane of Shade of perpendicular Lines. By which means, the contour of his Shadow is more convex than it can ever possibly appear, in any situation of the Luminary, and of the Spectator.

Plate XLVII. oppofite to Niches at each extreme. Suppofe, then, the Eye obliquely fituated to a Nich, in a Plane parallel to the Picture, on the fame fide with the Sun; I prefume, the convexity will not be fo great, as in direct oppofition; yet, it is cuftomary to fhade all alike, which is moft palpably abfurd.

Fig. 39. reprefents a Nich fo fituated and fhaded; the Luminary being on the left hand, its tranfprojected Image is, confequently, on the right, at ⊙[3], the Center of the Picture is at S, and the Diftance is SO.

Now, although the altitude of the Sun, in this, is lefs, and its Inclination greater, than in Fig. 38. yet, the contour of the Shadow is concave, where the other is convex; and this is folely owing to the fituation of the Eye, in refpect of the Nich; for, being feen directly oppofite, it would appear more convex than the other.

I think, none have ventured to defcribe this Shadow, when the Nich is in a Plane inclined to the Picture; which, in delineating, more frequently happens fo than otherwife, if the Picture be properly fituated. Yet, in fuch Cafe, many, not thinking properly about it, would give the contour of the Shadow the fame, which can never poffibly happen, except when the Rays of Light are very much inclined to the Plane of the Nich.

Fig. 40. Fig. 40. is the reprefentation of a Nich, in a Plane inclined to the Picture; C is the Center, the Diftance is CH, and ⊙ the Image of the Luminary.

The fame Letters of reference indicate the procefs the fame, as already defcribed, with only this difference; that, as, in the former, the Ordinates were parallel (being parallel to the Picture) fo, in this, they vanifh where C ⊙ cuts the Vanifhing Line of the Plane of the Nich.

The outline of the Nich being delineated, and feveral Points, in the hither fide of the Arch, taken at difcretion, draw the Ordinates, tending to their Vanifhing Point; from which Point, draw the Tangent, at A, where the Shadow begins.

Then, reprefentations of Semicircles being defcribed on thofe Diameters (Prob. 3. Sect. 8.) and Rays drawn from the feveral Points, to ⊙, cuting them refpectively, give the contour of the Shadow, as before; which, in this Cafe (as in the laft Example) the Eye being oppofite to the Surface where the Shadow is defined, is really concave*; and which, in other Points of view, would appear convex.

SCHOL. It muft have been obferved, that the Center and Diftance of the Picture are not neceffary in the projection of Shadows, except in determining the Vanifhing Point of the Rays; but, in this Example, as the Vanifhing Line C ⊙ is of Planes perpendicular to the Picture, it neceffarily paffes through the Center (Theo. 4.) and, the reprefentations of the Sections, through the head of the Nich, could not be defcribed without the Diftance.

I have now, I prefume, furnifhed the Reader with Rules and Examples, for the projection of Shadows, by the Sun, fufficient for any purpofe almoft whatever. The Shadows of ftreight Mouldings, on Planes, are Right Lines, fave the contour of the Profile; which, in Mouldings above the Horizon, in Cornices, &c. are almoft wholly fhaded by the projecting Fillets, &c. and for circular Mouldings, or others below the Eye, the projection of the Shadows of Circles, on plane or other Surfaces (as in this laft Section) contain all that can be done in fuch Subjects.

He who has well digefted what is there advanced, will never be much at a lofs, in the moft difficult Cafes (where it is poffible to project the Shadow, at all, by Rule) in complex, finifhed Subjects; of which, Examples may be feen, throughout the Work, particularly in the Frontifpiece; that being intended as a general Leffon. Yet, in that, I have defignedly deviated from the rigid obfervance of mathematical exactnefs; which, as I have obferved in the Preface, does not always produce the beft effect of Light and Shade. For, Example.

According to the general direction of the Rays of Light given, in that Picture, and the Shadows of perpendicular Lines, it muft be obvious, that, as the Columns,

* As the Line which projects the Shadow is concave, the Syftem of Rays, projecting it, form a cylindrical Surface; confequently, the Shadow is concave, towards the Eye, on whatever Surface it falls.

in the Front, ftand off, detached, their Shadows would cover more than the whole fpace between them, on the Wall; which, I am perfuaded, would make that part appear heavy, and too hollow, as receding more than it really does. A little Light is alfo introduced on the circular Colonade, on the right-hand, in order to relieve it from the Building; although it would be wholly in Shade from the Building, according to the fituation and altitude of the Luminary, the height, magnitude, and diftance of the Object, occafioning the Shade. Such liberties may freely be taken, where it manifeftly produces a better general Effect; and, when it does not clafh with, and evidently contradict the invariable Law of Nature.

In that Piece, as in moft other in this Work, the Luminary is on this fide of the Picture; and though, in general, it is productive of the beft effects, yet there may fometimes be reafons for fuppofing it on the other fide, or in the Plane of the Picture, as in Plate 27, of Chelfea College. If that Piece had been fhaded on the fuppofition of the Sun being on this fide, either it muft be much inclined, and confequently the Vanifhing Point at a great Diftance, or every Face, in that Object, would be illumined, being on the right-hand; or, being on the left, the whole Front would be immerfed in Shade; with very little Light, in the Picture; but on the Roof, and end of the Building. Befides, from the fituation of that Building, the Sun can never fhine on both, the Front and hither End; and I think that circumftance fhould always be attended to, when a real Object is reprefented, which is the fole reafon why the Queen's Palace (Plate 28.) is fhaded from the left; for I am of opinion, that it would have a better Effect being fhaded from the other hand; but, as the Sun cannot poffibly be on the right-hand, it has certainly the moft natural Effect.

Although there be no Face of that Object, as there reprefented, but what receives Light, immediately from the Sun, fave one Face of the Library; yet, as fome Faces are more oppofed to the Luminary than others, there are various Teints, which diftinguifh one Plane from another, though not fo ftrongly. For, according as the Rays of Light are lefs or more inclined to the Face of any Object, it may have all the variety of Teints, from the ftrongeft Light to total Shade; and till the inclination is fuch, that they are nearly parallel to the Plane, it is ftill illumined, though in a very fmall degree; and, when they become parallel, that is, when the Sun is in the continuation of any Plane, it is as much deprived of Light, as if it was, in reality, on the other fide; and perhaps more fo, as it then receives the leaft advantage from Reflection.

SECTION V.

Of SHADOWS projected by a Torch or Candle.

IF the Theory of Shadows, in general, (Sect. 2nd.) and the Practice by Sunfhine be well underftood, little need be faid refpecting Candle-light Shadows, for in reality there is no difference, except in the diftance of the luminous Body, or Point; which, on account of the fhort Diftance, and being fituated amidft various Objects, its Light is diffufed in all directions; as it is from the Sun, refpecting the whole Syftem of Planets: alfo, in refpect of any fingle Object, of the fame magnitude, or nearly, as the Flame, the Rays may be confidered as parallel.

In projecting Shadows by Candle-light, there is but one Cafe, or fituation of the luminous Point, in refpect of the Picture; for, it is always fuppofed on the other fide. In reality, it is an Object delineated in the Picture; and, when determined,

Plate XLVI. mined, it is the Center of the Rays of Light; from which, the Shadows of other Objects are projected all around, in every respect the same as by the Sun, in that Case. Therefore, the Theory being the same, and the Practice having but little variation, few Examples will suffice.

I shall not suppose it necessary to shew how to find the representation of the Flame of the Candle, the common Center of the Rays of Light, which is the same as finding any other Point, in any Object whatever.

The Shadows of Right Lines, perpendiculars, to any Plane, tend to the Seat of the luminous Point on that Plane; which, in this Case, is finite. In respect of the Sun, being supposed at an infinite Distance, it is, consequently, in the Vanishing Line of the Plane; but in this Case, it must be found on the Plane of projection; which being found, the Shadows are readily determined.

Shadows projected by a luminous Point, at a short Distance, it is manifest must necessarily be larger than the Object; seeing that, all the Rays diverge from that Point, the same as from the Sun, when it appears in the Picture; which, on account of its immense Distance, the Rays have not so much divergency. But, the Objects whose Shadows are projected by them, are generally of much greater magnitude; the one being confined to small Objects only, as Chairs, Tables, &c. within a Room, the other is of Buildings, &c. external.

PROBLEM I.

The representation of a luminous Point, being given, and its Seat on the Ground, or other horizontal Plane, together with the Interfections of other Planes, with that Plane, whether they be perpendicular, parallel, or inclined to the Picture, or to the Horizon, or to both; to determine its Seat on the other Planes, and the Shadows of Right Lines perpendicular to them.

Fig. 41. Let AB be the Interfection of a vertical Plane with the Ground Plane, parallel to the Picture, and BD of a vertical Plane perpendicular to the Picture. Also, let BD be the Interfection of the two vertical Planes; and DE, DH, EI, EF, &c. Interfections of various Planes with each other.

In short, let LABG, represent a Floor, GBDH a vertical Plane, HDEI the Cieling, and IEFK an inclined Cieling, as in a Garret; AFEDB is the farther End of the Room, parallel to the Picture, all the other Planes are perpendicular to it. O is the representation of the luminous Point (as the Flame of a Candle) and S is its Seat on the Floor.

First. To find the Seat of the luminous Point, on any of the other Planes.

OS, being perpendicular to the Floor, consequently parallel to the Picture, imagine a Plane passing through OS, parallel to the Picture, cuting all the Planes (save AFEDB) perpendicularly; the Section of which, with them, is GHIKLG.

Now, because the luminous Point is in that Plane, and it is perpendicular to the Planes BDHG, &c. a Perpendicular from O, to each Plane, must necessarily be in that imaginary Plane; and consequently, must cut each Plane, in its Interfection with the Plane; wherefore, Perpendiculars (Of) to each Interfection, GH, HI, IK, &c. give the Seat (f) of the luminous Point, on each Plane.

The process is so very obvious, that I shall avoid a further description.

To find its Seat, on the Plane AFEDB; draw SC, cuting the Interfection, AB, at r; draw rs, perpendicular, and OC, cuting it at s, the Seat of O on that Plane. For, OC represents a perpendicular to the Plane,† cuting it at s.

† Cor. to Theo. 4.

Secondly. To find the Shadows of Right Lines perpendicular to those Planes.

Through the Point *B*, or b, where the Perpendicular *AB*, or ab, cuts the Plane, draw S*B*, indefinite; and, through the extreme *A*, or a, draw OC, or Oc, cuting it at *C*, or c, the Shadow of the Point *A*, or a.

Consequently, *BC*, or bc, is the Shadow of *AB*, or ab.

This process, it is obvious, is the same in each Plane; and, changing S for ſ, the description, given, serves for all alike.

The Parallelopiped, on which the Candle stands, is right angled; wherefore, *AB*, &c. are perpendicular to the Floor; by which means, the Shadows, *C* and c of the Angles, *A*, &c. being obtained, *Cc* and *Cc* are the Shadows of Lines parallel to the Floor, as in the 3rd Problem.

The Seat of the Light, on the upper Face of the Parallelopiped, is in the middle of the foot of the Candlestick, which directs the Shadow of *ab*.

In the other Block, the side *AB*, perpendicular to the Floor, projects a Shadow on the Floor, to *D*, in the Intersection B*D*; from which it is cast on the Wall, parallel to itself. The other Side, which is perpendicular to the Wall, is produced till it cuts the Wall, at *B*, through which Point, draw ſ*C*, and O*C* through A, intersecting at *C*; or, till it cuts the perpendicular Shadow, *DC*.

The Shadow of the piece of Timber (X) projecting perpendicularly from the Wall, is managed as the Lines, simply; in which it is obvious, that, as the Shadows of the Sides tend to ſ, they are diverging from that Point. The Shadows of ad and de are parallel to them, respectively; wherefore, *de* has the same Vanishing Point as the Original Line (de) projecting the Shadow.

Perpendiculars to the horizontal Cieling are in the same position as on the Floor; to the vertical Planes they are horizontal Lines; and to the inclined Cieling, IEFK, they are (as in all other) perpendicular to any Right Line, drawn in the Plane, through the Point, in which the Perpendicular cuts the Plane.

The Perpendicular *ab*, to that Plane, is so situated, that the Eye is in the Plane of Shade; wherefore, its Shadow cannot be determined as the rest, being in a continuation of the Line, but may be thus found.

Through the Seat, ſ, of the Light on that Plane, draw ſ*B*, at pleasure; and, through *b*, where *ab* cuts the Plane, draw *bB*, perpendicular to *ab*, cuting ſ*B*, at *B*, and draw *Bd*, parallel to *ab*, and *ad* to *bB*. Then, through *d*, draw O*e*, cuting ſ*B* produced, at *e*, and draw *ce* parallel to *bB*; which determines, *bc*, for the Shadow of *ab*, which could not be determined by the Ray O*c*.

SECONDLY. When the Plane is vertical, and inclined to the Picture. Fig. 42.

Let AB, be the Intersection of a vertical Plane, with the Floor, and D its Vanishing Point. The Center of the Picture is C; O is the luminous Point, and S, its Seat on the Floor.

To determine its Seat on the vertical Plane, draw CE, perpendicular, and equal to the Distance of the Picture. Join DE, and draw EF, perpendicular to DE, cuting the Horizontal Line at F; the Van. Point of perpendiculars to the Plane.

Then, draw SF, cuting AB, at r, and rſ, perpendicular, cuting OF at ſ, the Seat of O, on that Plane; for, Oſ, represents a Perpendicular to the Plane.

Let ab, be a Line, perpendicular to the vertical Plane, its Vanishing Point is F.

To project its Shadow on the Plane; draw ſb, through the Point b, where it cuts the Plane, indefinite; and through a, draw Oc, cuting ſb, produced, at c.

bc, is the Shadow of ab, on the vertical Plane.

THIRDLY. When the Plane is inclined to the Horizon, and to the Picture.

Let AB, be the Intersection of a Plane, inclined to the Horizon and also to the Picture; C is the Center, and D the Vanishing Point of AB. Fig. 43.

O is the luminous Point, and S its Seat, on the Floor.

Find the Vanishing Point, F, of Lines perpendicular to the Intersection AB, (Prob. 4.) and draw SF, cuting it at r, as in the last.

Then, the Inclination of the Plane to the Horizon being known, find its Vanishing Line, DG, (by 5th of the same) and the Vanishing Point H, of Lines perpendicular to the Plane, (Prob. 2. Sect. 12.) which will be that Point, where a

 Perpendicular

Plate XLVII. Perpendicular to the Vaniſhing Line, paſſing through C, the Center of the Picture, is cut by a Perpendicular from F. Alſo, find the Vaniſhing Point G, of Lines, in the Plane, perpendicular to the Interſection AB.

Draw rG, and OH, cuting it at ſ, the Seat of O, (the luminous Point) on that Plane. For, H is the Vaniſhing Point of Perpendiculars to the Plane, and rG repreſents a perpendicular to AB; wherefore OH cuts the Plane at ſ.

Becauſe, OS being perpendicular to the Horizon, and F is the Vaniſhing Point of Sr, perpendicular to AB; conſequently, GH, paſſing through F, alſo perpendicular (that is, parallel to OS) is the Vaniſhing Line of a Plane OſrS, paſſing through OS, perpendicular to the inclined Plane; and H, being the Vaniſhing Point of Perpendiculars to the Plane, OH muſt cut it in rG, the common Section of the two Planes; therefore, ſ, where OH, cuts rG, is the Seat of the Point O, on the inclined Plane.

The Seat, ſ, of the luminous Point being found, and ab, a given Line perpendicular to the Plane, its Shadow is determined, by drawing ſc, through b, indefinite, and Oc, through a, interſecting at c; giving bc, the Shadow of ab, as in all the foregoing.

In this Problem is contained the whole Theory of projecting Shadows by Candle Light, in which there is not any difference to that of projecting them by the Sun, the Seat being obtained; in reſpect of Lines parallel or inclined to the Plane of projection, there is very little difference in the proceſs.

The chief difficulty in projecting Shadows, by Candle Light, is to find the Seat of the luminous Point; for which, the Rules given will anſwer in all Caſes, and in all poſitions of Planes, whatever, having the Interſection of the Plane, with ſome other, on which, the Seat is already determined, and the Inclination of the Planes, to each other, known.

PROBLEM II.

To project the Shadows of Right Lines, parallel to the Plane of projection, however ſituated to the Picture.

The Shadows of Right Lines, on Planes to which they are parallel, whether they are projected by the Sun or by Candle Light, it is obvious, are always parallel to the Lines projecting the Shadows.

Fig. 44. † 2. 6. El. For, however the luminous Point be ſituated, or at whatever Diſtance, the Rays OA, OB, &c. generating the Plane of Shade, are all cut proportionally by the Line and the Plane of projection †, conſequently, ſeeing the extreme Rays, OA, OB, or any other, are ſo cut, as OA is to AC, ſo is OB to BD, it neceſſarily follows, that CD is parallel to AB (Caſe 2. 2. 6. El.)

Wherefore, if the Line be parallel to the Picture, the Shadow is parallel on the Picture; that is, to the Vaniſhing Line of the Plane of projection; and conſequently, however otherwiſe the Line be ſituated to the Picture, the Shadow, being parallel to the Line, has neceſſarily the ſame Vaniſhing Point (Cor. Th. 3.)

Fig. 45. No. 1. Let ACD be a right angled Parallelopiped, having the Face AC parallel to the Picture; the Sides, CD, &c. are conſequently perpendicular to the Picture, and parallel to the Floor, on which the Shadow is projected.

Let O, be the luminous Point, and S its Seat, on the Floor.

Through A, draw Sb, indefinite, and OB cuting it at b; and, becauſe BC is parallel to the Floor, and to the Picture, its Shadow is parallel to the Line.

Wherefore, draw bc, parallel to BC, and Oc (through C) cuting it at c.

Then, becauſe CD is perpendicular to the Picture, and parallel to the Plane of projection, C (the Center) is the Vaniſhing Point of its Shadow; wherefore, draw cC, and OC, thro' D, which compleats the Shadow, as much as can be ſeen.

No 2. 2. EH is another Parallelopiped, obliquely ſituated to the Picture; having one Face ſo ſituated, that the luminous Point, is in its Plane, produced.

Produce SE, and, through F, draw Oſ, cuting it at ſ, the Shadow of f.

 Then,

Then, K being the Vanishing Point of FG, draw fK, and Og (through G) cuting it at g; fg is the Shadow of FG, to which it is parallel, having the same Vanishing Point. And, because GH vanishes at I, draw gI, and Oh (through H) cuting it; and lastly, draw hS, which compleats the Shadow Efgh.

To project the Shadows of Right Lines parallel to any other Plane, however situated to the Picture, has nothing more of variety in it; the seat of the Luminary being obtained on the Plane.

3. The Shadows of the Shelves, LM, &c. at the farther End of the Room, may be thus obtained, being parallel to the Picture. No. 3.

Through the feet of the upright pieces, R, and T, draw right Lines, from S, cuting the Interfection of the Wall with the Floor, at r, and the Door, at t; from which, draw Lines parallel to LR, and MT, indefinite. From O, draw through the extremes of the Shelves, L, N, &c. cuting the Wall, in the Shadow rl, at l, n, &c. from which Points, draw parallel to the Shelves, cuting the edge of the upright piece, at *l*, *n*, &c. and join, N*n*, O*o*, &c. as in the Figure.

4. The Shelves, against the side Wall, may have their Shadows projected after the same manner, with only this difference; that, the Shadows, instead of being drawn parallel to the edges of the Shelves, tend to the same Vanishing Point. No. 4.

As ab, the Shadow of AB, and cd of CD, tending to the Center.

5. For the Shadow of the single Shelf on the other side, find the Seat, f, of the Light on the Wall (Prob. 1.) then, ab being perpendicular to the Plane, draw fa, indefinite, and Ob, being produced, cuts it at *b*. Draw *b*C, cuting the Angle of the Room, at *c*, and join *cd*. No. 5.

After the same manner, the breadth of the Shadows of the Shelves on the other Side are determined.

6. For, the hanging Frame and Shelves, above, proceed thus. Fig. 45. No. 6.

Find *S*, the Seat of the Light on the Cieling; *AB* and *CD* being perpendicular, draw *SA*, *SC*, indefinite; and, through *B*, and *D*, draw O*B*, O*D*, cuting them at *b*, and *d*. Draw *bd*, the Shadow of *BD*; and, from the Center of the Picture (being the Vanishing Point of *BE* and *DF*) draw *be*, and *df*, the Shadows of *BE* and *DF*, on the Cieling.

Lastly; through *G* or *H*, draw O*g*, or O*h*, cuting *be*, or *df*, at *g* or *h*; through which, draw *gh*, parallel to *GH* (being parallel to the Picture) and, through the extremes of the Shelves, draw Lines from O, cuting *gh* at *a* and *c*.

The rest, for that Shelf, is too obvious to need further explanation.

The other, notwithstanding its inclination, is performed by the same means; only, instead of being parallel, the Shadow tends to the same vanishing Point, as the Shelf; or, both extremes being determined, the vanishing Point is useless. Their widths are determined after the same manner.

7. The Shadow of the Stool, on the Counter and Floor, is thus projected. No. 7.

From S, the Seat of the luminous Point, draw Right Lines through the feet of the Frame, a, b, &c. till they cut the Interfection, AC, of the Counter with the Floor, at *a*, *b*, &c. from which Points, draw Perpendiculars; which are the indefinite Shadows of the Frame, on the front of the Counter; and if, through the Angles, e, f, &c. Lines are drawn from O, cuting the Perpendiculars, at *e*, *f*, &c. respectively, their lengths are determined.

The Shadow of the cross Frame is determined, by drawing Lines, from O, through their extremes, cuting the Shadows of the Legs, at *i*, *k*, &c.

 8. The

Plate XLVII. No. 8. 8. The Box on which the Candle ſtands has its upper Face, only, illumined, and conſequently its Shadow, only, is projected, on the Floor, &c. by drawing Lines, from S, through the Seat of each Angle, on the Floor; and O m, O , being produced, till they cut them, at *m* and *n*. O p, cuts the other Box, in the perpendicular *rp*, and if *n o* be drawn, to the Vaniſhing Point of n p, till it cuts the Box, at *o*; and the point *q* being obtained, by the ſame means, *op* and *pq*, being joined, compleat its Shadow.

PROBLEM III.

To project the Shadows of Right Lines, any how inclined to the Plane of projection, and to the Picture.

This Problem I ſhall reſolve alſo, by Examples, as the foregoing.

The common Center of the Rays of Light, that is, the place of the luminous Point being determined, the proceſs is the ſame as by Sun-ſhine, having found the Vaniſhing Line of the Plane of Shade; which paſſes through the Vaniſhing Point of the Line, but not through the luminous Point.

Fig. 46. ABCD repreſents a high pair of Steps, and U, X, Y, Z, a large folding Screen, ſo ſituated, in reſpect of the Light, at O, that the Shadow of the inclined Sides, (AB and CD) of the Steps, fall on the Screen.

Their ſituation, in reſpect of the Picture, and alſo their poſition to each other, together with the Seat of the Light, on the Floor, are determined as in No. 2.

No. 2. Let U, X, &c. be the Seats of the Leaves of the Screen; A, D, E, F, of the feet of the Steps, and, S the Seat of the Light; BC, is the Seat of the top of the Steps, which being contracted, AB and CD, are the Seats of the inclination of the Sides, on the Floor, which caſt their Shadows on the Screen; let their inclination to the Horizon be the Angle CDG, equal BAG.

Let V be the Vaniſhing Point of the Side, AB, of the Steps.

Having found S, the Seat of the Light, alſo F, the Vaniſhing Point of the Seat of AB, on the Floor, draw FS, indefinite; VO, being produced, cuts it at G.

Draw GA, and produce it to the Horizontal Line, cuting it at L, and draw VL, the Vaniſhing Line of the Plane of Shade, of AB.

AL cuts the Interſection, of the Leaf U with the Floor, at *a*; through H, the Vaniſhing Point of that Interſection, HI being drawn, perpendicular, is its Vaniſhing Line, which cuts VL at I; draw *a*I, cuting the Interſection of the Leaves, U and X, at b.

Then, where the Vaniſhing Line of the Leaf, X, cuts VL, at J, draw bJ, cuting the Interſection of the Leaves X and Y, at c.

The Leaf Y, is parallel to the Picture; wherefore, draw cd parallel to the Vaniſhing Line, VL, cuting the next Interſection at d; and, laſtly, draw dK, to the Point, K, where the Vaniſhing Line of the Leaf Z, cuts VL, which compleats the Shadow of AB, on the Screen.

The remainder of its Shadow falls behind the Screen, out of ſight.

The Shadow of the other Side, CD, whoſe Vaniſhing Point is R; falls only on the two Leaves, Y, and Z; and on the Wall, to g, moſtly out of ſight.

RO cuts the Floor at *G*, and *G*D determines the Vaniſhing Point, U, of the Shadow of CD, on the Floor; wherefore, RU is the Vaniſhing Line of the Plane of Shade, of CD; to which, the Shadow of that Line, on the Leaf Y is parallel; and, on the Leaf Z it tends to X in the Vaniſhing Line of the Leaf, as the Shadow of AB, on the ſame Leaf, to K; the Points where the Vaniſhing Line of the Leaf is cut, by the Vaniſhing Lines of the Plane of Shade of each Line (AB and CD) reſpectively.

The Shadow of the Supporter, CE, is projected firſt on the Floor, to f, where it cuts the Wall; and, if the Shadow of any other Point, in the Line, be projected on the Wall, as at g, then, fg is its Shadow on the Wall.

For the Steps; from O, Lines are drawn through the extremes *A*, *B*, *C*, &c. on either Side, till they cut the Floor, or leaves of the Screen, at *a*, *b*, *c*, &c. and from, *a*, *b*, &c. to the Vanifhing Point of the Steps, on the Floor, and on the Leaves, being parallel to them. Otherwife, they are drawn to that Point in the Vanifhing Line, of each Leaf, in which, the Vanifhing Line of the Plane of Shade of the Step cuts it. Or, drawing through both extremes, of each Step, as *B* and *D*, &c. cuting the Shadows of both Sides, on the fame Plane, the Vanifhing Point is unneceffary; but when it falls within bounds, it is the moft correct.

In this Cafe (viz. of Lines inclined to the Plane of Projection) the Vanifhing Line of the Plane of Shade is neceffary, when the Shadow is projected on various Planes, but it is not fo eafily determined, as for Shadows projected by the Sun; nothing more being required than to draw a Right Line through the Vanifhing Points, of the Line and of the Rays, becaufe its diftance is fuppofed infinite; and confequently, the fame Vanifhing Line of any Plane of Shade, ferves for all Lines which are parallel. But, here, the Light being at a fhort diftance, it cannot poffibly be in the Vanifhing Line, which is at an infinite diftance; except apparently fo, when it happens to be fo fituated, in refpect of the Eye.

That V L is the Vanifhing Line of the Plane of Shade of A B, is manifeft; and is determined either by means of the horizontal Vanifhing Line (H L) or the vertical Vanifhing Line (M N) of the Wall, W, on which the Shadow would be projected, the Screen being out of the way. For, A B, cuts the Wall, being prodeced, at P, as the Floor at A; and V O reprefents a Line parallel to A B; which, is a Ray of Light in the fame Plane with A B, cuting the Floor, at G, and the Wall, at Q.

Confequently, G A, produced, is the Shadow of A B, on the Floor; and, P Q produced, is its Shadow on the Wall; the former cuts the Vanifhing Line of the Floor, at L, the latter cuts the vertical Vanifhing Line at M; both which, are in the fame Right Line paffing through V; and, fince L and M are Vanifhing Points of the Shadow, V L is, confequently, the Vanifhing Line of the Plane of Shade, occafioned by the Line A B (Theorem 10.)

2. The Shadow of the Screen, on the Wall and Cieling, may be thus projected.

Through, *A*, *B*, &c. at the bottom of the Screen, draw S *A*, S *B*, &c. and produce them to the Interfection of the Wall (W) with the Floor, cuting it at 1, 2, &c. from which Points, draw Perpendiculars up the Wall; and, through the Angles *F*, *G*, &c. at the top, draw O *F*, O *G*, &c. cuting the Perpendiculars, correfponding with *AF*, &c. at *f*, *g*, &c. and draw *fg*, *gh*, &c.

But the Ray O *I*, cuts the Cieling; wherefore, having found f, the Seat of the Light on the Cieling, draw f i, to that Point where the Perpendicular, from 4, cuts the Interfection of the Wall with the Cieling; and O *I*, produced, limits the Shadow, at *i*; let it be produced, alfo, till it cuts the Perpendicular at k, and join *h*k, cuting the Interfection at *j*, and join *ji*.

This procefs would be indifpenfibly neceffary, provided the Lines, H I, &c. were not parallel to the Ceiling; but being fo, they are drawn either parallel to the Line (as for *HI*) or to the fame Vanifhing Point (as for *IK*). *A 1 f g h j i k l*, are the extremes of the Shadow of the Screen, on the Floor, Wall, and Cieling.

By the fame means, the Shadow of A B, may be projected on the Screen, thus.

Draw S *A*, S *B*, &c. cuting A F, the Seat of A B on the Floor, at n, o, p, and q; from which, draw Perpendiculars, cuting A B, at *A*, *B*, &c. through which Points, draw O *A* a, &c. cuting the correfponding angles of the Screen, at a, b, c, &c. which, being joined by Right Lines, give the fame Shadow as before.

This procefs, though fhorter, is by no means fo correct and mafterly; but, as it is performed in lefs room, it may be applied when the inclination is fuch, that the Vanifhing Points are very remote.

3. The Shadow of the Table, on which the Candle ftands may be thus determined.

Having obtained the Seat (s) of the Light on the Table, and (S) on the Floor, through s, draw a b, parallel to the Horizon, alfo c d, and e f at pleafure, cuting the Horizontal Line, at H, and, F; and through S, on the Floor, draw H *c*, and F *e*, indefinite; alfo *ab*, parallel to a b.

Then, Rays drawn, from O, through the extremes of thofe Lines, *viz.* O a, O c, &c. (being produced) will cut the correfponding Lines in the Shadow of each, refpectively, by which means as many points may be obtained as are neceffary, for obtaining the true curve of the Shadow, which is a Circle, the Table being circular.

The

Plate XLVII. The Candle standing towards one edge of the Table, the Rays proceeding from it, around its Circumference, form a scalene, or oblique Cone; which being projected to the Floor (to which the Table is supposed parallel) the Section, thereon (which is its Shadow) is consequently a Circle; and, being seen oblique, its Representation is an Ellipsis (Theo. 2. Sect. 5.)

No. 4. 4. The Shadow of the concave edge of the hollow Cylinder (X) on the interior Surface, may be projected, in the following manner.

Find *s*, the seat of the Light on the Plane of its Base, and draw *sa*, *sb*, &c. at pleasure, cuting the circumference on both sides; from the Points, d, c, &c. draw Right Lines to the Vanishing Point of the sides of the Cylinder, and draw Oa, Ob, &c. cuting them, respectively, at *a*, *b*, &c. through which, the contour of the Shadow may be described.

No. 5. 5. For the Shadow of the edge, of the conical Vessel (W) on the interior Surface.

If Right Lines be drawn from *S*, the Seat of the Light, on the Plane of its upper Base, cuting it on both sides, and making vertical Sections through them, down the Sides, within; then, draw Oa, &c. cuting the opposite, corresponding lines, at *a*, *b*, &c. through which, the Shadow may be described, on the Side.
Where they fall on the Bottom, join *cd*, &c. Ob, Oc, &c. cuts those Lines, at *f*, *g*, &c. through which, the curve of the Shadow is described, thereon.

By taking several Points (g, h, &c.) in the exterior Curve, and finding their Seats on the Floor, its Shadow (*ghi*) may be described on the Floor.

No. 6. 6. The Shadows of Globes, it is not easy to describe, with certainty.

If a Tangent be drawn, from the Light (O) to any part of its Surface (as a) and, through that Point, a Section, by a Plane, be described, perpendicular to the Axis, (OC), (which, I freely own, is not easy to do, being obliquely situated*) then, take as many Points in its Circumference as are necessary (a, b, d, &c.) and find their Seats, (*A*, *B*, *D*, &c.) on the Floor, or Cieling; and, through S, or s, the Seat of the Light, draw S*A*, S*B*, &c. and Oa, Ob, &c. cuting them, respectively, at *a*, *b*, *d*, &c. a Curve described through *a*, *b*, &c. will be an Ellipsis, the true Shadow of the Sphere; being an oblique section of the Cone of Rays.

No. 7. 7. The Shadow in the Nich is described as by Sunshine, with little variation.

Find the Seat (*f*) of the Light on its Plane, (Prob. 1.) from which, draw the Ordinates, ab, cd, &c. on which Ordinates describe Sections through the head of the Nich, perpendicular to the Plane, and draw Oa, Oc, &c. giving the Points *a*, *c*, &c. a Curve described through those Points is the true contour of the Shadow, in the Head†. Draw *ef* perpendicular, and join *fg*.

The Shadow of the edge of the Vessel, def, falls within the Nich; the rest, with the Stool, falls on the Wall, the Legs being first projected on the Floor, through the Feet, by means of S, the Seat of the Light thereon.

* The Shadow of a Globe or Sphere, it is obvious, is no more than the Shadow of a plane Circle, which is not the full diameter of the Globe, because, Tangents to a Sphere, from the same Point cannot touch both extremes of the same Diameter, seeing they would be parallel. The Circle projecting the Shadow is that, whose Circumference is the Base of the Cone of Rays.

† In this Example, the Light is so situated as to cast very little Shadow into the Nich; but, if the Candle was more oblique to it, the contour of the Shadow would be nearly the same figure, the Light being below the head of the Nich; as a tangent to the curve of the Head would always touch it on that side of the Vertex towards the Light. Whereas, by Sunshine, the Light being elevated above the Nich, a Tangent touches it on the other side, and gives a different figure of the Shadow. And, if the Candle be elevated above it, the contour of the Shadow would be nearly the same; the difference arising only from the seat of the Light, on the Plane of the Nich, which, in this is finite, in the other, at an infinite distance; and also from the Rays of Light, which in this are diverging, in the other they are converging, the Light being on this side of the Picture.

SECTION VI.

Of reflected Light; and of the reflected Images of Objects, on Water, and polished, plane Surfaces; Keeping, &c.

Plate XLVIII.

IN treating on this Subject, it may be necessary to consider, in the first place, what is meant by Reflection, simply, or considered abstractedly.

REFLECTION, in a physical sense, signifies a rebounding of Matter, by Percussion; i. e. when an elastic Body,* in motion, strikes another Body, also elastic; it rebounds, from that other Body, in a different direction from that in which it was at first impelled; making an Angle, with its first direction, greater or less, according to the obliquity in which it strikes the Surface of the other Body.

If a Globe strikes a Plane, or the Surface of another Globe (or any Surface whatever) perpendicularly, it will rebound from the other Surface, perpendicularly, in direct opposition to its first, or incident motion; as if A falls, perpendicular, to the Plane E F, striking it at B, it will rebound again, from B, towards A. Fig. 47. But, it is found, from experiment, that if the incident motion be from C to B, oblique to the Plane, or other Surface, it will rebound, or be reflected from B towards D, also oblique; making the Angle ABD, with a perpendicular, at that Point (called the Angle of Reflection) equal to ABC (called the Angle of Incidence) or, which is the same thing, the Angle CBE, in which it inclines to the Plane, in its incident motion, is equal to DBF, in which it reflects from the Plane. And this is the same, in all positions of the Plane, whether horizontal, vertical, or inclined. Hence, Light is said to be reflected, from one Surface to another; and, probably, from that consideration it is imagined to be material.

Without taking Matter into consideration, it is certain, that any Surface (not wholly opake) being opposed to a luminous Body, becomes illumined itself; and, in an inferior degree, illumines other Objects, in vicinity with it.

The first and grand instance, of which, is the Moon and other Planets; as they are more or less illumined, by the Sun, or rather, the more their illumined Surfaces are towards the Earth, the more the Earth is illumined; even to such a degree (though but reflected) as to project Shadows, strongly defined. The Case is perfectly similar in respect of other Bodies, on the Earth. For, however any Surface be situated to the Sun, being illumined by its Light, that Surface illumines others, which are near it, more or less, according as that Surface is situated to the Sun, and as they are situated in respect of each other. Without which, Bodies, or the Surfaces of Bodies, which are not illumined, directly from the Sun or other luminous Body, would be so totally immersed in Shade as not to be visible, excepting their exterior Figure or Outline.

Now, admiting Rays of Light to be emited from any luminous Body, at S, and falling, directly, on another Body, or plane Surface, at A, they are said to be reflected, directly, again, towards S; but, falling on it oblique, as at B, they are reflected towards D, making the Angle DBE equal SBA. Fig. 48.

Hence arises an objection to the Newtonian System, respecting the reflection of Light, from one Body to another, and from that other Body to the Eye; by which means, only, it is conjectured, that Objects, not illumined, directly from the Sun, or other Light, become visible.

* By Elastic, in this place, is meant, simply, hard Bodies only, as Stones, Metals, &c. of which, some are more elastic than others. Lead, or pure Gold, yields to the stroke, and therefore, does not rebound like Tin, or Tin like Copper, nor that like Iron, or hard Steel. So Clay will not rebound like Free Stone, nor that like Marble; because, after Percussion, the more remote parts of Matter are still in motion, till the Body is compressed by the Stroke; which deprives it, either wholly or in part, of Motion; and consequently, it rebounds with less force, or not at all.

Plate XLVIII. Fig. 48.

Let AB be a Plane Surface, directly oppofed to the Sun; and, fuppofe AB the utmoft limits of the Plane. Let X be an Object, having one plane Face (CD) parallel to AB; and fuppofe no other Object, or Body, near.

Now, the diftance of the Sun being fuppofed infinite, its Rays, confequently, fall perpendicularly on AB, in every part.

Let SA, *S*B, &c. reprefent Rays of Light from the Sun. Then, according to the Maxim laid down, they are reflected, directly back again, towards S (which, being material, is fomewhat repugnant to our Ideas of Matter; feeing that, it is continually flowing, with equal and unremited velocity, from the Sun to AB, how can it return back, in direct oppofition?) confequently, the Object X being fituated fo, that none of the reflected Rays can poffibly fall on the Surface CD, that Surface is wholly invifible, to any Eye at E, or *E*. *Query*; whether it would be vifible or not?

I am of opinion that it would be clearly vifible, to any Eye, on this Side CD; not only in refpect of its Figure, but that, the Surface, CD, would be illumined, by means of Reflection, from AB; when, according to the general Maxim, it could not, feeing no Rays, from AB, are reflected to CD. Yet, I am fully perfuaded that it may be feen; not only, directly, at E, but alfo oblique, in any direction, as at *E*; although it is manifeft, that the Rays, from AB, muft be reflected oblique, to CD, and again to E, or *E*, in various Angles and Directions; and yet, the original Rays all fall perpendicularly on AB.

Now, if CD be feen, at all, it is manifeft it muft be illumined; and it is evident, that it cannot be fo from the Sun, directly, confequently, it muft be from Reflection; and fince no other Body is near, it muft be from the Surface AB. Hence, then, it is manifeft, that, Light is reflected in other directions, from one Object to another. How or by what means it is reflected, I will not attempt to enquire; but fhall only make a few Obfervations, refpecting the effects it produces, on Objects, fo effential to the perfecting a Picture, or true Portrait of Nature.

It had, formerly, been cuftomary, with many, to reprefent Objects, immerfed in Shade, fo very obfcurely, as fcarce to be diftinguifhable; whereas, it is not fo in Nature. In clear Day-light, when the Sun does not fhine out, we fee Objects in their true Colours, and every part is diftinct; but, when the Sun breaks out, and darts its Rays on thofe parts which are oppofite, the Colours are more intenfely vivid, occafioned by the refulgent luftre of the Sun-Beams. Neverthelefs, thofe parts which are prevented from receiving that additional Light, and appear to be in Shade, cannot poffibly be deprived of what Light they had before, but muft rather receive additional Light on them; the difference, then, can only arife, from the fplendor of the furrounding Light, which dazzles the Eye, and renders thofe parts obfcure, which, before, were diftinctly feen. But, when out of the full glow of Sun-fhine, we perceive every Object or parts of Objects, which are in Shade, as diftinctly as before; and in the fame Colours, though greatly different from thofe on which the Sun fhines.

Some, again, of late Years, run into the oppofite extreme, and make too little diftinction, between the fulleft Light and the ftrongeft Shade; by which means, their Pieces look flat, and do not produce, on the Mind, a juft Idea of what is intended. For, although it is certain, that, in Sunfhine, every part receives additional Light; yet, the contraft is fo ftrong, from the brilliancy of the Colours, where the Rays fall, that, the other Parts appear as if deprived of Light, in comparifon with thefe; and, fince that vivid luftre cannot be given by Art, and Colour, the difference muft be made by keeping the other parts fomewhat under.

Refpecting the degree of reflected Light on Objects, it is not eafy to determine; that being more or lefs, according to the fituation of the Object, in refpect of others; alfo, according as that other Object is fituated to the Light.

If one Object be fo fituated to another, that the Surface of one, being directly oppofed to the Light, is alfo much oppofed to the fhaded fide of the other Object; the

the reflected Light, on the other, will be ſtronger than when they are more obliquely ſituated; either the one, in reſpect of the Light, or the other, in reſpect of the illumined Surface.

For example. The Cylinder, AB, being ſituated near the Wall X, on which the Light is direct; the ſhaded Edge (AB) has a much ſtronger reflection, than if the Plane (X) was more inclined to the Light; or, being removed, and no other reflecting Surface near, there would be no Reflection on the Column, in this poſition of the Light, ſave what is received obliquely, from the Ground. Fig. 49.

Reſpecting plane Surfaces. We frequently ſee (in Prints and Drawings) the ſhaded face of a Pedeſtal, &c. very dark, at the hither edge, and gradated towards the other, ſo, as to give it the appearance of concavity; which cannot be, when no other Object is near. But, when one part of an Object projects from the other, as the Plane X from Z, the Plane Z, being much oppoſed to a ſtrong Light, and conſequently the Shadow of the Plane X, on Z, is very narrow; then, the Light, reflected from Z to X, will be ſtrongeſt where it joins to Z, and gradually leſſened, towards the other edge; but not in ſo extravagant a degree, as may be ſeen in various Prints; particularly in Mr. Kirby's Perſpective of Architecture. Fig. 50.

Now, the reaſon for this is extremely obvious; becauſe of the vicinity of the part which joins to Z, the Light reflected on it from Z is ſtronger than on thoſe parts which are more remote; and conſequently, the reflected Light is more languid on the remote parts. But, if the Light was very oblique on Z, ſo as to caſt a great breadth of Shadow, on the Plane; in that Caſe, the Light, reflected to the Plane X, will be more faint; inſomuch that, there is barely a diſtinction between the Shadow, and the Object occaſioning that Shadow; which there always ſhould be, in ſome degree, the Shadow being darkeſt; becauſe, the Shadow cannot receive any advantage from Reflection, which the Object does.

In reſpect of Mouldings; the edge of the Facia, AB, being in full Light, caſts its Shadow on the Cavetto below; conſequently, the Mouldings cannot have their effect from the direct Light, but from Reflection, only. On the returning Moulding; the under Facia or Planceer (BC) by means of Reflection, from below, is brighter than the vertical Facia, over it; alſo the vertical Face of each Fillet, is darker than the horizontal; which in other caſes, having the Light directly on them, is conſequently brighter.

In Figure 4, Plate 40, the great projecture of the Corona (X) deprives the Mouldings, below, of Light, entirely, from above, which are diſtinguiſhable only by means of Reflection, from below; conſequently, the Ovolo is brighteſt towards the lower edge, and the Cavetto, towards the upper; and, they are more or leſs ſo, according as they are oppoſed to ſome illumined Body, reflecting Light on them. It may, therefore, be made a general Rule, with very little exception, that, the effect of reflected Light on Objects is reverſe to the direct Light; particular regard being had to the ſituation of the Object which occaſions the reflection. For, certainly, in whatever degree an Object is illumined, or from whatever cauſe, whether directly or reflected, the effect of it, on the Object, is the ſame, in proportion to the degree of Light. But when (as is frequent) various Objects reflect Light on the ſame Object, no one being particularly predominant; the Light by that means is ſcattered and confuſed, ſo that, there is little or no Shade on the Object; inſomuch that, the parts can ſcarce be diſtinguiſhed, one from another. The ſame effect is produced from various luminous Bodies, variouſly ſituated to an Object, or from various inlets of Light, on Objects, each deſtroying the effect occaſioned by the other.

In a Nich (ſee Plate 43.) the Shadow is ſtrongeſt at the edge or outline, and gradually ſoftened towards that ſide of the Nich, which occaſions the Shadow; owing to the ſtrong Reflection from the other Side, on which the Light falls, direct.

Plate XLVIII. The Shade (without Sun-ſhine) on a convex or concave ſegment of a Cylinder, or Sphere, is the ſame; each may be the other, by ſuppoſing the Light on either Hand.

More might be ſaid in reſpect of reflected Light; but, if the Obſervations I have made be well attended to, it will be found ſufficient, for any purpoſe whatever.

Of the reflected Images of Objects on Water, and poliſhed Surfaces.

The Reflections of Objects on the Surface of ſtill Water gives (where Water is repreſented) tranſparency to the Water, and perfection to the Picture; which, if well managed, has a pleaſing effect.

Reflected Images of Objects on Mirrours, except when ſeen direct, is a real Anamorphoſis, or Diſtortion. On a horizontal Mirrour, the Object ſtanding upright, the Image appears the ſame, inverted; yet, the Picture is really on the Surface of the Water; for, an Object on the Water, or being cut by its Surface, always appears double, to an Eye at any diſtance from it, horizontally. For inſtance, the Eye, being at E, ſees the Image of the Object A B inverted, apparently in the Water; but it is manifeſt, that 'tis not ſo, in reality, for, the Repreſentation is on the ſurface of the Water; the part A C being repreſented by A c, which is drag'd out, to a prepoſterous length, whilſt b c repreſents the upper part, the Cone B C, which is much leſs than the Object; neverthelefs, I ſhall prove, that the Image is repreſented equal to the Object; but, being ſeen under unequal Angles, they do not appear exactly equal.

It is obvious, whether by means of Rays of Light, reflected to and again, from

Fig. 51. the Object to the Water, and from thence to the Eye, or otherwiſe, that, if A D be the Surface of Water (or a poliſhed Mirrour) and A B, an Object, ſtanding erect, we ſhall ſee the Image of the Point B, on the Water, at b (the Eye being at E) where Right Lines, drawn from B and E to the Water, at b, make equal Angles with its Surface. Conſequently, if Eb be produced; meeting B A, produced, at, *B*, A*B*, and, A*B*, equal will be to AB;* is the apparent place of the Object, which appears on the Surface of the Water. Hence it is manifeſt, that the Image of the Object A B, repreſented on the Surface of the Water (and which, in real length, is Ab) is repreſented equal to the Object itſelf. For, the Point A is common to both, and becauſe B, appears at *B*, it appears juſt as much within the Water, as its real place is from the Water, *viz.* A*B* equal AB, and A*C* equal AC, &c.

Hence, we have an unerring Rule, for repreſenting the reflected Images of Objects on poliſhed Surfaces. But it muſt be obſerved, that, if the Object be at ſome diſtance, beyond the Water, the meaſure of the Object muſt be applied from its Seat, on the Plane of the ſurface of the Water, not from the Water edge, as when it is in the Water or cloſe to it. Alſo, that the Image repreſented on the Water, is not ſimilar to the Repreſentation of the Object, on the Picture, which I ſhall ſhew; except when a plane Figure only is repreſented, parallel to the Picture.

E X A M P L E S.

Fig. 52. Let X be an Object in the Water, having one Face (X) parallel to the Picture.

No. 1. Produce the Sides, BA and CD, making A*B* and *C*D reſpectively equal to them; alſo, make the perpendicular *E*F equal to EF, and join *BE* and *CE*; i. e. make the Figure A*E*D ſimilar to AED, inverted, from the Line AD where the Surface of the Water cuts the Object. The Image of the Door, &c. is alſo ſimilar.

* For, the Angle BbA=EbF (the Anngle of incidence, to the Angle of reflection) and, Ab*B*=EbF (2. 1. El.) wherefore BbA=Ab*B* (Ax. 3. El.) then, becauſe the Angles at A are right (AB being ſuppoſed perpendicular to the Surface of the Water) and Ab is common to the two Triangles, ABb and A*B*b, conſequently, A*B* is equal to A*B*; for the Triangles are congruous (11. 1. El.)

For

For the Face AG, being perpendicular to the Picture, the horizontal Line, BG, vanifhes in the Center; wherefore, BG being parallel to the Water, its Image will be parallel to the Line, and confequently, it has the fame Vanifhing Point.

Draw *B*C, meeting GH, produced at *G*, which compleats the Image of that Face, and which, it is obvious, is not fimilar to the Reprefentation ABGH; feeing that, the Angles which are acute, in one, are obtufe in the other. Likewife, a fide of the Roof is reprefented in the Object, which, it is manifeft cannot be feen in the Water, by Reflection.

The Piles, at U, which are perpendicular, have their Images alfo equal, each to the Reprefentation; likewife, the Pile W is reflected equal, and equally inclined, being parallel to the Picture. But, being otherwife inclined, as Y and Z, one hanging towards, the other from the Picture, the Seats of their tops, on the furface of the Water, being determined, at a, produce the Perpendicular ba, making a*b* equal to ab, and draw from *b*, to the Timbers, where they are cut by the Surface. No. 2.

The Vanifhing Point of the reflected Image is at the fame diftance on the other fide of the horizontal Line, as that of the Object, whether it be above or below it.

The Image of the Bridge, on the Water, may be obtained after the fame manner, particularly the Piers. The reflected Images of all horizontal Lines tend to the fame Point, in the horizontal Vanifhing Line, as the Reprefentations; for, Right Lines, being parallel to the Surface of the Water, their Images are either parallel to them, or they have the fame Vanifhing Points, refpectively. No. 3.

The horizontal Line, HI, of the Bridge, vanifhes at O; wherefore, PQ, drawn perpendicular through O, is the Vanifhing Line of the Plane of the face of the Bridge; IK vanifhes at P, and the inclined Line at the hither end at Q, equally diftant from O, being equally inclined. Wherefore, k*I* being made equal to Ik, the horizontal Line has the fame Vanifhing Point (O) as its original, and the inclined Line *IK*, tends to Q, reprefenting a parallel to GH, at the hither end.

For the Arches; draw as many perpendicular Lines, ab, cd, &c. as are neceffary, and produce them in the Water; making each equal to its correfponding one, *a*b equal ab, &c. by which means, as many Points, in the Curve, as you pleafe, may be obtained, and the image of the Arch defcribed through them; each being a Semi-Ellipfis, of which, ik is a Diameter.

The Crane, &c. on the Wharf, hanging over the Water, has its Image reflected, by means of perpendicular Lines from each part, and finding their Seats on the furface of the Water (as *m*, of M) repeating the fame meafure downwards. No. 4.

For the Hogfhead; draw as many Lines (reprefenting lines parallel to the Horizon) as are neceffary, tending to the Vanifhing Point V of the horizontal Diameter ab; then, by means of perpendiculars from each extreme of thofe Lines, their Images are acquired, and the curve of the Head defcribed through them. If other Lines are drawn, from one head to the other, to the Vanifhing Point X, of its Axe, the whole Image may be defcribed, as in the Figure.

The Warehoufes, &c. ftanding at fome diftance from the Water, have their reflected Images defcribed as the other Objects; by fuppofing the furface of the Water produced, and the Seat of each Angle determined thereon; as of A, B, C, the Angles of the Warehoufe W; which are made equal, downward, from their Seats (*a*, *b*, *c*,) refpectively, and the horizontal Lines have the fame Vanifhing Points. No. 5.

The Spire of the Church, at T, is reflected on the Water; becaufe, its Seat (S) on the level of the Water, being obtained, the height (ST) being applied downward reaches the Water, otherwife it would not. No. 6.

Some Perfons have expreffed furprize, that, Objects, at a great diftance, fhould have their Images reflected on Water which is near. The moft diftant Objects, which can be feen, will be reflected; provided, that the Eye be properly fituated, near the Surface, and there be no other Objects intervening, i. e. when Right Lines

Plate XLVIII. from the Object and from the Eye, making equal Angles with the furface of the Water, meet on the Water; otherwife, the Objects cannot be reflected. It feems indeed rather furprizing, that a Surface, very much inclined to the Horizon, and nearly approaching to perpendicular with the Picture, as a gentle Affent can be reflected; but, the Eye being fo fituated, as above, and the furface of the Water being on a level with the foot of the hill, its inclined Surface will be reflected thereon.

It has been objected, that the Reflections are too ftrong, in the Plate; I grant they are, as a Picture; but my defign is to fhew the figure of the Image on the Water, rather than the effect, confidering the Water not fimply as Water, but as a Mirrour. Neverthelefs, in clear Water, perfectly at reft, the Images of Objects on its Surface will be found as diftinct as the Objects themfelves appear, according to their Diftance; fomewhat differing in Colour, rather darker, in the cleareft Water.

Thus much may fuffice, for Reflection on a horizontal Surface; which is the fame, whether it be a clear Fluid (as Water) or other plane Mirrour.

Secondly. Of Reflection on Mirrours, vertical and inclined.

Fig. 53. Let W be a Mirrour, parallel to the Picture, whofe Center is S; and X, an Object, whofe reflected Image is required.

In this Cafe, it muft be obferved (as in the former) that the Image, of any Point in an Object, is reflected to the fame diftance from its Seat on the Mirrour, as the original Point from its Seat; fo, that diftance is, here, reprefented, and its Seat obtained, perfpectively (in the former it is geometrical) and, becaufe the reflecting Surface is parallel to the Picture, the Center is, confequently, the Vanifhing Point of Lines producing its Seat, being perpendicular thereto.

Draw Lines, from the Feet, A, D, &c. of the Chair to S, the Center; and, where they cut the Interfection of the Mirrour with the Floor, at a, d, &c. make a*a* reprefent a length equal Aa, &c. (Prob. 8.) and draw *a*b and *d*c perpendicular; then, BS and CS, cuting thofe Perpendiculars, refpectively, give their reflected Images. For, as every Object appears to be as far on the other fide, as it really is on this fide, from the Mirrour; confequently, in this Cafe, it muft be fo perfpectively.

The reflected Images of horizontal Lines, as the rails of the Chair, &c. (in this Cafe) tend to a Vanifhing Point in the Horizontal Line, at the fame diftance from the Center of the Picture, on the contrary fide, as the Vanifhing Point of the Reprefentation. If V be the Vanifhing Point of BC, the front Rail; then, in SG, a diftance being taken equal to SV, is the Vanifhing Point of bc, the reflected Image of BC. I is the Vanifhing Point of the fide Rail CE; SK being made equal SI, K is the Vanifhing Point of its reflected Image. After the fame manner, the middle Point of the top Rail may be obtained, by a Perpendicular, ef, to the Floor, and as many other Points as are neceffary.

On a vertical Mirrour, inclined to the Picture, there is no difference in the procefs, but only in the Vanifhing Point of Perpendiculars to the Mirrour; and, if the Vanifhing Point of Perpendiculars be determined, 'tis the fame in all inclinations.

Fig. 54. Ler Z be a Mirrour inclined to the Horizon, and cafually inclined to the Picture. It is required to find the reflected Image of the Chair, on the Mirrour Z.

V being the Vanifhing Point of the bottom of the Mirrour, which is horizontal, find the Vanifhing Line (VL) of the Mirrour, its inclination to the Horizon being known (Prob. 5. Sect. 3.) and alfo, the Vanifhing Point (F) of Lines perpendicular to the Mirrour (Prob. 2. Sect. 12.)

Then, having found the Interfection (*CD*) of the Mirrour with the Floor, produced (its Vanifhing Point is V) and G, the Vanifhing Point of Perpendiculars thereto, draw AG, DG, &c. cuting *CD*, at *A*, *D*, &c. and through them, draw *A*b and *D*c, tending to the Vanifhing Point of the fides of the Mirrour*, indefinite.

* This Vanifhing Point is the Interfection of the Vanifhing Line (VL) of the Mirrour, and the vertical Vanifhing Line RG, produced.

Draw

Draw AF, DF, &c. cuting them, refpectively, in the correfponding Points, a, d, &c. (their Seats on the Mirrour*) beyond which, their Images are reprefented equal, in Perfpective; by making a *a*, reprefent an equal meafure as A a, &c. (Prob. 8.)

Find the reprefentation of the Interfection of the plane of the Front, with the Mirrour (which will pafs through V) and the Interfecting Points of AB and DC, therein, by producing them; from which Points, draw Lines through *a* and *d*, indefinite, and BF, CF, &c. cuting them in their refpective Images on the Mirrour.

Then, having found the Interfection *EB*, of the Plane of the fide of the Chair, with the Mirrour, and the interfecting Points, *E* and *B*, of *G*A and *F*B, draw *Ea* and *Bb* indefinite; and, from *F* and *G*, draw lines to F, the Vanifhing Point of Lines perpendicular to the Mirrour, cuting them at *f* and *g*.

Thus, as many Points (as e, at the top) may be determined as occafion requires, from their Seats on the Floor; by which means, the reflected Image of any Object, whatever, may be reprefented on the Mirrour.

BC is parallel to the Mirrour, wherefore, the Image (*bc*) of BC, has the fame Vanifhing Point, in the Horizontal Line; otherwife, it would not; for, although all horizontal Lines, whatever, vanifh in the Horizon of the Picture; in this Cafe, the Mirrour is a Picture, and the reprefentation of the Chair is confidered as the original Object, in refpect of that Picture, each Plane having its peculiar Vanifhing Line thereon.

PQ is the Interfection of the Plane of the Mirrour with the Cieling; by means of which, the Cage, hanging from the Cieling, at N, is reflected. If its Seat, on the Cieling be determined, the Image of each Angle on the Mirrour is reprefented, indefinite, and their reprefentative lengths are determined, by drawing lines from each extreme to F, the Vanifhing Point of perpendiculars to the Mirrour; as no is the Seat of NO, on the Mirrour, and *no* is its indefinite Image. After the fame manner, the corners of the Cage may be reprefented on the Mirrour.

Of Keeping; and the effect of Distance.

The Term, Keeping, in the Art of Painting, in general, is ufed to fignify a juft and proper fubordination of all the parts of a Picture to the principal Object; in refpect of Magnitude, Colour, and diftinctnefs of Parts.

The magnitudes of Objects, in refpect of each other, perfpectively, are various, according to the Station from which they are viewed; and confequently, the fubordination of Colour, &c. is not in proportion to the Objects, but to their Diftance from the Eye. It is abfolutely impoffible to lie down Rules, by which the Artift may, with certainty, produce the defired effect; feeing that, in hazy or foggy weather, or in a mifty morning, &c. Objects are lefs diftinct, at a fhort Diftance, than, in a clear Day, at a much greater; wherefore, no proportion or degree can be determined. If Objects, of known magnitude, appear not to be far diftant, in the Picture, and yet, their parts not diftinctly defined, in comparifon with others, in the Fore-ground; it implies, that the Air is more grofs and hazy, than if the parts were more perfect.

Keeping is, in a great meafure, fynonimous with Aerial Perfpective; which fignifies a diminution of Light, Colour, and diftinctnefs of the parts of Objects, in a regular gradation, as linear Perfpective of Magnitude; owing to the effect of Air, between the Eye and the Object; which, being a Medium, obftructs the fight, in fome degree, at any Diftance; confequently, as there muft be a greater quantity of Air between the Eye and diftant Objects than near ones, fo their parts are lefs difcernable; the Lights and Shades are infenfibly mixed, and, at a great Diftance, it is all a confufed jumble, of Light, Shade, and Colour, without diftinction.

It is cuftomary with many, in delineating, and may frequently be feen in Architectural Defigns, &c. to make a confiderable difference, in Teint, between one plane Surface and another; when the one, in the Original Object, recedes but a

* *A*b and *D*c are the Seats of AB and CD on the Plane of the Mirrour, produced.

few

Plate XLVIII. few Inches, which is abſurd; for, if that ſudden gradation of effect was continued to the diſtance of a few Yards, it would become totally black, before it was poſſible for the Air to have any apparent effect on it. Now it is far from being ſo in Nature, which is obvious to any Eye; for, if the materials are clean, and of an even and uniform Colour (without which no judgment can be made) I will venture to affirm, that two Plane Surfaces (V and Y) parallel between themſelves, at ſeveral Feet diſtance from each other, and having a full Light on them, cannot be diſtinguiſhed one from the other, nor ſo much as the Line or Edge (A D) ſeen, at a proper diſtance for delineating; provided the Light be not (on this ſide) ſo very oblique (as at S, No. 3.) as to ſhade the other, beyond where the Line OC cuts it. I ſay, the Eye being ſo ſituated (at O) that the Angle C, appears to cut the receding Plane, at B, they will appear as one continued Surface. Neverthelefs, it may be neceſſary, in delineating, to make a diſtinction, in proportion to the Diſtance; but, if the Plane V has Mouldings, &c. on it, being cut, apparently, by the edge of the other, the Line is ſufficiently defined without it.

Fig. 50.

No. 2.

As it muſt be obvious, that no poſitive Rule can be preſcribed, I ſhall juſt make a few obſervations, and conclude the Subject; and, with it, the Book.

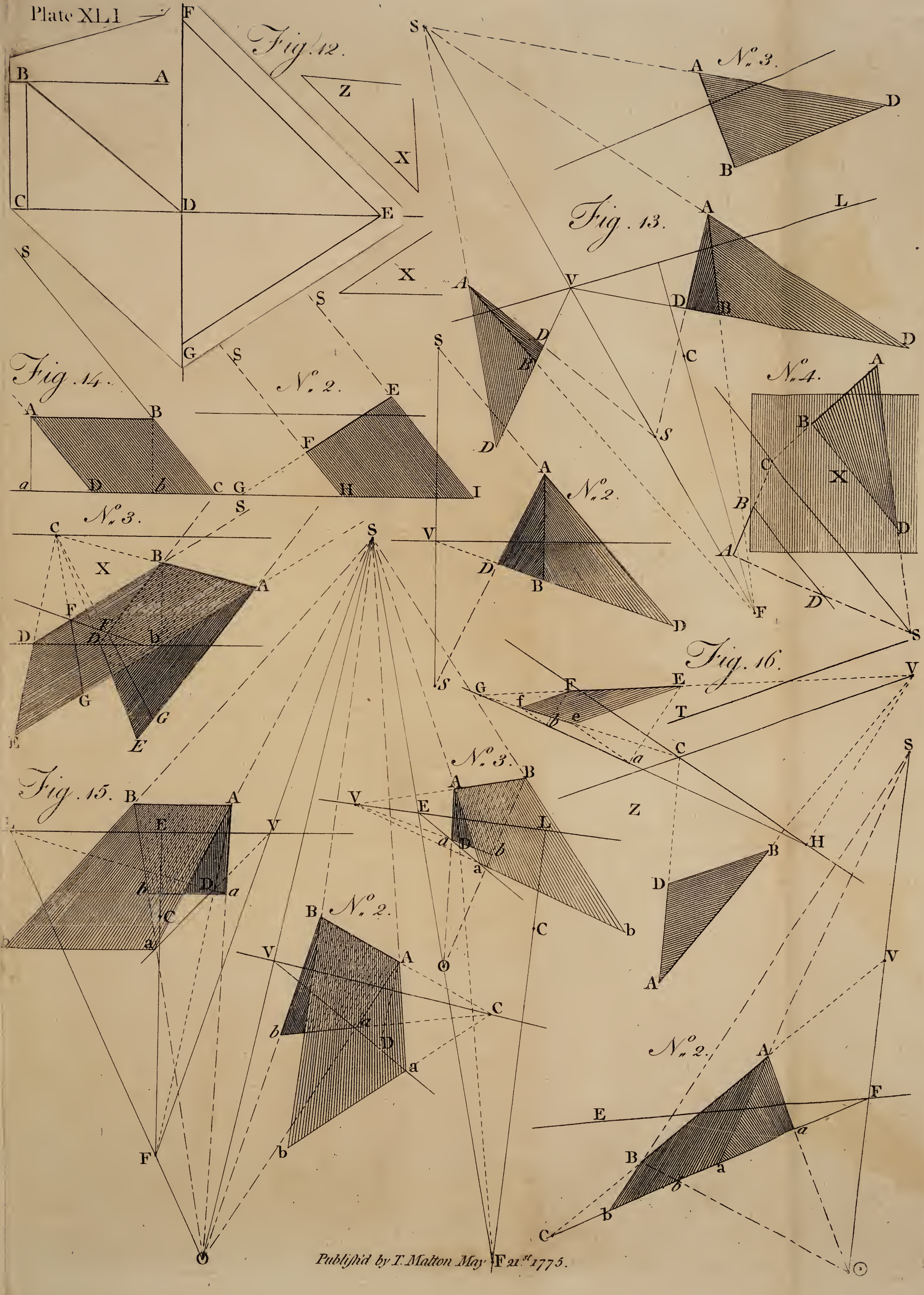
In reſpect of Magnitude; it is evident, that Objects mry appear, in Perſpective, to have any proportion to each other, although not greatly diſtant. For Example; in Plate 27, of Chelſea College, the fartheſt Building, with a Pediment, is not above two eleventh parts of the hither one (to which it is equal) in height; and yet it does not offend the Eye, nor appear at too great a Diſtance. But, if the gradation of Light was in proportion, it would be too great; becauſe, not merely the height is to be conſidered, but the ſquare of the height, which would reduce it to the proportion of thirty to one; but where we ſhall fix the ſtandard for unity, I am at a loſs to deviſe.

Now, if we were at twice the Diſtance from the hither Building (the diſtance between the two Objects being ſomewhat more than twice that) the proportion of one to the other, would not be much greater than one to three; the gradation of Light, in that caſe, would be nearly as one to ten, but one third part of the former. Yet, I preſume, that in two Pictures ſo delineated, it would not be adviſable to make that difference in the effect of Aerial Perſpective.

In ſhort, as it is abſolutely impoſſible to fix any criterion to determine the Ratio of the gradation of Light and the effect of Diſtance, but muſt ever be at the diſcretion of the Artiſt, I ſhall only obſerve, that the Objects in the Fore-ground, as the hither end of the Bridge (Plate 48) being ſuppoſed near the Picture, muſt have its parts diſtinct and perfect, with ſtrong Lights and Shades, which gradate to the other End. The Buildings on the Wharf, &c. ſhould be leſs perfect, as they recede from the Eye. The Church, at a Diſtance, the Trees and Hills, one beyond another, muſt conſequently be leſs and leſs diſtinct, according to their Diſtance; till at laſt they are ſcarce diſtinguiſhable from the Sky. For all which, the Artiſt can have no other Rule than, carefully and judiciouſly, to copy Nature (the beſt Miſtreſs) which, in that Caſe, is not always uniform, but infinitely variable.

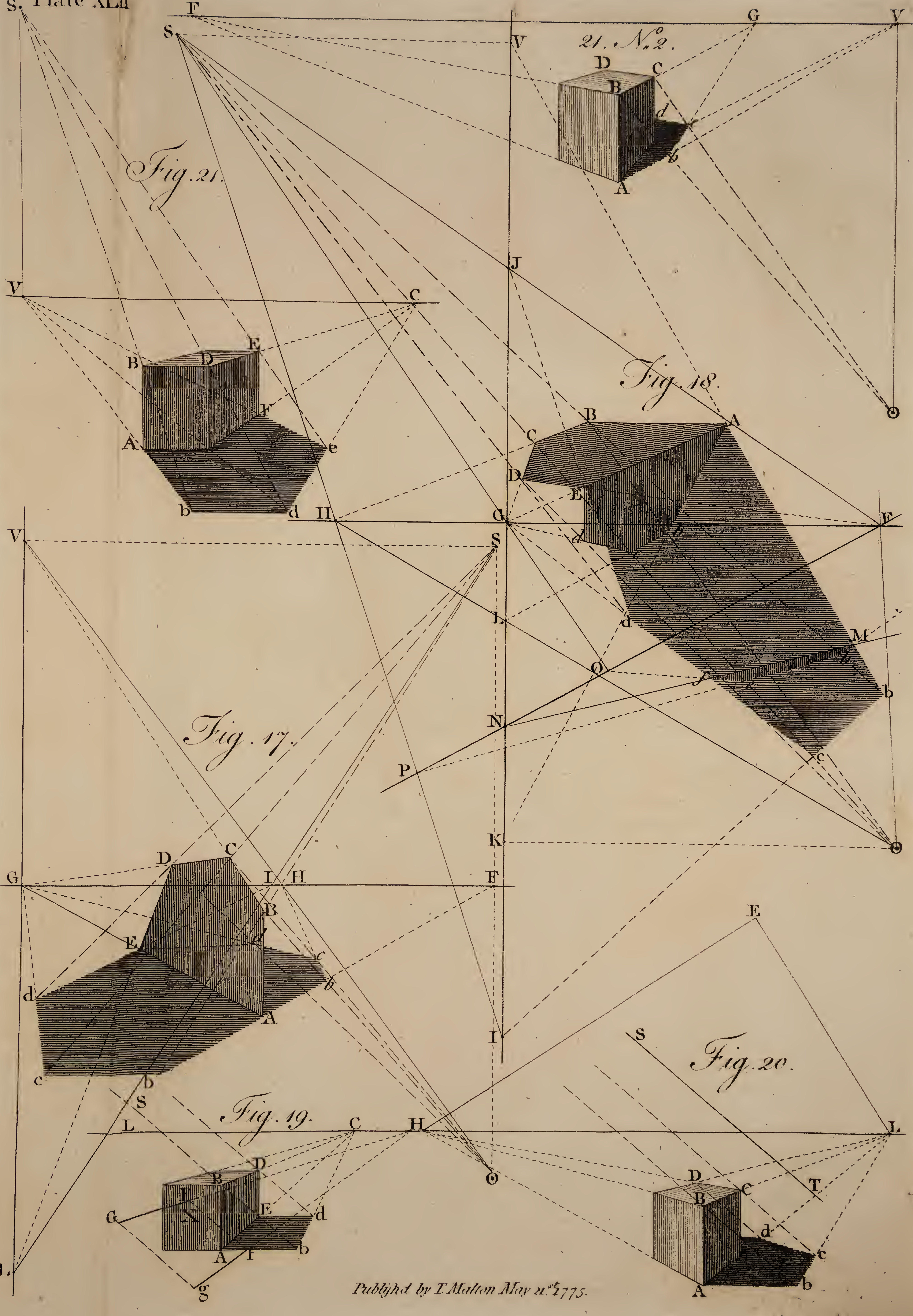
F I N I S.

Plate XLI

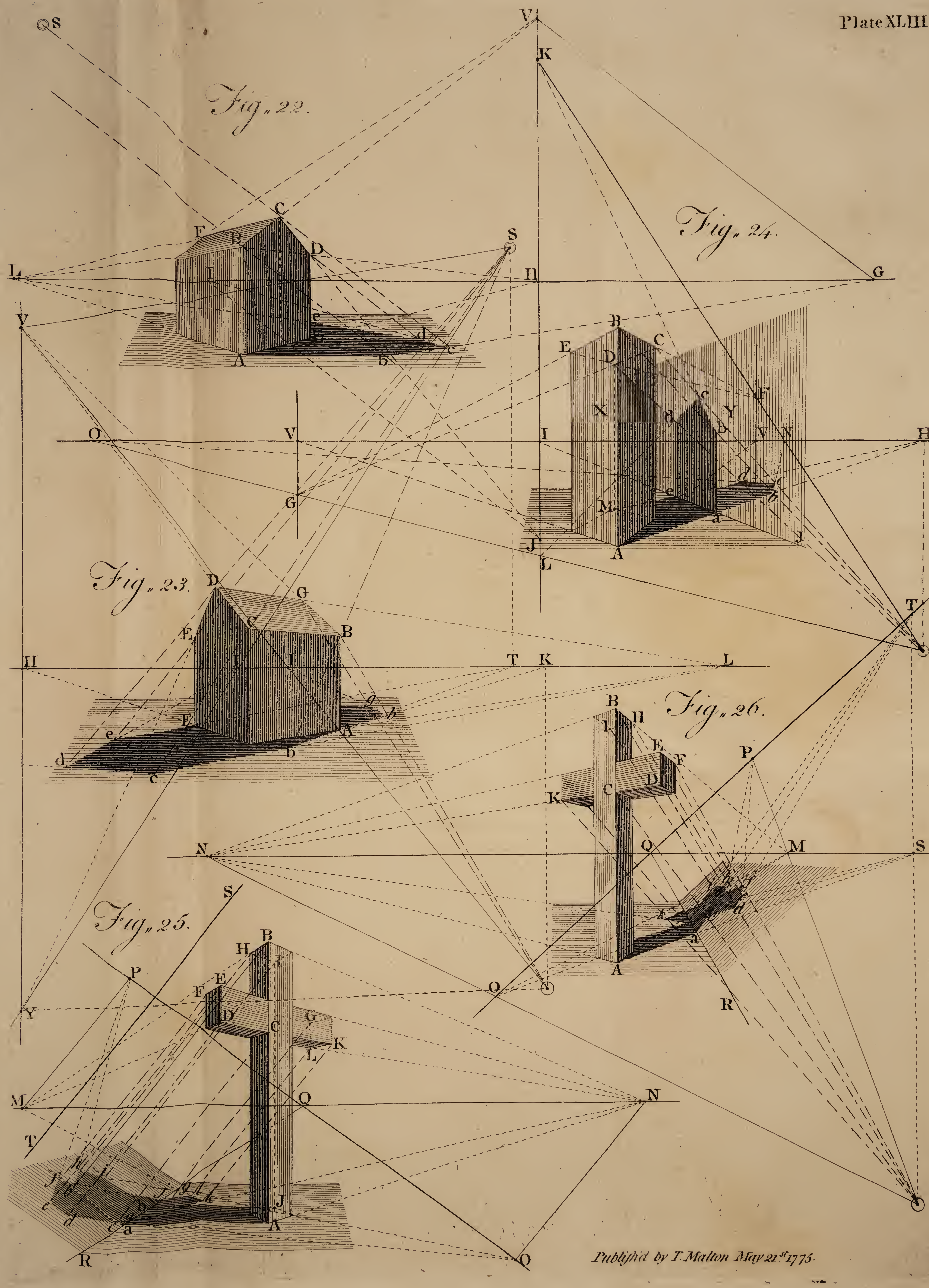


Publish'd by T. Malton May 21st 1775.

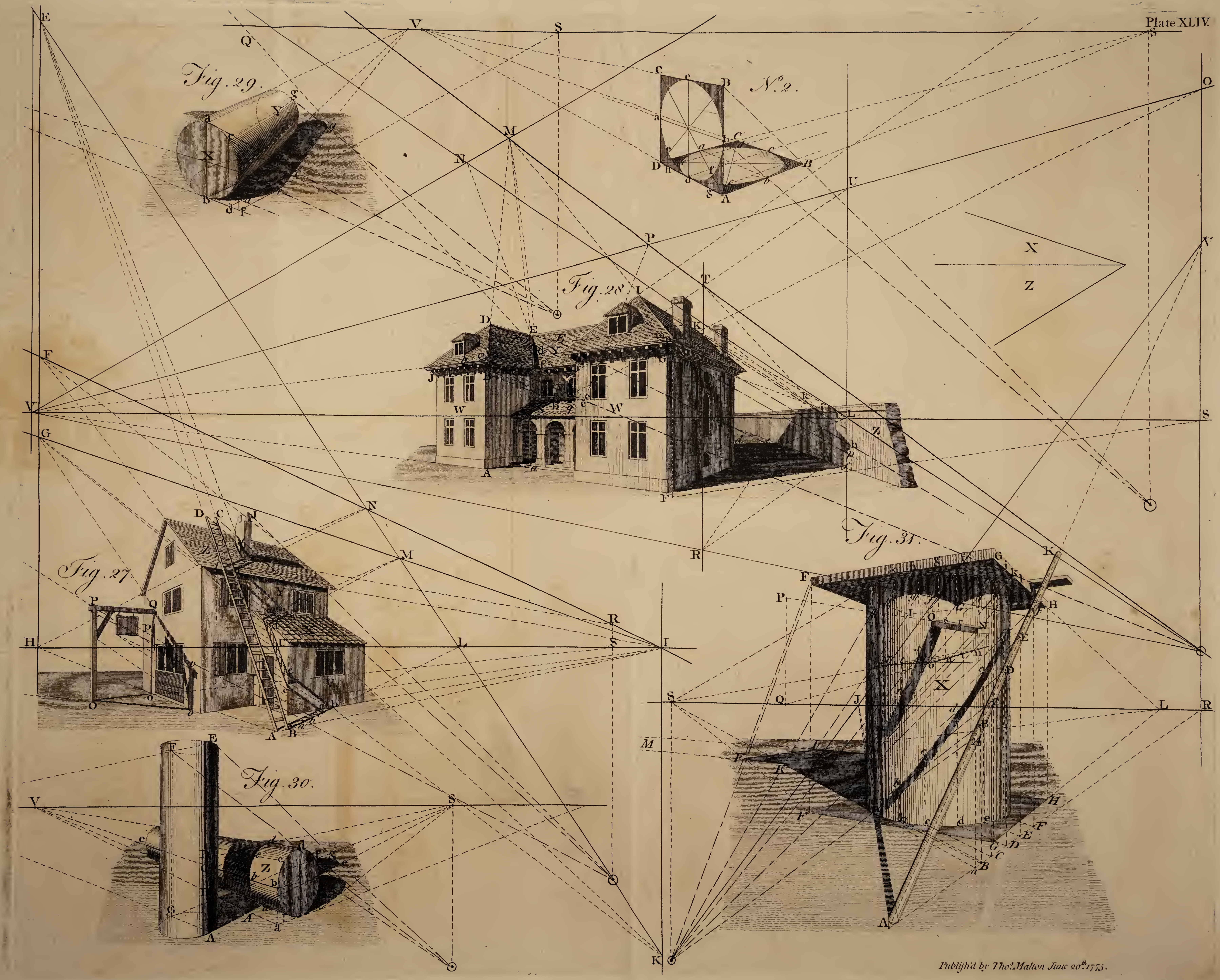
Plate XLII



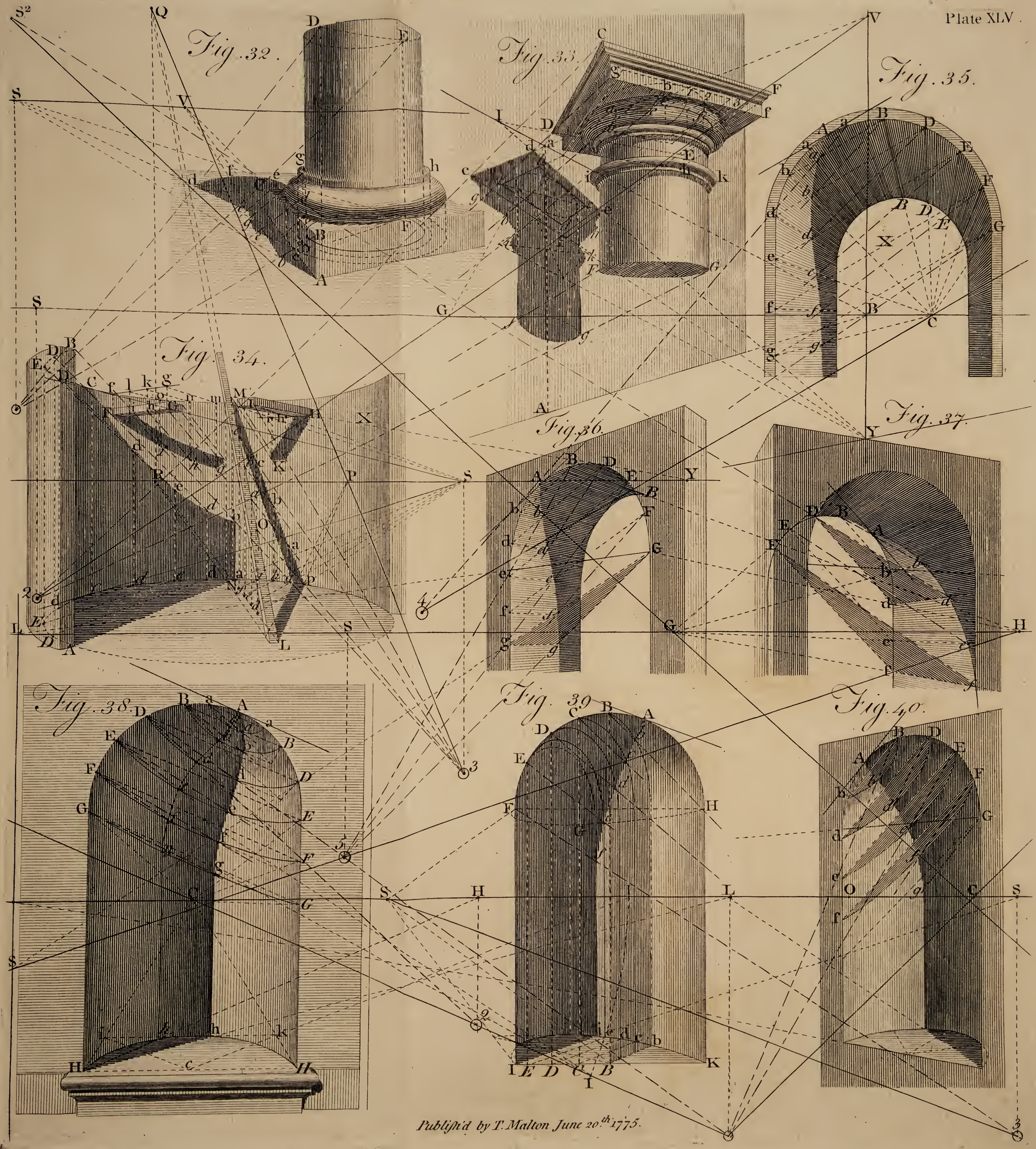
Publish'd by T. Malton May 21.st 1775.



Publish'd by T. Malton May 21st 1775.



Publish'd by Thos. Malton June 20th 1775.



Publish'd by T. Malton June 20th 1775.

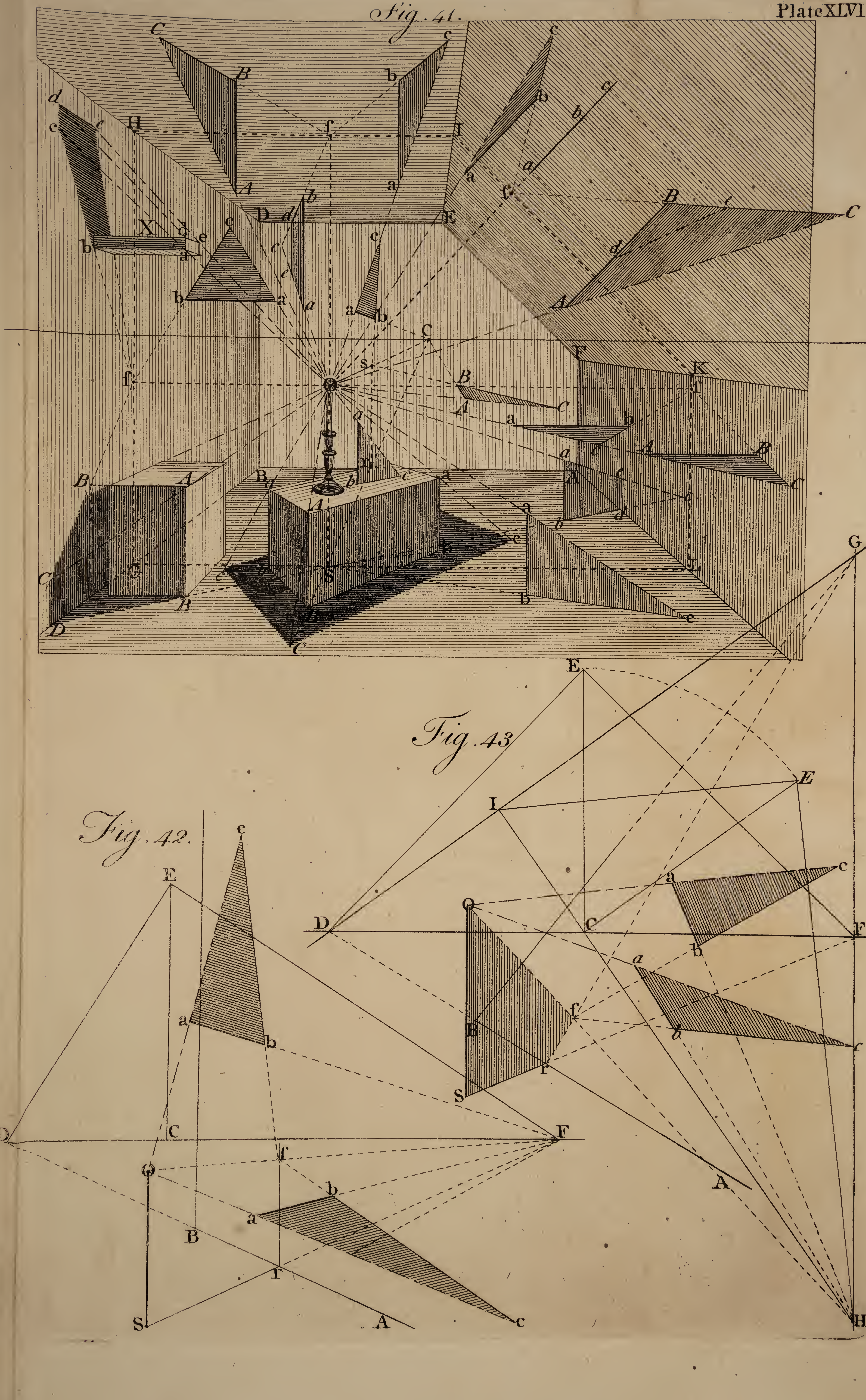
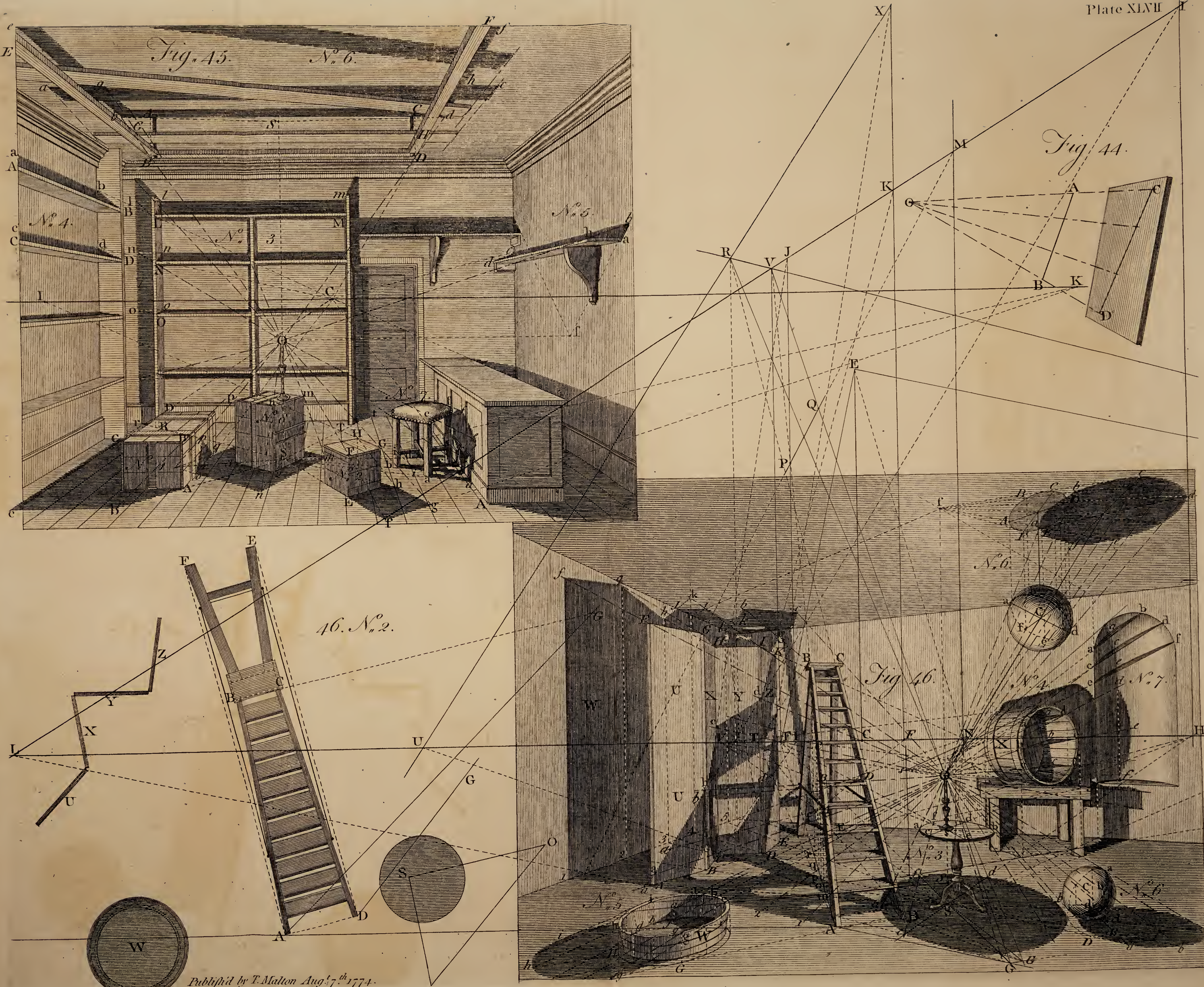


Fig. 41.
Fig. 42.
Fig. 43.



Publish'd by T. Malton Aug.st 7.th 1774.

Plate XLVIII

Miller sculp.

Publish'd by Thos. Malton Septr. 1st. 1775.